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How to Explain the Anchoring Formula in Behavioral Economics

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Abstract

According to the traditional economics, the price that a person is willing to pay for an item should be uniquely determined by the value that this person will get from this item, it should not depend, e.g., on the asking price proposed by the seller. In reality, the price that a person is willing to pay *does* depend on the asking price; this is known as the anchoring effect. In this paper, we provide a natural justification for the empirical formula that describes this effect.

1 Formulation of the Problem

What is anchoring effect? Traditional economics assumes that people know the exact value of each possible item, and this value determines the price that they are willing to pay for this item.

The reality is more complicated. In many practical situations, people are uncertain about the value of an item – and thus, uncertain about the price they are willing to pay for this item. This happens, e.g., when hunting for a house.

Interestingly, in many such situations, the price that the customer is willing to pay is affected by the asking price:

- if the asking price is higher, the customer is willing to pay a higher price, but
- if the asking price is lower, the price that the customer is willing to pay is also lower.

This phenomenon is known as the *anchoring effect*: just like a stationary ship may move a little bit, but cannot move too far away from its anchor, similarly, a customer stays closer to the asking price – which thus acts as a kind of an anchor; see, e.g., [2], Chapter 11, and references therein.

Comment. The anchoring effect may sound somewhat irrational, but it makes some sense:

- If the owner lists his/her house at an unexpectedly high price, then maybe there are some positive features of the house of which the customer is not aware. After all, the owner does want to sell his/her house, so he/she would not just list an outrageously high price without any reason.

- Similarly, if the owner lists his/her house at an unexpectedly low price, then maybe there are some drawbacks of the house or of its location of which the customer is not aware. After all, the owner does want to get his/her money back when selling his/her house, so he/she would not just list an outrageously low price without any reason.

A formula that describes the anchoring effect. Let p_0 be the price that the customer would suggest in the absence of an anchor. Of course, if the asking price a_0 is the same value $a = p_0$, there is no reason for the customer to change the price p that he/she is willing to pay for this item, i.e., this price should still be equal to p_0.

It turns out that each anchoring situation can be described by a coefficient $\alpha \in [0, 1]$ which is called an *anchoring index*. The idea is that if we consider two different asking prices $a' \neq a''$, then the difference $p' - p''$ between the resulting customer's prices should be equal to $\alpha \cdot (a' - a'')$.

This idea – in combination with the fact that $p = p_0$ when $a = p_0$ – enables us to come up with the formula describing the anchoring effect. Indeed, for anchor a, we difference $p - p_0$ between:

- the price p corresponding to the asking price a and

- the price p_0 corresponding to the asking price p_0

should be equal to $\alpha \cdot (a - p_0)$. Since $p - p_0 = \alpha \cdot (a - p_0)$, we thus have $p = p_0 + \alpha \cdot (a - p_0)$, i.e., equivalently,

$$p = (1 - \alpha) \cdot p_0 + \alpha \cdot a. \tag{1}$$

First natural question: how can we explain this empirical formula?

What are the values of the anchoring index. It turns out that in different situations, we observe different values of the anchoring index.

When people are not sure about their original opinion, the anchoring index is usually close to 0.5:

- For a regular person buying a house, this index is equal to $0.48 \approx 0.5$; see, e.g., [2, 3].
- For people living in a polluted city, when asked what living costs they would accept to move to an environmentally clean area, the anchoring index was also close to 0.5; see, e.g., [2].

For other situations, when a decision maker in more confident in his/her original opinion, we can get indices between 0.25 and 0.5:

- For a real estate agent buying a house, this index is equal to 0.41; see, e.g., [2, 3].
- For a somewhat similar situation of charity donations, this index is equal to 0.30; see, e.g., [1, 2].

Second natural question: how can we explain these values?

What we do in this paper. In this paper, we try our best to answer both questions. Specifically:

- we provide a formal explanation for the formula (1), and
- we provide a somewhat less formal explanation for the empirically observed values of the anchoring index.

To make our explanations more convincing, we have tried to make the corresponding mathematics as simple as possible.

2 Formal Explanation of the Anchoring Formula

What we want. We want to have a function that, given two numbers:

- the price p_0 that the customer is willing to pay in a situation in which the seller has not yet proposed any asking price, and
- the actual asking price a,

produces the price $p(p_0, a)$ that the customer is willing to pay for this item after receiving the asking price a.

First natural property. As we have mentioned, if $a = p_0$, then we should have $p(p_0, a) = p(p_0, p_0) = p_0$.

Second natural property. Small changes in p_0 and a should not lead to drastic changes in the resulting price. In mathematica terms, this means that the function $p(p_0, a)$ should be continuous.

Third natural property. Intuitively, the change from p_0 to p should be in the direction to the anchor, i.e.:

- if $a < p_0$, we should have $p(p_0, a) \le p_0$, and
- if $p_0 < a$, we should have $p_0 \le p(p_0, a)$.

Fourth natural property. Also, intuitively, when the changed value $p(p_0, a)$ moves in the direction of the asking price a, it should not exceed a, i.e.:

- if $a < p_0$, we should have $a \le p(p_0, a)$, and
- if $p_0 < a$, we should have $p(p_0, a) \le a$.

Comment. The first three property can be summarized by saying that for all p_0 and a, the price $p(p_0, a)$ should always be in between the original price p_0 and the asking price a.

Fourth natural property: additivity. Suppose that we have two different situations – e.g., a customer is buying two houses, a house to live in and a smaller country house for vacationing. Suppose that:

- for the first item, the original price was p'_0 and the asking price is a', and
- for the second item, the original price was p''_0 and the asking price is a''.

Then, the price of the first item is $p(p'_0, a')$, the price of the second item is $p(p''_0, a'')$, thus the overall price of both items is

$$p(p'_0, a') + p(p''_0, a''). \tag{2}$$

Alternatively, instead of considering the two items separately, we can view them as a single combined item, with the original price $p'_0 + p''_0$ and the asking price $a' + a''$. From this viewpoint, the resulting overall price of both items is

$$p(p'_0 + p''_0, a' + a''). \tag{3}$$

Since (2) and (3) correspond to the exact same situation, it is reasonable to require that these two overall prices should coincide, i.e., that we should have

$$p(p'_0, a') + p(p''_0, a'') = p(p'_0 + p''_0, a' + a''). \tag{4}$$

Now, we are ready to formulate and prove our main result.

Definition 1. *A continuous function $p : \mathbb{R}^+_0 \times \mathbb{R}^+_0 \to \mathbb{R}^+_0$ that transforms two non-negative numbers p_0 and a into a non-negative number $p(p_0, a)$ is called an* anchoring function *if it satisfies the following two properties:*

- *for all p_0 and a, the value $p(p_0, a)$ should always be in between p_0 and a, and*
- *for all possible values p'_0, p''_0, a', and a'', we should have*

$$p(p'_0, a') + p(p''_0, a'') = p(p'_0 + p''_0, a' + a'').$$

Proposition 1. *A function $p(p_0, a)$ is an anchoring function if and only if it has the form*

$$p(p_0, a) = (1 - \alpha) \cdot p_0 + \alpha \cdot a$$

for some $\alpha \in [0, 1]$.

Comment. This proposition justifies the empirical expression (1) for the anchoring effect.

Proof. It is easy to see that every function of the type (1) satisfies both conditions of Definition 1 and is, thus, an anchoring function. So, to complete the proof, it is sufficient to prove that every anchoring function – i.e., every function that satisfies both conditions from Definition 1 – indeed has the form (1).

Indeed, let us assume that the function $p(p_0, a)$ satisfies both conditions. Then, due to additivity, for each p_0 and a, we have

$$p(p_0, a) = p(p_0, 0) + p(0, a). \tag{5}$$

Thus, to find the desired function of two variables, it is sufficient to consider two functions of one variable: $p_1(p_0) \stackrel{\text{def}}{=} p(p_0, 0)$ and $p_2(a) \stackrel{\text{def}}{=} p(0, a)$.

Due the same additivity property, each of these functions is itself additive:

$$p(p_0' + p_0'', 0) = p(p_0', 0) + p(p_0'', 0)$$

and

$$p(0, a' + a'') = p(0, a') + p(0, a'').$$

In other word, both functions $p_1(x)$ and $p_2(x)$ are additive in the sense that for each of them, we always have $p_i(x' + x'') = p_i(x') + p_i(x'')$.

Since the function $p(p_0, a)$ is continuous, both functions $p_i(x)$ are continuous as well. Let us show that every continuous additive function is linear, i.e., has the form $p_i(x) = c_i \cdot x$ for some c_i.

Indeed, let us denote $c_i \stackrel{\text{def}}{=} p_i(1)$. Due to additivity, since

$$\frac{1}{n} + \ldots + \frac{1}{n} \ (n \text{ times}) = 1,$$

we have

$$p_i\left(\frac{1}{n}\right) + \ldots + p_i\left(\frac{1}{n}\right) \ (n \text{ times}) = p_i(1) = c_i,$$

i.e.,

$$n \cdot p_i\left(\frac{1}{n}\right) = c_i$$

and thus,

$$p_i\left(\frac{1}{n}\right) = c_i \cdot \frac{1}{n}.$$

Similar, due to additivity, since for every m and n, we have

$$\frac{1}{n} + \ldots + \frac{1}{n} \ (m \text{ times}) = \frac{m}{n},$$

we have

$$p_i\left(\frac{1}{n}\right)+\ldots+p_i\left(\frac{1}{n}\right)\ (m \text{ times})=p_i\left(\frac{m}{n}\right).$$

The left-hand side of this formula is equal to

$$m\cdot p_i\left(\frac{1}{n}\right)=m\cdot\left(c_i\cdot\frac{1}{n}\right)=c_i\cdot\frac{m}{n}.$$

Thus, for every m and n, we have

$$p_i\left(\frac{m}{n}\right)=c_i\cdot\frac{m}{n}.$$

The property $p_i(x)=c_i\cdot x$ therefore holds for every rational number, and since each real number x can be viewed as a limit of its more and more accurate rational approximations x_n ($x=\lim x_n$), and the function $p_i(x)$ is continuous, we thus conclude, in the limit, that $p_i(x)=c_i\cdot x$ for all non-negative numbers x.

Thus, $p(p_0,0)=p_1(p_0)=c_1\cdot p_0$, $p(0,a)=p_2(a)=c_2\cdot a$, and the formula (5) takes the form

$$p(p_0,a)=c_1\cdot p_0+c_2\cdot a. \tag{6}$$

For $p_0=a$, the requirement that $p(p_0,a)$ is between p_0 and a implies that $p(p_0,a)=p_0$. For $p_0=a$, the formula (6) means that $c_1\cdot p_0+c_2\cdot p_0=p_0$, thus that $c_1+c_2=1$ and $c_1=1-c_2$. So, we get the desired formula (1) with $c_2=\alpha$.

To complete the proof, we need to show that $0\le\alpha\le 1$. Indeed, for $p_0=0$ and $a=1$, the value $p(0,1)$ must be between 0 and 1. Due to the formula (1), this value is equal to $(1-c_2)\cdot 0+c_2\cdot 1=c_2$. Thus, $c_2\in[0,1]$.

The proposition is proven.

3 Explaining the Numerical Values of the Anchoring Index

First case. Let us first consider the case when the decision maker is not sure which is more important: his/her a priori guess – as reflected by the original value p_0 – or the additional information as described by the asking price a. In this case, in principle, the value α can take any value from the interval $[0,1]$.

To make a decision, we need to select one value α_0 from this interval. Let us consider the discrete approximation with accuracy $\frac{1}{N}$ for some large N. In this approximation, we only need to consider values

$$0,\frac{1}{N},\frac{2}{N},\ldots,\frac{N-1}{N},1,$$

for some large N. If we list all possible values, we get a tuple

$$\left(0,\frac{1}{N},\frac{2}{N},\ldots,\frac{N-1}{N},1\right).$$

We want to select a single tuple α_0, i.e., in other words, we want to replace the original tuple with a tuple $(\alpha_0, \ldots, \alpha_0)$. It is reasonable to select the value α_0 for which the replacing tuple is the closest to the original tuple, i.e., for which the distance

$$\sqrt{(\alpha_0 - 0)^2 + \left(\alpha_0 - \frac{1}{N}\right)^2 + \left(\alpha_0 - \frac{2}{N}\right)^2 + \ldots + \left(\alpha_0 - \frac{N-1}{N}\right)^2 + (\alpha_0 - 1)^2}$$

attains its smallest possible value. Minimizing the distance is equivalent to minimizing its square

$$(\alpha_0 - 0)^2 + \left(\alpha_0 - \frac{1}{N}\right)^2 + \left(\alpha_0 - \frac{2}{N}\right)^2 + \ldots + \left(\alpha_0 - \frac{N-1}{N}\right)^2 + (\alpha_0 - 1)^2.$$

Differentiating this expression with respect to α_0 and equating the derivative to 0, we conclude that

$$2(\alpha_0 - 0) + 2\left(\alpha_0 - \frac{1}{N}\right) + 2\left(\alpha_0 - \frac{2}{N}\right) + \ldots + 2\left(\alpha_0 - \frac{N-1}{N}\right) + 2(\alpha_0 - 1) = 0.$$

If we divide both sides by 2 and move the terms not containing α_0 to the right-hand side, we conclude that

$$(N+1) \cdot \alpha_0 = 0 + \frac{1}{N} + \frac{2}{N} + \ldots + \frac{N-1}{N} + 1,$$

i.e., that

$$(N+1) \cdot \alpha_0 = \frac{1 + 2 + \ldots + (N-1) + N}{N},$$

thus

$$\alpha_0 = \frac{1 + 2 + \ldots + (N-1) + N}{N \cdot (N+1)}.$$

It is known that $1 + 2 + \ldots + N = \dfrac{N \cdot (N+1)}{2}$, thus

$$\alpha_0 = 0.5.$$

This is exactly the value used when the decision maker is not confident in his/her original estimate.

Second case. What if the decision maker has more confidence in his/her original estimate than in the anchor? In this case, the weight $1-\alpha$ corresponding to the original estimate must be larger than the weight α corresponding to the anchor. The inequality $1 - \alpha > \alpha$ means that $\alpha < 0.5$.

Similarly to the above case, we can consider all possible values between 0 and 0.5, and select a single value α_0 which is, on average, the closest to all these values. Similar to above calculations, we can conclude that the best value is

$$\alpha = 0.25.$$

Correspondingly, intermediate cases when the decision maker's confidence in his original opinion is somewhat larger, can be described by values α between the two above values 0.5 and 0.25. This explains why these intermediate values occur in such situations.

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