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So How Were the Tents of Israel Placed? A Bible-Inspired Geometric Problem

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Comments:

So How Were the Tents of Israel Placed?
A Bible-Inspired Geometric Problem

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Abstract

In one of the Biblical stories, prophet Balaam blesses the tents of Israel for being good. But what can be so good about the tents? A traditional Rabbinical interpretation is that the placement of the tents provided full privacy: from each entrance, one could not see what is happening at any other entrance. This motivates a natural geometric question: how exactly were these tents placed? In this paper, we provide an answer to this question.

1 Formulation of the Problem

Tents of Israel: a Biblical story. In one of the Biblical stories, prophet Balaam, when asked to curse the Jewish people, instead blesses them: "How good are your tents, o Israel" (Numbers 24:6).

What attracted us in this story. This is an entertaining story, with a talking donkey, but this is not what attracted our interest: what we were attracted to was a Rabbinical interpretation of what Balaam meant by goodness of tents. According to the Talmud [1], what was good about the tents was that they guaranteed privacy: from each entrance one could not see what was happening at any other entrance.

A natural question. A natural question is: so how exactly were the tents of Israel placed?

2   Towards Formulating the Problem in Precise Terms

Main assumption. For simplicity, let us assume that the tents were close to each other, with practically no gaps — a natural idea in the old days, when you need to get together in a small space to protect the perimeter against possible enemies.

The simplest is when all the tents are square, when they all have the same size, and each of them has only one entrance – this way, it is easier to maintain privacy.

Resulting formulation. We assume that the whole plane is filled – without gaps – with squares of the same size. Each square has a point on its boundary that corresponds to the entrance. How can we place all these squares so that from each entrance one cannot see any other entrance?

3   Analysis of the Problem

Tents form rows and/or columns. Since there are no gaps, each piece of the tent's wall is also very close to the wall of another tent. Since the space between the tents is very small, we can safely ignore it, and consider the walls to be touching.
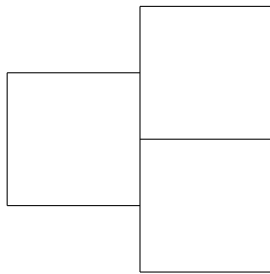
If each of the four walls is the whole wall of another tent, then the tents form a grid:
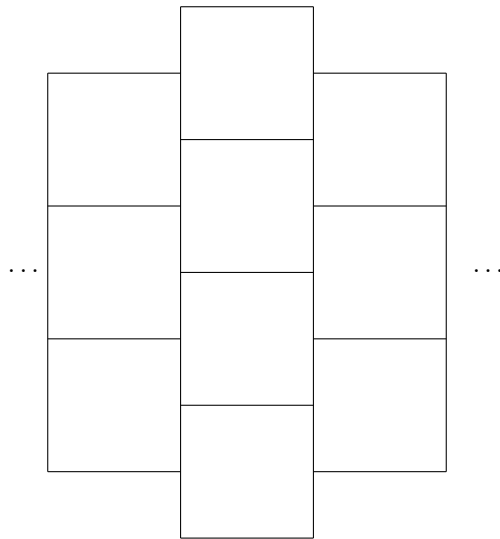
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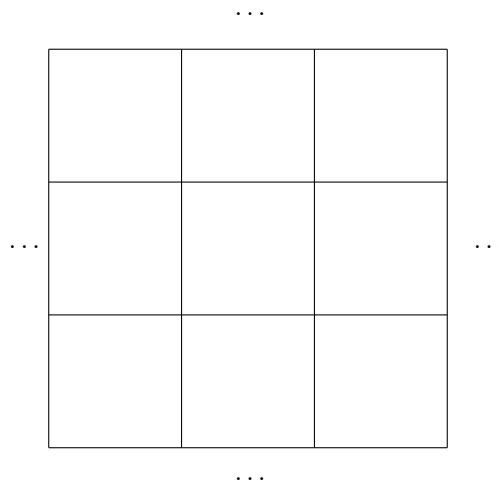
What if only a part of a wall is the wall of another tents, i.e., we have a configuration like this:
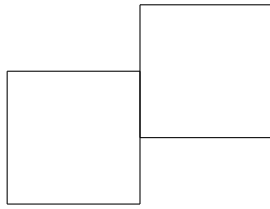
In this case, the only way to place a tent in the bottom right corner is to have it in such a way that the new tent and the original right tents – and hence, other tents – form a vertical column:


In general, we conclude that tents form rows or columns:

Without losing generality, it is dufficient to consider the case when the tents form columns.

Where can entrances be? If two tents from the same column have entrances opening to the right, then from these entrances, they can see each other – and

we want to only consider the case when from no two entrances, one can see each other:
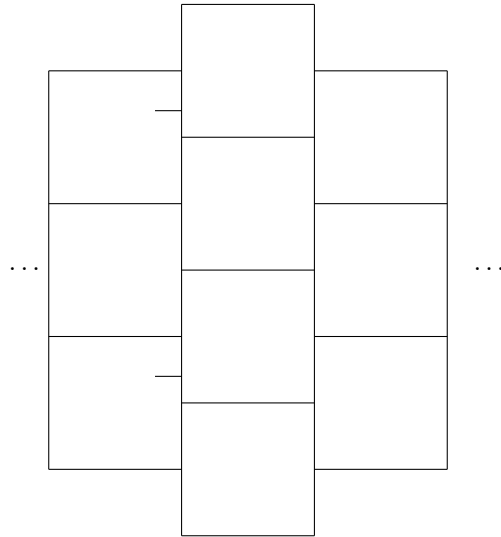

Thus, for all the tents from a column, no more than one entrance can be on the right. Similarly, no more than one entrance can be on the left. For all other tents from this column, the entrances must be either up or down.

If for one tent, the entrance is up, then for the next tent in this column, the entrance cannot be down: otherwise they will see each other:
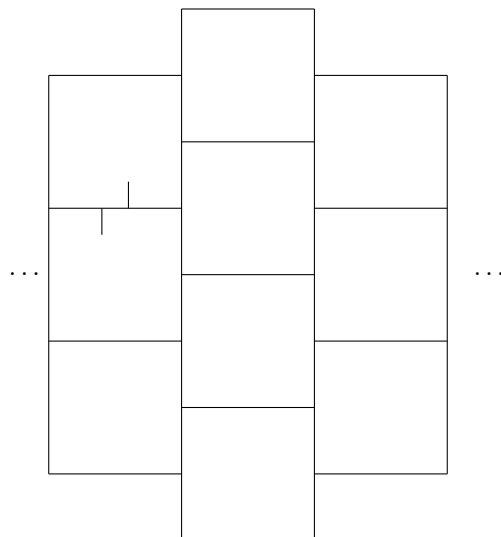
Thus, if one tent from a column has entrance up, all others – until we reach the exceptional tent which has entrance to the left or to the right – should also have entrances up. Similarly, if one tent from a column has entrance down, all others – until we reach the exceptional tent – should also be down.
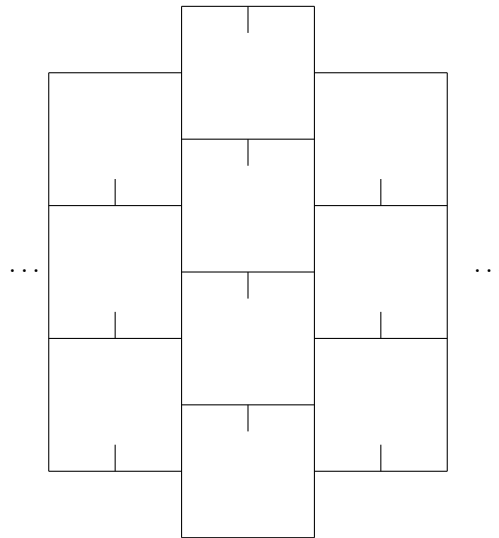
Comment. Of course, if one of the tents in a column has an entrance to the left or to the right, then it is possible that the vertical direction changes with this tent: e.g., all lower tents have entrances up, and all upper tents have entrances down:
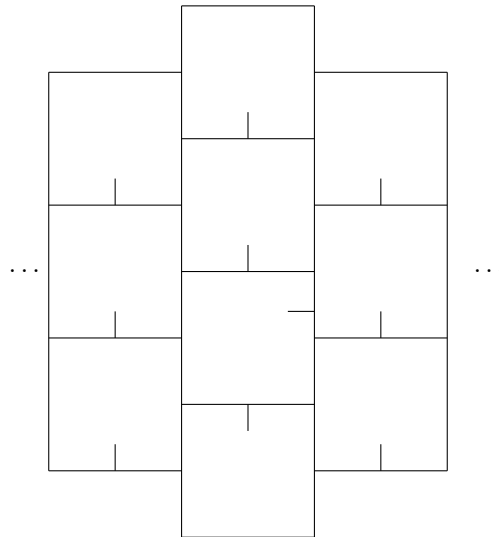
Neighboring columns must be shifted against each other. Because of the privacy requirement, we cannot have a grid: if we have entrances up in one column and up or down on the neighboring column, entrances from neighboring columns will see each other, and this is what we want to prevent:
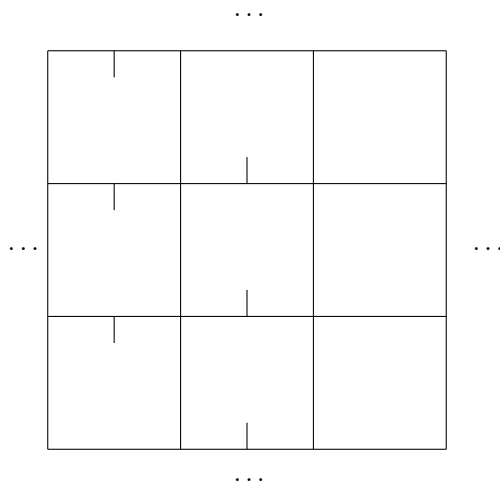
Thus, the neighboring columns must be shifted against each other.

Now, we are ready. Now, we are ready to describe all possible privacy-preserving tent placements.
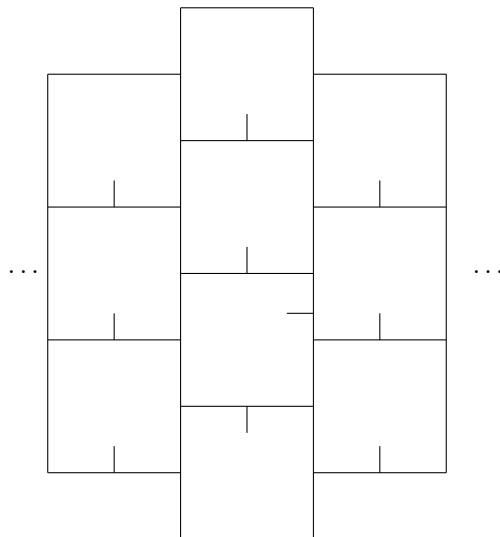
4   Resulting Solution

Resulting description of all privacy-preserving tent configurations: general rules.

- Tents form either columns or rows.

- Neighboring columns (rows) are shifted against each other.

- In each column (row), at most two tents can have entrances oriented toward a neighboring column (row). For all other tents, entrances should be toward the neighboring tent in the same column (row).

- With the possible interruption by tents oriented towards a neighboring column (row), all the tents in a column (row) should have entrances on the same side.

Examples. A typical tent placement should look like this:
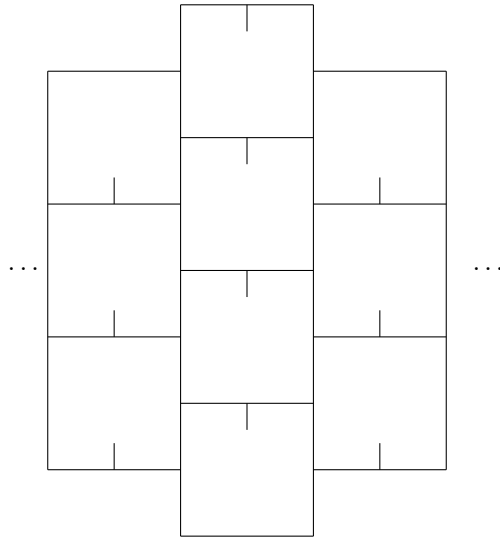
However, if one of the tents in a column (row) has an entrance towards the neighboring column (row), then it is possible that the direction of the entrances changes with this tent. Here is an example:

Acknowledgments

References

[1] Bava Batra 60a, https://www.sefaria.org/Bava_Batra.60a?lang=bi