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Abstract

People's preferences depend not only on the decision maker's immediate gain, they are also affected by the decision maker's expectation of future gains. A person's decisions are also affected by possible consequences for others. In decision theory, people's preferences are described by special quantities called utilities. In utility terms, the above phenomena mean that the person's overall utility of an action depends not only on the utility corresponding to the action's immediate consequences for this person, it also depends on utilities corresponding to future consequences and on utilities corresponding to consequences for others. These dependencies reflect discounting of future consequences in comparison with the current ones and to empathy (or lack of) of the person towards others. In general, many formulas involving utility are nonlinear, even formulas describing the dependence of utility on money. However, surprisingly, for discounting and for empathy, linear formulas work very well. In this paper, we show that natural symmetry requirements can explain this linearity.

1 Formulation of the Problem

Decision making: need for a general reminder. In this paper, we deal with decision making issues, namely, with how future consequences and effect on others affect human decision making. To formulate the corresponding problems, let us first recall how decision making is usually handled in decision theory. For more details, see, e.g., [6, 15, 17, 20, 22] and references therein.

Utility: the main concept of decision theory. One of the main objectives of decision theory is to help decision makers make decisions in complex situations – help by using computer-based decision support systems. For these systems to be able to help, we need to describe decision makers' preferences in computer-understandable form.

The natural language of computers is the language of numbers: processing numbers is what computers were originally designed for, processing numbers is what they can do with astronomical speed of several billion operations per second. Thus, we need to be able to describe human preferences in numerical form. This is done by using a special concept of *utility*.

To define utility, we need to select two extreme situations:

- a very bad situation A_- which is worse than anything that we will actually encounter, and
- a very good situation A_+ which is better than anything that we will actually encounter.

Then, for each number p from the interval $[0, 1]$, we can form a lottery – we will denote it by $L(p)$ – in which we get A_+ with probability p and A_- with the remaining probability $1 - p$.

For $p = 0$, this lottery coincides with the very bad alternative A_- and is, thus, worse than any actual alternative A; we will denote this relation between A_- and A by $A_- < A$. For $p = 1$, this lottery coincides with the very good alternative A_+ and is, thus, better than any actual alternative A: $A < A_+$. Clearly, the larger the probability p of the very good alternative, the better the lottery. Thus, we get a continuous scale that leads from the very bad alternative to the very good alternative.

For every actual alternative A, for small p, we have $L(p) \approx A_-$ and thus, $L(p) < A$. For $p \approx 1$, we have $L(p) \approx A_+$ and thus, $A < L(p)$. So, there must exist a threshold that separates values p with $L(p) < A$ from values for which $A < L(p)$. This threshold is equal to

$$\sup\{p : L(p) < A\} = \inf\{p : A < L(p)\}.$$

It is known as the *utility* $u(A)$ of the alternative A.

For this threshold $u(A)$, for every $\varepsilon > 0$, we have $L(u(A) - \varepsilon) < A < L(u(A) + \varepsilon)$. For very small ε, probabilities $u(A) - \varepsilon$, $u(A)$, and $u(A) + \varepsilon$ are practically indistinguishable. So, from the practical viewpoint, the alternative A is *equivalent* to the lottery $L(u(A))$. We will denote this equivalence by

$$A \equiv L(u(A)).$$

An important fact: utility is defined modulo a linear transformation. In the above procedure, to find a numerical value $u(A)$ for each alternative A, we need to select a pair (A_-, A_+) of extreme alternatives. What if we select a different pair (A'_-, A'_+), e.g. a pair for which $A_- < A'_- < A'+ < A_+$? How will the original utility $u(A)$ be related to the utility $u'(A)$ defined in terms of a new pair?

To answer this question, let us consider an alternative A with some utility value $u'(A)$. This utility value means that the alternative A is equivalent to a lottery $L'(u'(A))$ in which we get A'_+ with probability $u'(A)$ and A'_- with the remaining probability $1 - u'(A)$. On the other hand, since both new extreme situations A'_- and A'_+ are better than A_- and worse than A_+, each of them is also equivalent to an appropriate lottery $L(u(A'_-))$ (or $L(u(A'_+))$) in which we get A_+ with the probability $u(A'_+)$ (or $u(A'_-)$), and A_- with the remaining probability. Thus, the original alternative A is equivalent to a two-stage lottery, in which:

- first, we select A'_+ with probability $u'(A)$ or A'_- with probability $1-u'(A)$;
- then, if we selected A'_+ on the first stage, we select A_+ with probability $u(A'_+)$ and A_- with probability $1 - u(A'_+)$;
- and if we selected A'_- on the first stage, we select A_+ with probability $u(A'_-)$ and A_- with probability $1 - u(A'_-)$.

As a result of this two-stage lottery, we get either A_+ or A_-, and the probability of getting A_+ is equal to $u'(A) \cdot u(A'_+) + (1 - u'(A)) \cdot u(A'_-)$. By definition of utility, this probability is exactly the utility $u(A)$ with respect to the original pair (A_-, A_+) of extreme situations. Thus,

$$u(A) = u'(A) \cdot u(A'_+) + (1 - u'(A)) \cdot u(A'_-) = u(A'_-) + u(A) \cdot (u(A'_+) - u(A'_-)),$$

i.e., $u(A) = a \cdot u'(A) + b$, where we denoted $a \stackrel{\text{def}}{=} u(A'_+) - u(A'_-)$ and $b \stackrel{\text{def}}{=} u(A'_-)$.

So, utility is indeed defined modulo a linear transformation. This is the fact that we will actively use in this paper.

Need for discounting. A person's attitude to an alternative depends not only on the immediate gain, it is also affected by future consequences of an action.

Let us describe this dependence in precise terms. Suppose that we know the exact consequences of an action, both immediate and future consequences – this happens, e.g., when we take a fixed-interest loan. Let u_0 denote the utility corresponding to the immediate consequences of an action, and let u_t, for $t = 1, 2, \ldots$, demote the utility of consequences t years from now. Based on this information, we need to describe the overall person's attitude to this action.

In general, the attitude is described by utility values. So, what we need is a method $u(u_0, u_1, \ldots)$ that would describe the overall utility of an action based on the values u_0, u_1, $\ldots$. Such a method is known as *discounting*, since the future consequences affect the decision maker less than the current ones.

Empirical fact: all known discounting formulas are linear. There exist several different discounting formula that provide a reasonably good description of how people actually make decisions; see, e.g., [5, 7, 11, 12, 13, 18, 19, 21, 26]. Interestingly, all these formulas are linear, i.e., have the form

$$u(u_0, u_1, \ldots, u_n) = c + p_0 \cdot u_0 + p_1 \cdot u_1 + \ldots + p_n \cdot u_n,$$

for some coefficients c and p_i.

First question. This empirical linearity is strange, since, in general, many formulas involving utility are non-linear: e.g., even the dependence of utility on money is non-linear; see, e.g., [10, 16] and references therein. So, a natural question is: why is empirical discounting linear?

Need to take empathy into account. Another important factor that affects people's decision making is the need to take into account how other people feel. For example, it is difficult to enjoy good food in a restaurant if poor hungry people sit outside begging for food. Since preferences are described in terms of utility, this means that the utility of a person depends on the utilities of other people.

In other words, the overall utility u of an action should depend not only on the utility u_0 of the decision maker, it should also depend on the utilities u_1, u_2, ..., of other people affected by this decision: $u = u(u_0, u_1, \ldots, u_n)$.

The idea that a utility of a person should depend on utilities of others was first explicitly formalized in [23, 24]; it was further developed by Nobelist Gary Becker; see, e.g., [1]; see also [2, 4, 3, 8, 9, 14, 20, 25].

Empirical fact: all known empathy-related formulas are linear. There exist several different formulas that provide a reasonably good description of how people's utility depend on feeling (i.e., utilities) of others – see above references. Interestingly, all these formulas are also linear, i.e., also have the same linear form $u(u_0, u_1, \ldots, u_n) = c + \sum\limits_{i=0}^{n} p_i \cdot u_i$.

Second question and what we do in this paper. This empirical fact raises a natural second question: why is empathy well described by a linear formula?

In this paper, we provide a possible explanation of why in both cases – of discounting and of empathy – we observe linear dependencies.

2 Towards an Explanation

What we want: a reminder. In both discounting and empathy cases, we need to describe a utility $u(u_0, u_1, \ldots, u_n)$ that corresponds to a situation described by several utility values:

- In the case of discounting, we want to describe how the overall person's utility depends on the utility value u_0 coming from current gains and on the values u_t ($1 \le t \le n$) describing the person's expected utility t years into the future.

- In the case of empathy, we want to describe how the overall person's utility depends on the utility value u_0 coming from this person's gains and the utilities $u_1, \ldots, u_n$ of other people.

To describe which dependencies are reasonable, let us analyze what are the reasonable requirements on the corresponding function, and then let us look for the dependencies that satisfy all these requirements.

First natural requirement: smoothness. When changes in $u_0, u_1, \ldots, u_n$ are so small that they are barely noticeable (or even not noticeable at all), we expect that the overall utility will also undergo a barely noticeable change. This idea is formalized in mathematics into the notion of smoothness (differentiability). Thus, in precise mathematical terms, we expect the function $u(u_0, u_1, \ldots, u_n)$ to be smooth (differentiable).

Second natural requirement: invariance. We want to combine the degrees of satisfaction, but what we actually combine are utility values. We have already mentioned that the same degree of satisfaction can be described by different numerical utility values – because these numerical values depend on the selection of the alternatives A_- and A_+ that are used in defining utility.

If we replace the original pair (A_-, A_+) with a different pair (A'_-, A'_+), then, as we have shown earlier, all the numerical values will change according to an appropriate linear transformation $u \to u' = a \cdot u + b$ with $a > 0$. The coefficients of this transformation depend on the utilities of the alternatives $A_\pm$ and $A'_\pm$ with respect to each other. These utilities – and thus, the corresponding coefficients a and b – may change with time and may change from one person to another. As a result, each utility u_i is, in general, transformed differently, as $u_i \to u'_i = a_i \cdot u_i + b_i$ with coefficients $a_i > 0$ and b_i which are, in general, different for each i.

It seems reasonable to require that the overall degree of satisfaction should not change if we simply re-scale the corresponding utility values. In other words, it seems reasonable to require that it we linearly re-scale each utility value u_i, then the resulting utility $u(u_0, u_1, \ldots, u_n)$ should also be linearly re-scaled.

Unfortunately, this idea does not work. From the common sense viewpoint, invariance may sound reasonable, but, as well show, it does not work.

Definition 1. *We say that a differentiable function $u(u_0, u_1, \ldots, u_n)$ is* fully invariant *if for every tuple of values $(a_0, b_0, a_1, b_1, \ldots, a_n, b_n)$ with $a_i > 0$, there exist values $a > 0$ and b for which, for all possible values of $u_0, u_1, \ldots, u_n$, we have*

$$u(a_0 \cdot u_0 + b_0, a_1 \cdot u_1 + b_1, \ldots, a_n \cdot u_n + b_n) = a \cdot u(u_0, u_1, \ldots, u_n) + b.$$

Proposition 1. *A function $u(u_0, u_1, \ldots, u_n)$ is fully invariant if and only it is a linear function of only one of the variables $u_0, u_1, \ldots, u_n$.*

Proof.

1°. Full invariance means, in particular, that:

- for each b_0, we have the values $a(b_0)$ and $b(b_0)$ for which, for all $u_0, u_1, \ldots, u_n$, we have
$$u(u_0 + b_0, u_1, \ldots, u_n) = a(b_0) \cdot u(u_1, u_1, \ldots, u_n) + b(b_0); \qquad (1)$$
and

- for each a_0, we have the values $a'(a_0)$ and $b'(a_0)$ for which, for all $u_0, u_1, \ldots, u_n$, we have
$$u(a_0 \cdot u_0, u_1, \ldots, u_n) = a'(a_0) \cdot u(u_1, u_1, \ldots, u_n) + b'(a_0). \qquad (2)$$

Let us fix all the values $u_1, \ldots, u_n$ and consider the dependence on u_0 only. In other words, let us consider a function $F(u_0) \stackrel{\text{def}}{=} u(u_0, u_1, \ldots, u_n)$ of only one variable.

In terms of this function, the formulas (1) and (2) take the form
$$F(u_0 + b_0) = a(b_0) \cdot F(u_0) + b(b_0); \qquad (1a)$$
$$F(a_0 \cdot u_0) = a'(a_0) \cdot F(u_0) + b'(a_0). \qquad (2a)$$
We will show that these two equalities imply that $F(u_0)$ is a linear function.

2°. Let us start with the equality (1a). Let us fix two different values $u_0 = u_0'$ and $u_0 = u_0''$ of u_0, for which $F(u_0') \neq F(u_0'')$. Then, we get the following two linear equations with constant coefficients for two unknowns $a(b_0)$ and $b(b_0)$:
$$F(u_0' + b_0) = a(b_0) \cdot F(u_0') + b(b_0);$$
$$F(u_0'' + b_0) = a(b_0) \cdot F(u_0'') + b(b_0).$$
The solutions to this equation are linear combinations of the expressions
$$F(u_0' + b_0) \text{ and } F(u_0'' + b_0).$$

Since the original function $u(u_0, u_1, \ldots, u_n)$ is differentiable, the function $F(u_0)$ is also differentiable and thus, both functions $a(b_0)$ and $b(b_0)$ are differentiable – as linear combinations of differentiable functions.

3°. Now that we know that all three functions $F(u_0)$, $a(b_0)$, and $b(b_0)$ involved in formula (1) are differentiable, we can differentiate both sides of this formula by b_0, and get $DF(u_0 + b_0) = Da(b_0) \cdot F(u_0) + Db(b_0)$, where Df denotes the derivative of a function f. In particular, for $b_0 = 0$, we get following new formula: $DF(u_0) = A \cdot F(u_0) + B$, where we denoted $A \stackrel{\text{def}}{=} Da(0)$ and $B \stackrel{\text{def}}{=} Db(0)$. In other words, we get
$$\frac{dF}{du_0} = A \cdot F + B. \qquad (3)$$

We can separate variables in this equation if we multiply both sides by du_0 and divide both sides by $A \cdot F + B$:

$$\frac{dF}{A \cdot F + B} = du_0. \tag{4}$$

Here, we have two possible cases: $A = 0$ and $A \neq 0$. Let us consider them one by one.

4°. If $A = 0$, then (4) takes the form $\dfrac{dF}{B} = du_0$. Integrating both sides, we get $\dfrac{F}{B} = u_0 + C$, where C is the integration constant and thus, $F(u_0) = B \cdot u_0 + B \cdot C$. So, in this case, the function $F(u_0)$ is indeed linear.

5°. If $A \neq 0$, then for $G \stackrel{\text{def}}{=} A \cdot F + B$, we get $dG = A \cdot dF$, thus, $dF = \dfrac{dG}{A}$, and the formula (4) takes the from $\dfrac{1}{A} \cdot \dfrac{dG}{G} = du_0$. Integrating both sides, we get $\dfrac{1}{A} \cdot \ln(G) = u_0 + C$, so $\ln(G(u_0)) = A \cdot u_0 + A \cdot C$. By applying $\exp(x)$ to both sides, we get $G(u_0) = \exp(A \cdot u_0 + A \cdot C)$, hence

$$F(u_0) = \frac{G(u_0) - B}{A} = \frac{\exp(A \cdot u_0 + A \cdot C) - B}{A}. \tag{5}$$

6°. Let us now utilize the formula (2a). If we plug in the expression (5) into the formula (2), we will see that the left-hand side is proportional to $\exp(a_0 \cdot A \cdot u_0)$, while the right-hand side is proportional to $\exp(A \cdot u_0)$. For $a_0 > 1$, the left-hand side grows faster than the right-hand side, so they cannot be equal.

Thus, the case $A \neq 0$ is impossible, so $A = 0$, and by Part 4 of this proof, $F(u_0)$ is a linear function.

7°. For a linear function $F(u_0) = p_0 \cdot u_0 + q_0$, as one can easily see, we have $F(u_0 + b_0) = F(u_0) + p_0 \cdot b_0$. So, in the formula (1a), we have $a(b_0) = 1$ and $b(b_0) = p_0 \cdot b_0$. Thus, the formula (1) takes the form

$$u(u_0 + b_0, u_1, \ldots, u_n) = u(u_0, u_1, \ldots, u_n) + p_0 \cdot b_0. \tag{6}$$

In particular, for $u_0 = 0$, we have

$$u(b_0, u_1, u_2, \ldots, u_n) = u(0, u_1, u_2 \ldots, u_n) + p_0 \cdot b_0,$$

i.e., renaming b_0 into u_0:

$$u(u_0, u_1, u_2, \ldots, u_n) = u(0, u_1, u_2 \ldots, u_n) + p_0 \cdot u_0.$$

Similarly, we can prove that

$$u(0, u_1, u_2, \ldots, u_n) = u(0, 0, u_2 \ldots, u_n) + p_0 \cdot u_1,$$

and thus

$$u(u_0, u_1, u_2, \ldots, u_n) = u(0, 0, u_2, \ldots, u_n) + p_0 \cdot u_0 + p_1 \cdot u_1$$

for some p_1. Performing the same procedure with u_2, etc., we can conclude that

$$u(u_0, u_1, \ldots, u_n) = c + p_1 \cdot u_0 + p_1 \cdot u_1 + \ldots + p_n \cdot u_n$$

for some p_i, where we denoted $c \stackrel{\text{def}}{=} u(0, 0, \ldots, 0)$.

8°. Let us show, by contradiction, that this expression depends on one input, i.e., that $p_i \neq 0$ only for one index i.

If this expression depended on two or more inputs, this would mean that at least two of the coefficients p_i are different from 0. Let us denote the corresponding indices by i and j. Then, once we take $u_k = 0$ for all other k, then for $F(u_i, u_j) \stackrel{\text{def}}{=} u(0, \ldots, 0, u_i, 0, \ldots, 0, u_j, 0, \ldots, 0)$, we get $F(u_i, u_j) = c + p_i \cdot u_i + p_j \cdot u_j$ for some $p_i \neq 0$ and $p_j \neq 0$.

In this case, for $u'_i = p_j$, $u'_j = u''_i = 0$, and $u''_j = p_i$, we have $F(u'_i, u'_j) = F(u''_i, u''_j) = c + p_i \cdot p_j$. Due to full invariance, we have $F(2u_i, u_j) = a \cdot F(u_i, u_j) + b$ for some constants a and b. Thus, we should have

$$F(2u'_i, u'_j) = a \cdot F(u'_i, u'_j) + b = a \cdot F(u''_i, u''_j) + b = F(2u''_i, u''_j)$$

and therefore, $F(2u'_i, u'_j) = F(2u''_i, u''_j)$, but here

$$F(2u'_i, u'_j) = c + 2p_i \cdot p_j \neq F(2u''_i, u''_j) = c + p_i \cdot p_j.$$

This contradiction completes the proof of the proposition.

Discussion. In both cases – of discounting and of empathy – the utility should depend on the value u_0. So, if we require full invariance, then the overall utility cannot depend on anything else. In the discounting case, this would mean that we do not take future gains into account at all – only the current ones. In the empathy case, this means that we completely ignore happiness or suffering of others and concentrate exclusive on our own happiness. Such behaviors happen, but they are on the edge of pathology, this is *not* a normal behavior.

To describe normal behavior, we thus need to ease some of the symmetry restrictions. Let us see what is the most natural way to do it.

Analysis of the problem. In our first attempt, we use invariance under all possible linear transformation, i.e., under shift $u \to u + b$, under scaling $u \to a \cdot u$ and thus, under any combination of these two transformations.

Invariance under shift makes perfect sense. Indeed, we can take, as point 0 (corresponding to the utility A_-) either the status quo point or what happened several years ago, when the situation was worse – the resulting decision should not change. This is similar to how we measure the person's income:

- we can measure it directly in dollars,

- we can add the amount of indirect subsidies provided by the state to everyone (e.g., in the form of state support for low food prices and/or low public transportation prices), and thus, get a larger amount,
- or, alternatively, we can subtract the living minimum from all the salaries and consider only what is left for extra things.

On the other hand, invariance with respect to scaling is less convincing. True, if we want a decision procedure that does not depend on the choice of monetary units, then we should treat, e.g., the difference between 100 dollars and 200 dollars the same way as the difference between 100 million and 200 million. However, in practice, there is a big difference:

- for a person dealing with such sums as 100 and 200 dollars, there is a huge difference between these two amounts, while
- for a billionaire dealing with hundreds of millions all the time, an extra hundred million is probably an icing on a cake, something that he/she is often willing to donate to charity.

With this is mind, let us see what will happen if instead of full invariance, we only require invariance with respect to shifts.

Definition 2. *We say that a differentiable function $u(u_0, u_1, \ldots, u_n)$ is* shift-invariant *if for every tuple of values $(b_0, b_1, \ldots, b_n)$, there exist a value b for which, for all possible values of $u_0, u_1, \ldots, u_n$, we have*

$$u(u_0 + b_0, u_1 + b_1, \ldots, u_n + b_n) = u(u_0, u_1, \ldots, u_n) + b.$$

Proposition 2. *A function $u(u_0, u_1, \ldots, u_n)$ is shift-invariant if and only it is linear.*

Discussion. Thus, we indeed have *a symmetry-based explanation of* why both in discounting and in empathy, *linear functions* provide a good description of people's decision making.

Proof. The proof of this proposition is similar to the proof of Proposition 1.

1°. For the case when $b_1 = \ldots = b_n = 0$, shift-invariance means that

$$u(u_0 + b_0, u_1, \ldots, u_n) = u(u_0, u_1, \ldots, u_n) + b(b_0). \tag{7}$$

So, if we fix the values $u_1, \ldots, u_n$, and consider a function $F(u_0) \stackrel{\text{def}}{=} u(u_0, u_1, \ldots, u_n)$, then for this function, the formula (7) takes the form

$$F(u_0 + b_0) = F(u_0) + b(b_0). \tag{7a}$$

2°. Since the function $F(u_0)$ is differentiable, the function $b(b_0)$ is also differentiable – as the difference $F(u_0 + b_0) - F(u_0)$ of two differentiable functions. Thus, we can differentiate both sides of the formula (7a) with respect to b_0 and get the expression $DF(u_0 + b_0) = Db(b_0)$. In particular, for $b_0 = 0$, we get $DF(u_0) = p_0$, where we denoted $p_0 \stackrel{\text{def}}{=} Db(0)$, i.e., $\dfrac{dF}{du_0} = p_0$. Integrating both sides, we conclude that $F(u_0) = p_0 \cdot u_0 + C$. Thus,

$$b(b_0) = F(u_0 + b_0) - F(u_0) = (p_0 \cdot (u_0 + b_0) + C) - (p_0 \cdot u_0 + C) = p_0 \cdot b_0.$$

3°. Substituting the above formula for $b(b_0)$ into the expression (7), we conclude that

$$u(u_0 + b_0, u_1, \ldots, u_n) = u(u_0, u_1, \ldots, u_n) + p_0 \cdot b_0.$$

In particular, for $u_0 = 0$, we conclude that

$$u(b_0, u_1, \ldots, u_n) = u(0, u_1, \ldots, u_n) + p_0 \cdot b_0,$$

i.e., renaming b_0 to u_0:

$$u(u_0, u_1, \ldots, u_n) = u(0, u_1, \ldots, u_n) + p_0 \cdot u_0.$$

Similar to the proof of Proposition 1, we can similarly conclude that

$$u(0, u_1, u_2, \ldots, u_n) = u(0, 0, u_2, \ldots, u_n) + p_1 \cdot u_1$$

for some value p_1 and thus, that

$$u(u_0, u_1, u_2, \ldots, u_n) = u(0, 0, u_2, \ldots, u_n) + p_0 \cdot u_0 + p_1 \cdot u_1,$$

and, in general, that

$$u(u_1, \ldots, u_n) = c + p_0 \cdot u_1 + \ldots + p_n \cdot u_n,$$

where we denoted $c \stackrel{\text{def}}{=} u(0, 0, \ldots, 0)$.

The proposition is proven.

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