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Abstract

In many applications – e.g., in health care – it is desirable to make robots behave human-like. This means, in particular, that robotic control should not be optimal, it should be similar to human (suboptimal) behavior. People's decisions are based on bounded rationality: since we cannot compute an optimal solution for all possible situations, we divide situations into groups and come up with a solution appropriate for each group. What is optimal here is the division into groups. It is therefore desirable to implement a similar algorithm for robots. To help with such algorithms, we provide techniques that help optimally divide situations into groups.

1 Formulation of the Problem

Need for robots that look and act like humans: a brief reminder. In many practical applications, it is desirable to have robots that look and act like humans. For example, if we want to create a robot that takes care of small children, it is desirable to have a human-like robot, to utilize the children's natural affinity towards human beings and their natural fear of unusual creatures. Similarly, a medical robot that looks and acts like a human will hopefully help the patients to be somewhat more relaxed in an already stressful situation of an illness. A robot that takes care of older people will sound warmer if this robot is more human-like. And, of course, if something happens to a human operator (e.g., human driver, human pilot, etc.), it would be great for the robot to be able to fit into the control seat and take over.

What was the main challenge in the past. In the past, the main problem was to make a robot look and behave like a human.

What is the main problem now. Nowadays, we have robots that look and sometimes behave remarkably like humans – we have robotic TV announcers that are difficult to distinguish from the real ones, robotic performers, etc.

So now we face a different problem – that the robots can be made too good. Their movements can be made optimal up to the minute detail, their decisions can be made optimal. As a result, while these robots may look like humans, they do not behave like humans – to be more precise, instead of behaving like us humans, the robots behave like idealized never-making-a-mistake superhuman beings.

Clarification: sometimes we do need superhuman robots. Robots with superhuman abilities are definitely needed in many applications – we need robots that can bravely go where humans cannot, that can explore space, rescue victims of earthquakes, repair nuclear reactors.

In many other applications, we need more human-like robots. Having superhuman ability in a humanoid robot defeats the very purpose of a humanoid robot – to look and behave like a human.

Of course, we want this robot to act like a very good human – e.g., we do not want a medical robot to make mistake on purpose. However, in their movements, in their appearance, we do not want these robots to be perfect, we want them to be like us. Misdiagnosing a patient is a big no-no, but why not make a robot "accidentally" bump into a chair and slightly move it (as a human being would) if this will make this robot (and thus, this robot's advice) more acceptable to the patients.

What we do in this chapter. In this chapter, we provide algorithmic foundations that will (hopefully) help in designing such human-like robots.

2 Analysis of the Problem

Main idea. How can we simulate sub-optimality of human behavior? According to modern psychology, as discovered and emphasized by the Nobelists Herbert Simon and Daniel Kahneman (in his collaboration with Amos Tversky), this sub-optimality is mostly due to *bounded rationality* – i.e., to the fact that we humans have limited ability to process information; see, e.g., [2].

Details. As a result, e.g., when optimizing, we do not exactly find *the* value of the parameters for which the objective function attains its largest value – instead:

- we first discretise the problem, by dividing the range of possible situations (i.e., of possible values of the parameters) into finitely many subranges, and

- then, in each subrange, we select a typical situation, and we find the decision which is optimal for this typical situation; this decision will be used for all situations from this subrange.

This is how we deal with most real-life problems; for example:

- in a big class, where individual approach is not realistically possible, an instructor deals separately with A students (they need extra assignments), with C students (they need encouragement), and with potentially failing students (they need help);
- a medical doctor diagnoses a patient, and then prescribes the medicine corresponding to this particular diagnosis and this particular group of patients, etc.

How should we select subranges? Once we apply the same solution to all the situations from a given subrange, our solutions become suboptimal. For some divisions into subranges, we may have, in some situations, a big deviation from optimality. For other divisions, the deviations are not that large.

It is reasonable to select a division which is *optimal* – in the sense that the resulting decisions are as close to the optimal one as possible for the given fixed number of subranges.

What we do in this chapter. In this chapter, we formulate the corresponding optimization problem – of selecting the optimal division into subranges – in precise terms, and provide a solution to this problem.

We will consider two possible cases:

- the case when we do not know the probabilities of different situations; in this case, the natural way to gauge sub-optimality is by the worst-case difference between the optimal and suboptimal values of the corresponding objective function, and
- the case when we know the probabilities of different situations; in this case, it is more natural to gauge sub-optimality by the *average* difference between the optimal and suboptimal values.

Comment. Some of the corresponding mathematics will be similar to the one used in the book [4] to describe human behavior. In particular, this book shows that the idea of bounded rationality explains why humans use imprecise ("fuzzy") natural-language terms when making decisions, and why fuzzy control – that takes these words into account – often performs better than a probabilistic approach; see, e.g., [1, 3, 5, 6, 7, 8]. The explanation is that the fuzzy approach implicitly takes into account not only the probability of different alternatives, but also their utility.

The main difference of what we do in this chapter from the mathematical analysis presented in [4] is that there, our main goal was to *describe* human behavior, while here, the objective is to *recommend* (*prescribe*) the robot's behavior.

3 Case When We Do Not Know the Probabilities of Different Situations

General description of the control situation. To describe a situation, we need to describe the values of the quantities $x = (x_1, \ldots, x_n)$ that describe this situation. For example, the state of a doctor's patient can be described by the patient's body temperature, age, blood pressure, etc. The state of a student can be characterized by the student's grades on different assignments. The state of a mobile robot can be characterized by the coordinates describing its location and by the components of the velocity vector. If the robot has arms, we should also describe the angles between different parts of the robot's arm and the corresponding angular velocities.

Not all possible combinations of parameters are usually realistically possible. Let X denote the set of possible values of the tuples x.

To improve the situation, we can apply different controls. Control can also be characterized by the values of the corresponding parameters $u = (u_1, \ldots, u_m)$. For example, we can slow down or speed up the robot, change its direction, lift of lower its arm, etc.

We usually know the *objective function*, i.e., we know the gain $G(x, u)$ that we will get if we are in the state x, and we apply the control u. For example, if the goal is for a robot to reach the patient within a certain period of time t_0 (e.g., if the patient fell down), and the sooner the better, then $G(x, u)$ is the difference between t_0 and the time that the robot in the original state x will take to reach the patient after applying the control u.

In general, our objective is to maximize this gain.

Ideal case: optimal control. In the ideal case, for each situation x, we should selected the *optimal* control $u^{\rm opt}(x)$, i.e., control for which the gain is the largest:

$$G\left(x, u^{\rm opt}(x)\right) = \max_u G(x, u). \tag{1}$$

Case of human-like behavior. In the case of human-like behavior, we divide the range X into subranges X_j. In each subrange, we select a typical representative situation $x^{(j)}$, and apply the control $u^{\rm opt}\left(x^{(j)}\right)$ to all situations from the subrange X_j.

How to describe the degree of sub-optimality. For each situation $x \in X_j$, the best we can do is to apply the control $u^{\rm opt}(x)$ which is optimal for this situation. Then, we will get the gain $G\left(x, u^{\rm opt}(x)\right)$. Instead, we get the gain $G\left(x, u^{\rm opt}\left(x^{(j)}\right)\right)$. The difference between these gains is equal to

$$\Delta G_j(x) = G\left(x, u^{\rm opt}(x)\right) - G\left(x, u^{\rm opt}\left(x^{(j)}\right)\right). \tag{2}$$

For a close-to-optimal control, the subranges are small, and all the situations within each subrange are close to each other, so

$$x = x^{(j)} + \Delta x, \tag{3}$$

for some small Δx, and, correspondingly,

$$u^{\rm opt}(x)=u^{\rm opt}\left(x^{(j)}\right)+\Delta u, \tag{4}$$

for some small Δu. We can therefore substitute the expression

$$u^{\rm opt}\left(x^{(j)}\right)=u^{\rm opt}(x)-\Delta u \tag{5}$$

into the formula (2):

$$\Delta G_j(x)=G\left(x,u^{\rm opt}(x)\right)-G\left(x,u^{\rm opt}(x)-\Delta u\right), \tag{6}$$

expand this expression in Taylor series, and keep the largest non-zero terms in this expansion. In general, we have

$$G\left(x,u^{\rm opt}(x)-\Delta u\right)=G\left(x,u^{\rm opt}(x)\right)-\sum_{i=1}^m \frac{\partial G(x,u)}{\partial u_i}\cdot\Delta u_i+$$

$$\frac{1}{2}\cdot\sum_{i=1}^m\sum_{j=1}^m \frac{\partial^2 G(x,u)}{\partial u_i\partial u_j}\cdot\Delta u_i\cdot\Delta u_j+\ldots \tag{7}$$

By definition (1) of the optimal control $u^{\rm opt}(x)$, the function $G(x,u)$ attains its maximum at this control, so all the partial derivatives are equal to 0:

$$\frac{\partial G(x,u)}{\partial u_i}=0, \tag{8}$$

thus

$$G\left(x,u^{\rm opt}(x)-\Delta u\right)=G\left(x,u^{\rm opt}(x)\right)+\frac{1}{2}\cdot\sum_{i=1}^m\sum_{j=1}^m \frac{\partial^2 G(x,u)}{\partial u_i\partial u_j}\cdot\Delta u_i\cdot\Delta u_j+\ldots \tag{9}$$

Substituting this expression into the formula (6), we conclude that the main term in the difference (6) is quadratic:

$$\Delta G_j(x)=-\frac{1}{2}\cdot\sum_{i=1}^m\sum_{j=1}^m \frac{\partial^2 G(x,u)}{\partial u_i\partial u_j}\cdot\Delta u_i\cdot\Delta u_j. \tag{10}$$

Here,

$$\Delta u_i=\sum_{a=1}^n \frac{\partial u_i^{\rm opt}(x)}{\partial x_a}\cdot\Delta x_a. \tag{11}$$

Thus, the formula (1) takes the form

$$\Delta G_j(x)=\sum_{a=1}^n\sum_{b=1}^n c_{ab,j}\cdot\Delta x_a\cdot\Delta x_b, \tag{12}$$

where we denoted

$$c_{ab,j} = c_{ab}\left(x^{(j)}\right)$$

and

$$c_{ab}(x) = \sum_{i=1}^{m}\sum_{j=1}^{m} \frac{\partial^2 G(x,u)}{\partial u_i \partial u_j}_{|u=u^{\rm opt}(x)} \cdot \frac{\partial u_i^{\rm opt}(x)}{\partial x_a} \cdot \frac{\partial u_j^{\rm opt}(x)}{\partial x_b}. \quad (13)$$

The overall quality of division into subranges is described by the worst-case value of $\Delta G_j(x)$, i.e., by the value

$$\max_j \max_{x\in X_j} \Delta G_j(x). \quad (14)$$

We want to find the division into subranges for which the quantity (14) is the smallest possible.

Towards finding the optimal division into subranges. For each region X_j, let $v_j = \max\limits_{x\in X_j} \Delta G_j(x)$ be the largest value of $\Delta G(x)$ for all points $x \in X_j$. Then, it makes sense to assign, to this region, all the points x for which $\Delta G_j(x) \le v_j$ – adding these points to X_j will not increase the worst-case sub-optimality.

According to the formula (12), the value $\Delta G_j(x)$ is a quadratic function of x_a, so each region $\{x : \Delta G_j(x) \le v_j\}$ is an ellipsoid. Thus, in the first crude approximation, each subrange X_j is an ellipsoid whose axes are eigenvectors of the matrix a_{ij}. However, these subranges needs to fill the whole space, so we need to make them parallelepipeds with axes parallel to the axes of the ellipsoid – i.e., to the eigenvectors of the matrix a_{ij}.

If for some j_0, we have $v_{j_0} < \max\limits_j v_j$, then we can increase the subrange X_{j_0} and decrease the size (and thus, the values v_j) for other subranges, thus decreasing the value $\max\limits_j v_j$. Thus, in the optimal division into subranges, we should have all values v_j equal. Let us denote the common value of all these v_j by v.

What value v should we select? Suppose that we want to divide the whole range X into N subranges. At each point x, the volume of the subrange containing x is the volume of the corresponding parallelepiped X_j. In the coordinate system $y_1, \ldots, y_n$ formed by unit eigenvectors of the matrix c_{ab}, this matrix has a diagonal form $c'_{aa} = \lambda_a$ and $c'_{ab} = 0$ for $a \ne b$, where λ_a is the corresponding eigenvalue of the original matrix c_{ab}. In these coordinates, the condition $\Delta G_j(x) \le v$ takes the form $\sum\limits_a \lambda_a \cdot (\Delta y_a)^2 \le v$. Thus, each axis has half-length $\sqrt{\dfrac{v}{\lambda_a}}$ and, correspondingly, length $2\sqrt{\dfrac{v}{\lambda_a}}$. The volume of the box X_j is equal to the product of these lengths, i.e., to

$$2^n \cdot v^{n/2} \cdot \sqrt{\frac{1}{\prod\limits_a \lambda_a}}. \quad (15)$$

The product of all the eigenvalues of a matrix is equal to its determinant, so $\prod_a \lambda_a = \det(c_{ab})$. Thus, the volume of each subrange is equal to

$$v = 2^n \cdot v^{n/2} \cdot \frac{1}{\sqrt{\det(c_{ab})}}. \tag{16}$$

So, in a unit volume close to the point x, we have $1/v$ such subranges, i.e.,

$$\sqrt{\det(c_{ab}(x))} \cdot 2^{-n} \cdot v^{-n/2} \tag{17}$$

subranges. The overall number of subranges can be obtained if we add these numbers over all unit-volume parts of the range X, i.e., if we consider the integral

$$\int \sqrt{\det(c_{ab}(x))} \cdot 2^{-n} \cdot v^{-n/2}\, \mathrm{d}x. \tag{18}$$

The number of subranges should be equal to N, so we conclude that

$$\int \sqrt{\det(c_{ab}(x))} \cdot 2^{-n} \cdot v^{-n/2}\, \mathrm{d}x = N, \tag{19}$$

and thus, that

$$v^{n/2} = \frac{2^{-n}}{N} \cdot \int \sqrt{\det(c_{ab}(x))}\, \mathrm{d}x, \tag{20}$$

so

$$v = \frac{1}{4 \cdot N^{2/n}} \cdot \left(\int \sqrt{\det(c_{ab}(x))}\, \mathrm{d}x \right)^{2/n}. \tag{21}$$

Thus, we arrive at the following optimal division into subranges.

Solution: optimal division into subranges. Suppose that we can have N subranges. Then, we compute the value v by using the formula (21), where the matrix $c_{ab}(x)$ is determined by the formula (13). This value v is the largest possible difference between the optimal and suboptimal values of the objective function $G(x, u)$.

The corresponding subranges have the following form. Around each point x, we find the unit eigenvectors and eigenvalues λ_a of the matrix $c_{ab}(x)$. In the local coordinate system $y_1, \ldots, y_n$ formed by the unit eigenvectors, the subrange X_j is the following box:

$$\left[y_1^{(j)} - \sqrt{\frac{v}{\lambda_1}}, y_1^{(j)} + \sqrt{\frac{v}{\lambda_1}} \right] \times \ldots \times \left[y_n^{(j)} - \sqrt{\frac{v}{\lambda_n}}, y_n^{(j)} + \sqrt{\frac{v}{\lambda_n}} \right]. \tag{22}$$

4 Case When We Know the Probabilities of Different Situations

Description of the case. Suppose that we also know the relative frequency of different situations, i.e., we know the probability density function $\rho(x)$ describing how frequently we will encounter different situations.

Analysis of the problem. In this case, as one can show, locally, we have a similar division. The difference is that instead of the same value v_j for all the subranges, we may have different values $v(x)$ for different subranges: to decrease the average measure of difference $\Delta G_j(x)$, it makes sense to make it larger for scarcely populated subranges and smaller for densely populated subranges.

Once we know $v(x)$ for each x, we can determine the corresponding division into subranges. So, the main remaining problem is finding the optimal function $v(x)$. The main constraint is the overall number N of subranges, which, similar to formula (19), has the form

$$2^{-n} \cdot \int \sqrt{\det(c_{ab}(x))} \cdot (v(x))^{-n/2}\,\mathrm{d}x = N. \tag{23}$$

Under this constraint, we want to minimize the average difference $\Delta G(x)$. For each subrange, the average difference is proportional to $v(x)$, so minimizing the average difference is equivalent to minimizing the average values of $v(x)$:

$$\int \rho(x) \cdot v(x)\,\mathrm{d}x \to \min. \tag{24}$$

By using the Lagrange multiplier method, we can reduce this constraint optimization problem to the unconstrained problem of minimizing the functional

$$\int \rho(x) \cdot v(x)\,\mathrm{d}x + \lambda \cdot \left(2^{-n} \cdot \int \sqrt{\det(c_{ab}(x))} \cdot v(x)^{-n/2}\,\mathrm{d}x - N\right), \tag{25}$$

where λ is the Lagrange multiplier. Differentiating this expression with respect to $v(x)$ and equating the derivative to 0, we conclude that

$$\rho(x) - \lambda \cdot 2^{-n} \cdot \frac{n}{2} \cdot \sqrt{\det(c_{ab}(x))} \cdot (v(x))^{-n/2-1} = 0, \tag{26}$$

i.e., that:

$$(v(x))^{-n/2-1} = C \cdot \frac{\rho(x)}{\sqrt{\det(c_{ab}(x))}}, \tag{27}$$

where we denoted

$$C \stackrel{\text{def}}{=} \frac{1}{\lambda \cdot 2^{-n} \cdot \dfrac{n}{2}}. \tag{27}$$

Thus,

$$(v(x))^{-n/2} = \left((v(x))^{-n/2-1}\right)^{n/(n+2)} =$$

$$C^{n/(n+2)} \cdot \frac{(\rho(x))^{n/(n+2)}}{\left(\sqrt{\det(c_{ab}(x))}\right)^{n/(n+2)}}. \tag{28}$$

Substituting this expression for $(v(x))^{-n/2}$ into the formula (23), we conclude that

$$2^{-n} \cdot C^{n/(n+2)} \cdot \int (\rho(x))^{n/(n+2)} \cdot (\det(c_{ab}(x)))^{1/(n+2)}\,\mathrm{d}x = N, \tag{29}$$

hence

$$C^{n/(n+2)} = \frac{2^n \cdot N}{\displaystyle\int (\rho(x))^{n/(n+2)} \cdot (\det(c_{ab}(x)))^{1/(n+2)}\,\mathrm{d}x}, \tag{30}$$

and

$$C = \frac{2^{n+2} \cdot N^{1+2/n}}{\left(\displaystyle\int (\rho(x))^{n/(n+2)} \cdot (\det(c_{ab}(x)))^{1/(n+2)}\,\mathrm{d}x\right)^{1+2/n}}. \tag{31}$$

From (27), we can then conclude that

$$v(x) = \frac{(\det(c_{ab}(x)))^{1/(n+2)}}{C^{2/(n+2)} \cdot (\rho(x))^{2/(n+2)}}. \tag{32}$$

So, we arrive at the following solution.

Solution: optimal division into subranges. First, we compute the auxiliary value C by using the formula (31). Then, the corresponding subranges have the following form. Around each point x, we find the unit eigenvectors and eigenvalues λ_a of the matrix $c_{ab}(x)$. In the local coordinate system $y_1, \ldots, y_n$ formed by the unit eigenvectors, the subrange X_j is the following box:

$$\left[y_1^{(j)} - \sqrt{\frac{v(x)}{\lambda_1}}, y_1^{(j)} + \sqrt{\frac{v(x)}{\lambda_1}}\right] \times \ldots \times \left[y_n^{(j)} - \sqrt{\frac{v(x)}{\lambda_n}}, y_n^{(j)} + \sqrt{\frac{v(x)}{\lambda_n}}\right], \tag{33}$$

where $v(x)$ is determined by the formula (32).

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