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Students Who Took the Class Help Studenrs Who Are Taking It: What Is the Best Arrangement?

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STUDENTS WHO TOOK THE CLASS HELP STUDENRS WHO ARE TAKING IT: WHAT IS THE BEST ARRANGEMENT?

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Abstract

The more help students get, the better. It is therefore reasonable to ask students who took the class to help students who are currently taking this class. This arrangement also help the helpers: it is known that the best way to learn the material is to teach it. An important question is: how to pair the students to get the maximal effect? In this paper, we show that, under reasonable conditions, the best effect is when we match the best performing "older" student with the worst performing "younger" one, the second best with the second worst, etc.

Keywords: *students helping students, optimal arrangement, optimal student performance*

MAIN IDEA AND THE FORMULATION OF THE PROBLEM

Students need help. Studying at a university if not easy – as judged, e.g., by the fact that many initially promising students do not succeed. It is therefore necessary to provide as much help as possible to the students.

Helping students is one of the main obligations of instructors and of their Teaching Assistants. However, the available time of instructors and Teaching Assistants is limited. So, especially for big classes, students need additional help.

Students helping students. Who can provide such help? Naturally, only people who already know this material can meaningfully help. In addition to instructors and Teaching Assistants, there is another group of folks who know this material – and who can thus help: students who have already taken this class. So, a natural idea is to have these students help students who are currently taking this class – i.e., help students who are currently struggling with this class's material.

This benefits helpers too. Such help clearly benefits students who are currently taking this class, but why should other students help them? Some universities can afford to pay student helpers, but such ability to pay is rare.

Good news is that such arrangement helps helpers as well. Every instructor knows that the best way to learn the material is to teach it: by repeating each part of the material many times, by answering numerous questions, the instructors gets a better and better mastery of the material. Similarly, in the process of helping other students, student helpers get a better mastery of the material.

Because helping is beneficial to both groups of students, at some universities – e.g., at the University of California at Barkeley – it is, in effect, institutionalized: students are supposed to spend some time during their next class on helping students who are currently taking the prevous class.

A natural question. A natural question is: what is the best way to pair the students? Who should help whom?

This is the question that we will answer in this paper.

MATHEMATICAL MODEL OF THE SITUATION

To answer the above question, let us reformulate this question in precise terms.

Numerical description of helpers. The better was the helping student's performance in the class, the more this student can help students who are taking this class right now. Let us denote the helping ability of the i-th helping student by $h(i)$.

As a measure of such an ability, we can take, e.g., the student's grade in the previous class – or, better yet, the 0-to-100 number of points that led to this specific grade. For simplicity, we can normalize this numerical characteristic – by dividing it by its largest possible value. This way, the corresponding numerical characteristic takes values from 0 to 1.

Numerical description of helpees. Students who currently take this class also have different success rates. Let us denote the success level of the i-th helpee student by $s(i)$.

Again, as a numerical measure of this success, we can take the student's grade-so-far, or the student's average grade in the previous classes. This grade can also be normalized, so that it takes values from 0 to 1. The smaller the value $s(i)$, the larger the student's need for help. We can estimate this need for help $n(i)$ as the difference between the ideal value 1 and the current value $s(i)$: $n(i) = 1 – s(i)$.

How effective is help? The effectiveness of help cannot exceed the ability of the helping student: if a student is not very strong in this material, this student is not of much help. Similarly, if a helpee already knows everything – or at least his knowledge gap is small, then the help cannot exceed this gap. On the other hand, if the helpee's gap is exactly equal to the knowledge that the helper has, and both the helper and the helpee work hard, they get the maximum effect: all gaps are covered.

So, in general, the efficiency of a helper-helpee arrangement, when the helper i is paired with the helpee p(i), is proportional to the smallest of the two values, i.e., to:

min(h(i), n(p(i))).

We want to maximize the overall effectiveness, i.e., the sum of the effectivenesses of all the helper-helpee pairs.

Thus, we arrive at the following precise formulation of the problem.

Resulting formulation of the mathematical problem. We have s students potentially needing help, and we have s students who can potentially deliver this help.

- We know how much help h(i) each helper can provide.
- We also know how much help n(i) each helpee needs.
- We need to fnd the permutation

  p: {1, …, s} -> {1, …, s}

for which the following sum is the largest possible:

min(h(1), n(p(1))) + … + min(h(s), n(p(s))).

ANALYSIS OF THE PROBLEM AND THE RESULTING SOLUTION

Resulting solution. Let us prove that the largest possible value of the above objective function corresponds to the case when:

- the best performing helper is matched to the worst performing helpee,
- the second best helper is matched to the second worst helpee, etc.

In other words, if we sort helpers in the increasing order of their helping ability h(i), and we sort helpees in the increasing order of their helping need n(i), then:

- the student with the smallest possible value h(1) of the helping ability is matched with the student with the smallest possible value n(1) of the helping need,
- the student with the second smallest value h(2) of the helping ability is matched with the student with the second smallest possible value n(2) of the

helping need,

- …
- and finally, the student with the largest possible value h(s) of the helping ability is matched with the student with the largest possible value n(s) of the helping need.

How to prove that this arrangement is optimal. In the optimal arrangement, if h(i) < h(j), then we should have n(p(i)) <= n(p(j)). To prove that such an arrangement is optimal, it is sufficient to prove that:

- if for two pairs (i. p(i)) and (j, p(j)), we have an opposite relation, i.e., we have h(i) < h(j) and n(p(i)) > n(p(j)),
- then, by swapping p(i) and p(j), we will increase (or at least not decrease) the value of the objective function.

So, whatever arrangement we start with, by applying this swap again and again, we will arrive at the arrangement in which the orders of h(i) and n(p(i)) are the same – and the value of the objective function will only increase (or remain the same). Thus, the largest possible value of the objective function is indeed attained when the orders coincide.

We want to prove that when h(i) < h(j) and n(p(i)) > n(p(j)), then the swap – after which the i-th helpee is marched with helper p(j) and the j-th helpee is matched with the helper p(i) -- always increases (or leaves the same) the corresponding part of the objective function, i.e., that we always have v <= V, where we denoted

v = min(h(i), n(p(i))) + min(h(j), n(p(j))) and V = min(h(i), n(p(j))) + min(h(j), n(p(i))).

To prove this inequality, let us consider all possible locations of the values

n(p(j)) < n(p(i))

with respect to h(i) and h(j).

Case 1. First, let us consider the cases when n(p(j)) <= h(i) < h(j). In this case, the value n(p(i)) can be: (1) either efore h(i), (2) or between h(i) and h(j), (3) or after h(j). Let us consider these three subcases one by one.

Subcase 1.1. In this subcase, n(p(j)) < n(p(i)) <= h(i) < h(j). Here,
v = n(p(i)) + n(p(j)) and V = n(p(i)) + n(p(j)), so v = V: the decided inequality is satisfied.

Subcase 1.2. In this subcase, n(p(j)) <= h(i) < n(p(i)) <= h(j). Here,
v = h(i) + n(p(j)) and V = n(p(j)) + n(p(i)). Since h(i) < n(p(i)), we have v < V.

Subcase 1.3. In this subcase, n(p(j)) <= h(i) < h(j) <= n(p(i)). Here,

$v = h(i) + n(p(j))$ and $V = n(p(j)) + h(j)$. Since $h(i) < h(j)$, we also have $v < V$.

Case 2. Let us now consider the case when $h(i) <= n(p(j)) < h(j)$. In this case, the value $n(p(i))$ can be: (1) either between $h(i)$ and $h(j)$, (2) or after $h(j)$. Let us consider these two subcases one by one.

Subcase 2.1. In this subcase, $h(i) <= n(p(j)) < n(p(i)) <= h(j)$. Here,
$v = h(i) + n(p(j))$ and $V = h(i) + n(p(i))$. Since $n(p(j)) < n(p(i))$, we have $v < V$.

Subcase 2.2. In this subcase, $h(i) <= n(p(j)) < h(j) <= n(p(i))$. Here,
$v = h(i) + n(p(j))$ and $V = h(i) + h(j)$. Since $n(p(j)) < h(j)$, we have $v < V$.

Case 3. Finally, we need to consider the case when $h(j) <= n(p(j))$. In this case, there is only one possible arrangement: $h(i) < h(j) <= n(p(j)) < n(p(i))$. Here,
$v = h(i) + h(j)$ and $V = h(i) + h(j)$. So, $v = V$.

Summarizing. In all possible cases, we have $v <= V$. Thus, indeed, in all the cases, swapping either increases the value of the objective function or leave it intact.

Thus, we have proved that the above arrangement -- in which the orders between h(i) and n(i) coincide -- is indeed optimal. In other words, we arrive at the following conclusion.

CONCLUSION

The optimal pairing between helpers and helpees is as follows:

- We sort all potential helpers in the increasing order of their ability to help, i.e., in the increasing order of the grade that they got when they took this class:
  $h(1) <= h(2) <= … <= h(s)$.
- Then, we sort all potential helpees in the increasing order of their need for help, i.e., equivalently, in the decreasing order of the grade-so-far:
  $n(1) <= n(2) <= … <= n(s)$.
- Then, we match the 1st helper with the 1st helpee, the 2nd helper with the 2nd helpee, …, and the s-th helper with the s-th helpee.

WHAT IF WE USE A MORE ACCURATE MODEL OF HELPING EFFICIENCY

More accurate models are possible. In the above arguments, we used a simplified model of the helping efficiency, according to which the efficiency of each helping-helper pair is equal to min(h(i), n(p(i)).

This seems like a good first approximation model.

Natural question. However, a natural question is: what if we use a more accurate model? Will the recommendation change?

One possible alternative model is to estimate the helping efficiency not as the minimum, but as the product of the values h(i) and n(p(i)), i.e., as h(i) * n(p(i))?

We can use even more sophisticated models, in which the efficiency of a helper-helpee pair is described as f(h(i), n(p(i))) for some function f(a, b).

Analysis of the problem. For which functions f(a, b) is the above argument applicable? It is applicable if for all a < A and b < B, we have

f(a, b) + f(A, B) >= f(a, B) + f(A, b).

This inequality can be reformulated in the equivalent form

f(a, B) – f(a, b) <= f(A, B) – f(A, b).

If we take B = b + h, divide both sides by h, and take a limit h -> 0, then we get an inequality between the values of the partial derivative with respect to b:

df/db(a, b) <= df/db(A, b).

Vice versa, if this inequality between the values of the partial derivative holds for all a, A, and b, then by integrating over b, we can get the desired inequality between the differences.

The inequality between the values of the partial derivative simply means that the partial derivative is an increasing function of the first argument a. According to calculus, this is equivalent to the condition that the derivative of this expression with respect to a is always non-negative, i.e., that

$d^2f/da\ db >= 0$ for all a and b.

One can easily check that this inequality holds for the original function f(a, b) = min(a, b), for the product function f(a, b) = a * b (for which the above double derivative is always equal to 1), and for many other models. Thus, we can arrive at the following conclusion.

Conclusion. The optimality of the above arrangement holds not only for the simplest model min(h(i), n(p(i))) of helping efficiency, it holds for any model f(h(i), n(p(i)))) for which $d^2f/da\ db >= 0$ – in particular, for the product model f(a, b) = a * b.

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