12-2020

Why Physical Processes Are Smooth Or Almost Smooth: A Possible Physical Explanation Based on Intuitive Ideas Behind Energy Conservation

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Comments:
Technical Report: UTEP-CS-20-124

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Abstract

While there are some non-smooth (and even discontinuous) processes in nature, most processes are smooth or almost smooth. This smoothness help estimate physical quantities, but a natural question is: why are physical processes smooth or almost smooth? Are there any fundamental reasons for this ubiquitous smoothness? In this paper, we provide a possible physical explanation for emirical smoothness: namely, we show that smoothness naturally follows from intuitive ideas behind energy conservation.

1 Formulation of the Problem

Smoothness: empirical fact. Most physical processes are smooth. There are cases of non-smooth transitions – phase transitions, shock waves, explosions, etc. – but most processes are smooth; see, e.g., [1, 3].

On the macrolevel, we can have non-smooth (e.g., fractal) shapes – starting with the famous example of the shape of the shoreline of Britain [2] – but on the micro-level, on the level of fundamental physical equations, most everything is smooth.

Smoothness helps. A large part of analysis of physical systems is based on smoothness – to be more precise, on the fact that in each small-size region, all dependencies can be accurately approximated by linear ones; see, e.g., [1].

But why almost everything is smooth? What is the fundamental reason for this empirical smoothness?

A seemingly natural explanation does not work. At first glance, the answer to this question seems straightforward: physics is described in terms of differential equations, and differential equations imply smoothness. However, this "explanation" is not fully convincing, for the following two reasons:

- first, most physical differential equations can be equivalently reformulated in integral form;
- second, even if we consider differential equations, nothing wrong with infinite values of some quantities; for example, for Newton's law $m \cdot \ddot{x} = F$, why not have a force which is infinite at some point?

For example, many formulas of probability theory are described in terms of the probability density function $\rho(x)$ – the derivative of the cumulative distribution function $F(x)$ – but it does not mean that all empirical cumulative distribution functions are smooth.

What we do in this paper. In this paper, we provide a possible fundamental physical explanation for empirical smoothness – via (informally understood) energy conservation law.

2 Our Explanation

Simplest case of finitely many non-relativistic particles: a brief description of the case. Let us consider the simplest case of a system consisting of finitely many non-relativistic particles of masses $m_1, \ldots, m_n$ located at points $x_1, \ldots, x_n$. For this system, the overall energy E is equal to the sum of its potential energy $V(x_1, \ldots, x_n)$ and the overall kinetic energy of all the particles:

$$E = V(x_1, \ldots, x_n) + \sum_{i=1}^{n} \left(\frac{1}{2} \cdot m_i \cdot (\dot{x}_i)^2 \right).$$

Physical meaning of energy conservation law. An intuitive physical description of energy conservation law means that perpetuum mobile is impossible, i.e., that we cannot extract infinite energy from any given system. No matter how much energy we extract, at some point, we will exhaust it.

In other words, for each system, there is a limit L to the amount of work that can be extracted from this system. In particular, this means that no matter what dynamics happens within this system – without any outside pumping of energy – the overall kinetic energy

$$E_{\rm kin} = \sum_{i=1}^{n} \left(\frac{1}{2} \cdot m_i \cdot (\dot{x}_i)^2 \right) \quad (1)$$

is limited by this value L.

This intuitive energy conservation law implies smoothness. Indeed, if the sum (1) of several non-linear terms cannot exceed the value L, this implies that each of these terms cannot exceed L. Thus, we have $\frac{1}{2}\cdot m_i\cdot(\dot x_i)^2\le L$ and thus, $(\dot x_i)^2\le\frac{2L}{m_i}$, hence

$$|\dot x_i|\le\sqrt{\frac{2L}{m_i}}.$$

In other words, the derivative of the trajectory $x_i(t)$ of each particle i is bounded by a finite constant – this exactly means that each trajectory is smooth.

What happens if we take relativistic effects into account. If we take into account the effect of special relativity theory, then smoothness is even easier to explain: it follows from the fact that, according to this theory, all velocities are bounded by the speed of light c: $|\dot x_i|\le c$.

General case of field theories. In this case, the overall energy E is equal to the integral of energy density $\rho_e(x)$:

$$E=\int\rho_e(x)\,dx,$$

energy density is proportional – at least in the first approximation – to the square of the corresponding field (e.g., electric field), $\rho_e\sim(\vec E)^2$, and the field $\vec E$ is proportional to the derivatives of the corresponding potential φ:

$$\vec E\sim\frac{\partial\varphi}{\partial x}.$$

Thus, while we can have this derivative infinite at some points, the integral E of this derivative's square is finite. This means that in almost all locations, the value of the derivative $\frac{\partial\varphi}{\partial x}$ is finite – and the larger bound B we take for this derivative, the smaller the volume V of the area where this derivative can exceed B.

Indeed, the integral of the values exceeding B^2 on area of volume V cannot be smaller that $V\cdot B^2$. Since the overall integral of non-negative energy density is equal to E, we thus conclude that $V\cdot B^2\le E$, hence

$$V\le\frac{E}{B^2}.$$

When the bound B increases, the volume of the area where the derivative exceeds B tends to 0. In this sense, the derivative is almost always finite and, thus, the corresponding potential function $\varphi(x)$ is almost everywhere smooth.

Acknowledgments

This work was supported in part by the National Science Foundation grants 1623190 (A Model of Change for Preparing a New Generation for Professional Practice in Computer Science), and HRD-1834620 and HRD-2034030 (CAHSI Includes).

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