

11-2020

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Technical Report: UTEP-CS-20-119

Published in *Applied Mathematical Sciences*, 2020, Vol. 14, No. 19, pp. 919-924.

Recommended Citation

Kosheleva, Olga and Kreinovich, Vladik, "Yet Another Possible Explanation of Egyptian Fractions: Motivated by Fairness" (2020). *Departmental Technical Reports (CS)*. 1518.

https://scholarworks.utep.edu/cs_techrep/1518

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Yet Another Possible Explanation of Egyptian Fractions: Motivated by Fairness

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Abstract

Ancient Egyptians represented fractions as sums of inverses of natural numbers, and they made sure that all these natural numbers are different. The representation as a sum of inverses makes some sense: it is known to lead to an optimal solution to the problem of dividing bread between workers, a problem often described in the Egyptian papyri. However, this does not explain why the corresponding natural numbers should be all different: some representations with the same natural number repeated several times lead to the same smallest number of cuts as the representations that the ancient Egyptians actually used. In this paper, we provide yet another possible explanation of Egyptian fractions – based on fairness; this idea explains also why all the natural numbers should be different.

1 Mystery of Egyptian Fractions: Reminder

How we usually represent fractions. We are accustomed to representing fractions as ratios $\frac{p}{q}$:

- whether in the explicit form like $\frac{2}{3}$,
- or in the implicit form like 0.25 which means $\frac{25}{100}$.

The fact that we are actively using this representation seems to indicate that this representation is convenient for computations.

So, not surprisingly, this is the way fractions have been represented in many cultures.

Egyptian exception. Interestingly, ancient Egypt was a major exception. In ancient Egypt, fractions were represented as sums of inverse of different natural numbers; see, e.g., [1, 2, 3, 4, 12] and references therein. For example, $\frac{2}{3}$ was represented as

$$\frac{2}{3} = \frac{1}{2} + \frac{1}{6}.$$

It does not seem to be very convenient to operate with fractions presented this way.

The use of this unusual representation is even more surprising if we take into account that ancient Egypt left behind a lot of computations – probably more than any other ancient civilization. So why were fractions represented in this strange way?

Existing (partial) explanation. A possible explanation for using Egyptian fractions comes from the fact that they seem to provide an optimal solution of dividing several loaves of bread between several people, a problem solved in many of the ancient Egyptian papyri; see, e.g., [6] (see also [5, 8, 9, 10, 11]).

For example, if we want to divide 7 loaves between 12 people, then a usual representation $\frac{7}{12}$ of the corresponding fraction seems to indicate that we divide each of 7 loaves into 12 parts. To divide a loaf into 12 parts, we need to make 11 cuts. Thus, overall, we need to make $7 \cdot 11 = 77$ cuts.

On the other hand, if we represent $\frac{7}{12}$ as

$$\frac{7}{12} = \frac{1}{2} + \frac{1}{12},$$

this means that we need 12 halves and 12 (1/12)-parts. Thus, we:

- divide 6 loaves into 2 parts each, to get 12 halves – this requires 6 cuts, and
- we divide the remaining loaf into 12 pieces – which requires 11 cuts.

Overall, we thus need $6 + 11 = 17$ cuts – much fewer than 77.

One can show that, in general, minimizing the number of cuts is equivalent to representing each fraction $\frac{p}{q}$ as the Egyptian-style sum

$$\frac{p}{q} = \frac{1}{n_1} + \dots + \frac{1}{n_k}$$

with the smallest possible number of terms k .

Remaining problem. The above explanation explains *some* features of Egyptian fractions, but not all of them. For example, the fraction $\frac{2}{3}$ can be represented as the sum of two inverses in two different way:

- in the usual way

$$\frac{2}{3} = \frac{1}{3} + \frac{1}{3}$$

and

- in the Egyptian way:

$$\frac{2}{3} = \frac{1}{2} + \frac{1}{6}.$$

In both cases, to divide 4 loaves between 6 people, we need the exact number of cuts:

- in the first case, we divide each of 4 loaves into 3 parts, which requires 2 cuts each, to the total of 8, and
- in the second case, we divide 3 loaves into 2 parts each – which requires 3 cuts, and we divide the remaining loaf into 6 parts, which requires 5 cuts, to the total of $3 + 5 = 8$ cuts.

However, surprisingly, ancient Egyptians never use the standard representation, only their unusual one.

Why? There exist attempts to explain this phenomenon; see, e.g., [7, 8]. However, no absolutely convincing explanation has been found so far.

What we do in this paper. In this paper, we provide yet another possible explanation – based on the notion of fairness.

2 Fairness and How It Leads to Egyptian Fractions

Fractions mean that we need to divide some loaves (or other objects). The fact that a person gets a fraction of a loaf and not the whole loaf means that some loaves have to be divided between people.

What is fair division. If we want to divide a loaf (or any other object) between n people, then a fair way to divide is to give every person the exact same portion of the loaf: a portion of $\frac{1}{n}$.

What a person has may come from several fair divisions. A person may accumulate his/her bread, by combining several pieces that he/she got in several fair divisions. As a result, the amount of bread that this person will get is equal to the sum

$$\frac{1}{n_1} + \frac{1}{n_2} + \dots + \frac{1}{n_k},$$

where:

- n_1 is the number of people in the first division,

- n_2 is the number of people in the second division,
- \dots , and
- n_k is the number of people in the k -th division.

Need for fairness explains the Egyptian-style representation. So, if a person wants to show that he/she has got all his bread fairly, he/she can represent the amount of bread he/she has in the above form. Thus, the idea of fairness explains the need for such an Egyptian-style representations.

But this idea goes further.

Need for fairness also explains why all the integers in the representation should be different. If a person has $\frac{2}{3}$ of a loaf, and represents it as

$$\frac{2}{3} = \frac{1}{3} + \frac{1}{3},$$

this is not very convincing:

- maybe he/she got each of the two thirds in a fair division, or
- maybe there was one division, in which he/she unfairly grabbed two parts and left only one part to the other person.

On the other hand, if this person represents his/her amount of bread as

$$\frac{2}{3} = \frac{1}{2} + \frac{1}{6},$$

then this representation itself does not cause any doubt in this person's fairness: since it shows that his/her amount was obtained in two divisions:

- in the first one, a loaf was fairly divided between two folks, and
- in the second one, a loaf was fairly divided between six folks.

Comment. Need for fairness also explains why we should have few inverses in the Egyptian-style representation. Indeed:

- it is believable that a person participated in two or three fair divisions, but
- 100 divisions is already not believable at all – and
- in general, the more terms, the least believable it is.

Thus, to make everyone convinced that the person's amount comes from fair divisions, it is desirable to find a representation with the smallest possible number of terms.

Acknowledgments

This work was supported in part by the National Science Foundation grants 1623190 (A Model of Change for Preparing a New Generation for Professional Practice in Computer Science), and HRD-1834620 and HRD-2034030 (CAHSI Includes).

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