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# Predictably (Boundedly) Rational: Examples of Seemingly Irrational Behavior Can Be Quantitatively Explained by Bounded Rationality

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## Abstract

Traditional economics is based on the simplifying assumption that people behave perfectly rationally, that before making any decision, a person thoroughly analyzes all possible situations. In reality, we often do not have enough time to thoroughly analyze all the available information, as a result of which we make decisions of bounded rationality – bounded by our inability to perform a thorough analysis of the situation. So, to predict human behavior, it is desirable to study how people actually make decisions. The corresponding area of economics is known as *behavioral economics*. It is known that many examples of seemingly irrational behavior can be explained, on the *qualitative* level, by this idea of bounded rationality. In this paper, we show that in many cases, this qualitative explanation can be expanded into a *quantitative* one, that enables us to explain the numerical characteristics of the corresponding behavior.

## 1 Introduction

### **Traditional economic models are based on the rationality assumption.**

Traditional models in economics are based on the assumptions that we humans are rational human beings, that each of our decisions is motivated by the desire to maximize whatever objective function we select: whether it is maximizing the profit or maximizing our pleasure.

**This assumption is not always true.** In many practical situations, people exhibit seemingly irrational behavior: they make decisions that they would not have made if they rationally analyzed the situation. This phenomenon has

been observed in numerous situations. Several Nobel prizes in economics have been awarded for analyzing this phenomenon, and behavioral economics – that takes into account how humans actually behave – is one of the fastest growing branches of economics-related research.

**Bounded rationality – a natural (qualitative) explanation for seemingly irrational behavior.** There is a natural explanation of why people do not always make most rational decisions, why they make decisions that they would not have made if they spent some time and effort analyzing this situation rationally. The explanation is very straightforward: often, when we make decision, we do not have the time for a thorough analysis. Our ability to process information fast is limited, so we make fast decisions whose rationality is bounded by our ability to optimize fast.

On the qualitative level, this idea of *bounded rationality* indeed explains many observed phenomena of seemingly irrational behavior.

**It is important to have a quantitative explanation as well.** To be able to make successful economic decisions, we need to know how people will actually behave. We want to know how much they are willing to pay for different goods and service, with what frequency they will choose different alternatives, etc. In other words, it is desirable to have a *quantitative* explanation of human behavior.

**What is known and what we do in this paper.** In some cases, it has been possible to come up with a *quantitative* explanation of seemingly irrational human behavior. In other words, in some cases, it was possible to explain not only why our decisions differ from the ideal rational ones, but also explain the exact numerical decisions that we make: e.g., what price we are willing to pay, with what frequency we make this or that choice, etc.

However, as of now, there are only few such quantitative explanations of seemingly irrational behaviors. Many books and papers are filled with examples of seemingly irrational human behavior – examples for which there is a convincing qualitative explanation – but only for a few of them, there is a quantitative explanation.

In this paper, we analyze these examples and we show that in many cases, a quantitative explanation is indeed possible: a quantitative explanation based on a few basic principles of human decision making under uncertainty. The fact that many seemingly irrational human behaviors can be this quantitatively explained make us hope that a quantitative explanation will be found for all such behaviors.

*Comment.* As some readers may guess, we selected the title of this paper to relate the title of a popular book [2] that cites many cases of seemingly irrational behavior. While we cite many examples from this book (and from similar books and papers), our emphasis is *not* just to show how irrational our decisions may seem, but to explain that, taking into account our bounded abilities, most of these decisions are actually perfectly rational.

## 2 How Decision Making Is Described in Traditional Decision Theory

**How can we describe perfectly rational decision making?** To better understand cases of seemingly irrational behavior, let us recall how rational behavior can be described. This description forms the basis of traditional decision theory. Readers interested in more details are welcome to see [11, 23, 28, 37, 40].

**How do we describe preference: the notion of utility.** The main objective of the traditional decision theory is to help people make decisions. To be able to do that, we need to understand what each person wants, what he or she prefers.

How can we elicit this information from the user? A natural idea is to provide the user, several times, with several hypothetical alternatives, and each time ask what the user prefers.

Of course, we want computers to automate – partly or fully – the advising process. It is a known fact that computers process numbers much more efficiently than any other type of information – after all, processing numbers is what computers were originally designed to do. From this viewpoint, it is desirable to describe our knowledge about preferences also in terms of numbers.

The notion of utility provides a natural way to do it. Utility is defined as follows. Let us select two alternatives:

- a very bad alternative  $A_-$ , which is much worse than what any alternative  $A$  that the user will actually encounter; we will denote this preference by  $A_- < A$ ; and
- a very good alternative  $A_+$ , which is much better than what any alternative  $A$  that the user will actually encounter:  $A < A_+$ .

For each real number  $p$  from the interval  $[0, 1]$ , we can then design a lottery  $L(p)$  in which:

- we get  $A_+$  with probability  $p$  and
- we get  $A_-$  with the remaining probability  $1 - p$ .

Then, to find a numerical value corresponding to a given alternative  $A$ , we compare it with lotteries  $L(p)$  corresponding to different probabilities  $p$ .

- When  $p = 0$ , the lottery  $L(p)$  coincides with  $A_-$  and is, thus, worse than the given alternative  $A$ .
- When  $p = 1$ , the lottery  $L(p)$  coincides with  $A_+$  and is, thus, better than the given alternative  $A$ .

The larger the probability of the very good alternative  $A_+$ , the better. Thus:

- If  $A < L(p)$  and  $p < p'$ , we have  $L(p) < L(p')$  and therefore,  $A < L(p')$ .

- Similarly, if  $L(p) < A$  and  $p' < p$ , then  $L(p') < A$ .

So, one can prove that there exists a threshold

$$p_0 = \sup\{p : L(p) < A\} = \inf\{p : A < L(p)\}$$

such that  $L(p) < A$  for all  $p < p_0$  and  $A < L(p)$  for all  $p > p_0$ .

In particular, for every  $\varepsilon > 0$ , we have  $L(p_0 - \varepsilon) < A < L(p_0 + \varepsilon)$ . From the practical viewpoint, when  $\varepsilon$  is small enough, lotteries  $L(p_0)$  and  $L(p_0 \pm \varepsilon)$  are practically indistinguishable. Thus, from the practical viewpoint, we can say that the lottery  $L(p_0)$  is equivalent to the original alternative  $A$ ; we will denote this by  $L(p_0) \equiv A$ . This value  $u_0$  is known as the *utility* of the alternative  $A$ .

**For different selections of  $A_-$  and  $A_+$ , we get different numerical values of utility.** The above definition depends on the selection of two extreme alternatives  $A_-$  and  $A_+$ . What if we select another pair, e.g., we select  $A'_-$  and  $A'_+$  for which  $A'_- < A_- < A_+ < A'_+$ .

The fact that an alternative  $A$  has utility  $u$  with respect to the original pair  $(A_-, A_+)$  means that  $A$  is equivalent to a lottery in which:

- we get  $A_+$  with probability  $u$  and
- we get  $A_-$  with probability  $1 - u$ .

With respect to the pair  $(A'_-, A'_+)$ :

- the alternative  $A_-$  has some utility  $u'_-$ , and
- the alternative  $A_+$  has some utility  $u'_+$ .

This means that  $A$  is equivalent to a two-stage lottery, in which:

- first, we selected either  $A_+$  with probability  $u$  or  $A_-$  with probability  $1 - u$ , and then,
- if on the first stage we selected  $A_+$ , then on the second stage we select  $A'_+$  with the probability  $u'_+$  and  $A'_-$  with the remaining probability  $1 - u'_+$ , and
- if on the first stage we selected  $A_-$ , then on the second stage we select  $A'_+$  with the probability  $u'_-$  and  $A'_-$  with the remaining probability  $1 - u'_-$ .

As a result, we get either  $A'_+$  or  $A'_-$ , and the probability of selecting  $A'_+$  is equal to

$$u \cdot u'_+ + (1 - u) \cdot u'_- = u \cdot (u'_+ - u'_-) + u'_-$$

By definition, this probability is the utility  $u'$  of the alternative  $A$  with respect to the pairs  $(A'_-, A'_+)$ .

Thus, when we change the underlying pair of extreme events, the utility changes according to a linear formula  $u \rightarrow u' = c_1 \cdot u + c_0$ , for appropriate values  $c_0$  and  $c_1$ . In our case,  $c_1 = u'_+ - u'_-$  and  $c_0 = u'_-$ .

**Which action should we select: case when we know probabilities.**

Suppose that for each action  $a$ , we know possible consequences, and we know their probabilities  $p_1, \dots, p_n$ . Let  $u_i$  denote the utility of the  $i$ -th alternative.

By definition of utility, it means that each alternative  $i$  is equivalent to a lottery  $L(u_i)$  in which:

- we get  $A_+$  with probability  $u_i$ , and
- we get  $A_-$  with the remaining probability  $1 - u_i$ .

So, the analyzed action  $a$ , in which we get each alternative with probability  $p_i$ , is equivalent to a two-stage lottery in which:

- first, we select one of the alternatives, with the probability to select the  $i$ -th alternative being  $p_i$ , and
- second, depending on which alternative we select on the first stage, we select  $A_+$  with probability  $u_i$  and  $A_-$  with the remaining probability.

In this two-stage lottery, we end up with either  $A_+$  or  $A_-$ , and the probability  $p$  of getting  $A_+$  can be easily computed as

$$p = \sum_{i=1}^n p_i \cdot u_i.$$

Since the action  $a$  is equivalent to a lottery in which we  $A_+$  with probability  $p$  and  $A_-$  with the remaining probability, the utility of the action is equal to this value  $p$ .

Clearly, if an action is equivalent to a lottery  $L(p)$  with a higher probability  $p$  of getting a very good outcome  $A_+$ , then this action is better. So, among all possible actions, we should select the one with the largest possible utility  $p$ . From the mathematical viewpoint, the above formula for the utility is computing the expected value of the utility. Thus, we should select the action with the largest value of expected utility.

**Subjective probabilities.** What if we do not know the actual (objective) probabilities of different possible outcomes? In this case, we can get *subjective* probabilities of these outcomes, i.e., estimates of these probabilities.

To find a subjective probability of an event  $E$ , we can consider an auxiliary lottery lottery  $L(E)$  in which:

- we get  $A_+$  if  $E$  happens and
- we get  $A_-$  if the event  $E$  does not happen.

Then, we compare this auxiliary lottery with lotteries  $L(p)$  corresponding to different values  $p$ . Similarly to the utility case, we conclude that there is a threshold value  $p_0$  for which  $L(E) \equiv L(p_0)$ . This value  $p_0$  – which is actually the utility of the lottery  $L(E)$  – is then taken as the subjective probability of the event  $E$ .

**Important: utility is a non-linear function of money.** It is important to note that, according to the above arguments, a rational person should maximize expected *utility*, and *not* expected amount of money. The utility  $u(m)$  that one gets when getting amount of money  $m$  definitely depends on money, but, in general, the dependence is not linear and thus, the option with the largest value of expected utility does not necessarily lead to the largest value of expected money gain.

There is a good reason why utility non-linearly depends on money (see, e.g., [19]): it can be shown that if the dependence was linear, people would either not save for retirement at all, or save the maximal amount of money, while from the commonsense viewpoint, it makes sense to save some amount in between, this is what most people do. The mathematica behind this argument is very simple:

- if the dependence of utility on money is linear,
- then maximizing utility means maximizing a linear function of the money amount  $m$ , and
- for a linear function on an interval, maximum is always attained at one of the endpoints of this interval.

Empirical data shows that utility is approximately proportional to the square root of the money amount  $u(m) = c \cdot \sqrt{m}$ ; see, e.g., [17, 27].

**Taking time into account: first approximation.** In some decisions, we need to select between gains now or gains in the future. If for some decision, now we get utility  $u_0$ , in one year, we plan to get utility  $u_1$ , in two years, we plan to get utility  $u_2$ , etc., what is the equivalent utility of this decision? This utility  $u$  should depend on all the utilities  $u_0, u_1, u_2$ , etc.:  $u = u(u_0, u_1, u_2, \dots)$ .

In general, any continuous function can be approximated by a polynomial – for example, an analytical function can be expanded in Taylor series, and we can use the first few terms of this expansion to get a good approximation; this is a usual technique in physics; see, e.g., [10, 50]. Usually, linear terms are the largest, quadratic terms are next in size, etc. So, in the first approximation, we can keep only linear terms and thus get the following formula:

$$u = c + c_0 \cdot u_0 + c_1 \cdot u_1 + c_2 \cdot u_2 + \dots + c_t \cdot u_t + \dots$$

As we have mentioned earlier, utility is defined modulo a linear transformation. It makes sense to select this transformation in such a way that when all the utility is concentrated at the current moment of time, we get  $u = u_0$ . After this transformation, the above formula takes the following form:

$$u = u_0 + c_1 \cdot u_1 + c_2 \cdot u_2 + \dots + c_t \cdot u_t + \dots,$$

for some values  $c_t$ .

Clearly, getting some gain now is better than getting the same gain in a distant future, so we should have  $c_t < 1$  and we should have  $c_t$  decreasing with time  $t$ .

This formula is called *discounting*, because of the analogy with a similar process related to money, where, too, \$1 now is better than the same amount \$1 given  $t$  years from now. For money, there is a simple reason for this: we can invest this \$1 in the bank that every year provides us with  $i\%$  interest, and get  $(1 + i/100)^t$  dollars by year  $t$ . So, 1 dollar now is equivalent to  $(1 + i/100)^t$  dollars in year  $t$ . By the same logic, \$1 at year  $t$  is equivalent to  $q^t$  dollars now, where we denoted  $q \stackrel{\text{def}}{=} 1/(1 + i/100)$ . So, if in some scheme, we get an amount  $m_0$  now, an amount  $m_1$  next year, an amount  $m_2$  in 2 years, etc., then this scheme is equivalent to getting the following sum of money now:

$$m = m_0 + q \cdot m_1 + q^2 \cdot m_2 + \dots + q^t \cdot m_t + \dots$$

Because of this analogy, for utilities, sometimes, the same formula is used, with  $c_t = q^t$ :

$$u = u_0 + q \cdot u_1 + q^2 \cdot u_2 + \dots + q^t \cdot u_t + \dots,$$

for some appropriate value  $q$ .

**Taking time into account: towards a more accurate description.**

Strictly speaking, in the money case, the discounting coefficient  $q$  should depend not only on the interest rate now, but also on how stable we believe the bank to be, i.e., on the probability  $p_t$  that the bank will remain standing by year  $t$ . From this viewpoint, the expected overall amount of money – taking discounting into consideration – is equal to

$$E[m] = m_0 + p_1 \cdot q \cdot m_1 + p_2 \cdot q^2 \cdot m_2 + \dots + p_t \cdot q^t \cdot m_t + \dots$$

Similarly, when we apply discounting to utility, we need to take into account that we are never 100% sure about the future. We can describe this uncertainty about the future by a (possibly subjective) probability  $p_t$  that the analyzed scheme will still provide us with the expected utility  $u_t$  at time  $t$ . With these probabilities in mind, the equivalent expected utility is equal to

$$u = u_0 + p_1 \cdot c_1 \cdot u_1 + p_2 \cdot c_2 \cdot u_2 + \dots + p_t \cdot c_t \cdot u_t + \dots$$

**Where do coefficients  $c_t$  come from?** As we have mentioned earlier, a rational reformulation of our preferences is that we want to maximize the expected utility.

This general idea can explain how we select the coefficients  $c_t$ :

- if we are happy now, i.e., if our value  $u_0$  is large, and we do not expect such happiness in the future, then we should de-emphasize the future-related part of the above formula and select smaller values of  $c_t$ ;
- on the other hand, if we are unhappy now, i.e., if our value  $u_0$  is small, but we expect to be happier in the future, then we should emphasize the future more, i.e., select larger values of  $c_t$ .



**How to make decisions under uncertainty?** In many practical situations, we do not know the exact consequences of each action  $a$  – and thus, we do not know the exact value of the corresponding expected utility  $u(a)$ . At best, for each action  $a$ , we know the interval  $[\underline{u}(a), \bar{u}(a)]$  of possible values of  $u(a)$ . Which action should we then select?

In such situations, the traditional decision theory recommends to choose some parameter  $\alpha \in [0, 1]$  and then select an action for which the expression  $u(a) = \alpha \cdot \bar{u}(a) + (1 - \alpha) \cdot \underline{u}(a)$  is the largest possible. This idea was first proposed by a Nobelist Leo Hurwicz [15] and is thus known as Hurwicz optimism-pessimism criterion. Indeed:

- if  $\alpha = 1$ , then  $u(a) = \bar{u}(a)$ , so we base our decisions on the most optimistic case and ignore the possibility of all other situations;
- if  $\alpha = 0$ , then  $u(a) = \underline{u}(a)$ , so we base our decisions on the most pessimistic case and ignore the possibility of all other situations;
- if  $\alpha$  is between 0 and 1, then take both optimistic and pessimistic estimates into account.

**What is the best group decision?** Let us assume that we have several people who want to make a joint decision – e.g., people from a city deciding whether to build a stadium or a museum. Let  $u_i^{(0)}$  be the initial utility of the  $i$ -th person. Then, reasonable arguments (see above references) show that the best group decision is to select an action for which the product of the utility increases  $\prod_{i=1}^n (u_i - u_i^{(0)})$  is the largest possible. This idea was first proposed and justified by a Nobelist John Nash in [36]; it is known as *Nash's bargaining solution*.

**How to take empathy into account.** People's preferences are not determined only by their own gains or losses, they are also affected by how other people feel. Thus, the actual utility  $u_i$  of a person  $i$  is not just equal to the “self-utility”  $s_i$ , i.e., to the utility this person would get if we only take into account his or her gains and losses, but also on the utilities  $u_j$  of other people:  $u_i = u_i(s_i, u_1, u_2, \dots)$ . Similarly to the case of discounting, in the first approximation, we can restrict ourselves to linear terms in this dependence, i.e., take

$$u_i = a_0 + c_{ii} \cdot s_i + \sum_{j=1}^n c_{ij} \cdot u_j.$$

Similarly to the discounting case, we can apply a linear transformation to this utility to make sure that in the absence of other people, we have  $u_i = s_i$ . This leads us to the following formula:

$$u_i = s_i + \sum_{j=1}^n c_{ij} \cdot u_j.$$

Here:

- when  $c_{ij} > 0$ , this means that the happier the  $j$ -th person, the happier the  $i$ -th person will be; this is the case of empathy (sympathy);
- when  $c_{ij} < 0$ , this means that the happier the  $j$ -th person, the more unhappy the  $i$ -th person will be; this is the case of antipathy.

**Where do coefficients  $c_{ij}$  come from?** Similarly to the coefficients  $c_t$  used in the discounting formula, it is natural to conclude that the coefficients  $c_{ij}$  (that describe our sympathy or antipathy) come from our desire to maximize our utility.

From this viewpoint, it is reasonable to expect that if one of the people we know is very happy, i.e., has a large utility  $u_j$ , then we should have a larger  $c_{ij}$  with respect to this person – i.e., we should empathize more with him/her. This is indeed a known phenomenon (see, e.g., [39], pp. 233–234), e.g.:

- there is a public fascination with (supposedly) very happy real-life princes and princesses, especially when are involved in happy activities such as marriage;
- there is a general phenomenon that “everybody loves a lover”.

*Comment.* Now that we finished describing the main ideas of the traditional decision making theory, let us explain how these ideas can explain different aspects of our seemingly irrational behavior.

### 3 Non-Linear Dependence of Utility on Money Already Explains Some of the Seemingly Counterintuitive Observations About Human Decision Making

Let us show that the non-linear dependence of utility  $u$  on the money amount can explain some of examples that are sometimes presented as examples of irrational behavior.

**Observed phenomenon.** For example, [38] and [49] cite the results of the 1990 experiment performed by R. H. Thaler. In this experiment, a person had to select between the following two options:

- in Option A, this person wins \$4 with probability  $8/9$ , and
- in Option B, the person wins \$40 with probability  $1/9$ .

In this experiment, most people selected Option A. From the viewpoint of expected money gain, this may sounds counterintuitive, since:

- for Option A, the expected gain is  $89 \cdot \$4 \approx \$3.56$ , while

- the expected money gain in Option B, is  $19 \cdot \$40 \approx \$4.44$ , much larger.

**Our explanation.** Let us show that this result is clearly rational if we take into account that the utility is proportional to the square root of money amount. Thus:

- for Option A, the expected utility is  $89 \cdot \sqrt{4} \approx 1.78$ , while
- for Option B, the expected utility is equal to  $19 \cdot \sqrt{40} \approx 0.70$  – much much smaller.

*Comment.* Interestingly, not all the people preferred Option A – only about 2/3 did. Similarly, when people were asked instead of compare the expected money gains, also not everyone correctly concluded that Option B is better – only about 2/3 did. These proportion can also be explained – we will provide this explanation later.

## 4 Why Some Billionaires Are Liked And Some Are Not

**Observed phenomenon.** The public attitude to rich people varies: some are liked, some are not; see, e.g., [26]. At first glance, this may seem like a psychological mystery.

**Our explanation.** From the viewpoint of utilities, the explanation is simple:

- if a person became rich by benefiting many people in the process – e.g., by producing movies that many of us enjoy, or by producing software products that many people use – what is not to like?
- on the other hand, if a person became rich by hurting others – e.g., by a drastic increase in prices of medicine, this may be perfectly legal, but what is there to like?

## 5 Why Happy People Are Less Worried About the Future

**Observed phenomenon.** It is known (see, e.g., [39], pp. 233–234), that:

- when we are happier, we think less about the future consequences of our actions, while
- when we are unhappy, we pay more attention to the future consequences.

**Our explanation.** This is in line with the fact that we can change the values  $c_t$  describing discounting so as to increase the overall utility value.

*Comment.* This phenomenon of happy people ignoring the future can, by the way, lead to bad (= irrational) behavior, when temporary happiness makes us jump into unhealthy relationships or do unhealthy things like taking drugs – without thinking about long-term consequences.

## 6 Joy and Fear

**Observed phenomenon.** Kids like to be repeatedly thrown up in the air and caught. Clearly, they experience fear, a negative emotion, when they are up in the air, so why? This phenomenon is not limited to kids: in general, fear and joy are often interrelated; see, e.g., [43], p. 24, and [52], pp. 198 and 405. Why?

**Our explanation.** Let us show that this phenomenon can be explained if we take into account that we can modify the coefficients  $c_t$  (that describe our attitude to the future) so as to achieve larger utility values. So:

- when the kids are in the air, they experience a negative feeling of fear, but since they expect to be caught in the nearest future, they place higher weight  $c_1$  on the resulting future joy – thus feeling overall happier;
- on the other hand, when the kids are safely caught and experience related positive emotions, they place a smaller weight on the future fear and thus, do not let this future fear to ruin their current happiness.

## 7 Why Giving Makes People Happier?

**Phenomenon.** According to studies cited in [44], a person who is giving measures their happiness higher after giving. How can we explain this?

**Our explanation.** Let us assume that:

- this person had the original amount of money  $m_1$ , so this person's self-utility was  $s_1 = \sqrt{m_1}$ , and
- the recipient had the original amount of money  $m_2 < m_1$ , so this person's utility was  $u_2 = \sqrt{m_2}$ .

In this case, the actual utility of the giver is

$$u_1 = s_1 + c_{12} \cdot u_2 = \sqrt{m_1} + c_{12} \cdot \sqrt{m_2}.$$

Here, we assume that  $c_{12} > 0$  – one does not give money to whose he or she hates.

Once an amount  $\delta$  passes from the giver to the receiver, then:

- the giver will have the amount of money  $m_1 - \delta$ , so the giver's self-utility becomes  $s_1 = \sqrt{m_1 - \delta}$ , and
- the recipient will have the amount of money  $m_2 + \delta$ , so the recipient's utility will be  $u_2 = \sqrt{m_2 + \delta}$ .

In this case, the actual utility of the giver will become

$$u_1 = s_1 + c_{12} \cdot u_2 = \sqrt{m_1 - \delta} + c_{12} \cdot \sqrt{m_2 + \delta}.$$

If we differentiate this expression with respect to  $\delta$ , we get

$$\frac{da_1}{d\delta} = -\frac{1}{2\sqrt{m_1 - \delta}} + c_{12} \cdot \frac{1}{2\sqrt{m_2 + \delta}}.$$

For small  $\delta \ll m_1$ , this derivative becomes equal to

$$\frac{da_1}{d\delta} = -\frac{1}{2\sqrt{m_1}} + c_{12} \cdot \frac{1}{2\sqrt{m_2}}.$$

This derivative is positive – i.e., the utility of the giver indeed increases with giving – if

$$c_{12} \cdot \frac{1}{\sqrt{m_2}} > \frac{1}{\sqrt{m_1}},$$

i.e., equivalently, if  $m_2 < c_{12}^2 \cdot m_1$  – a very realistic assumption.

## 8 Probabilities Usually Come from Observations

According to the above description of decision theory, to make a decision, we need to have some information about the corresponding probabilities. In general, probabilities usually come from observations:

- We know that the probability of a coin falling heads is 1/2, since we tested coins many times, and indeed, it fell heads in approximately half of the times.
- When the weather forecast says that today there is 30% chance of rain, this usually means that it rained in about 30% of similar situations in the past.

Let us show how this simple observation can explain some observed phenomena in human decision making.

## 9 Why Observing a Good Deed Changes Our Attitude Towards Future Events

**Observed phenomenon.** In experiments cited in [8], people were given two choices:

- a certain amount of money  $m$  after some time  $t$ , or
- a smaller amount  $s$  now.

The smaller amount was selected in such a way that:

- most people preferred it to the future gain, but
- a further decrease in this amount would cause them to prefer the future amount instead.

Because of this selection of the amount  $s$ , people consistently selected the current-gain alternative to the future-gain one.

After this, the participants saw a good deed. When the same experiment was repeated, most people selected the future gain.

**Our explanation.** In utility terms, people select the current value  $s$  if the corresponding utility  $u(s) = \sqrt{s}$  is larger than the discounted future utility  $p_t \cdot c_t \cdot \sqrt{m}$ . In these terms, the above selection of the smaller amount  $s$  means that  $s$  was selected in such a way that  $\sqrt{s} > p_t \cdot c_t \cdot \sqrt{m}$  but  $\sqrt{s} \approx p_t \cdot c_t \cdot \sqrt{m}$ .

In this formula, the probability  $p_t$  is the probability that the promise will be followed at a future moment  $t$ . According to the above general idea of how we estimate probability, we estimate  $p_t$  by dividing:

- the number  $K$  of cases when people were good in following their promises – implicit or explicit – by
- the overall number  $P$  of cases when someone made a promise (implicit or explicit) in the past:

$$p_t \approx \frac{K}{P}.$$

After we observe a good deed, the overall number of cases increases by 1, and the number of good cases increases by 1, so the new estimate is  $p'_t = \frac{K+1}{P+1}$ .

One can easily check that

$$\frac{K+1}{P+1} > \frac{K}{P}.$$

Indeed, if we bring both fraction to the common denominator and delete the common terms  $K \cdot P$ , we see that the desired inequality is equivalent to the true inequality  $P > K$ . So,  $p'_t > p_t$ , and thus, the new utility for the future options becomes larger than the utility of selecting the current-gain option:

$$p'_t \cdot c_t \cdot \sqrt{m} > p_t \cdot c_t \cdot \sqrt{m} \approx \sqrt{m}.$$

This explains why, after observing a good deed, we change our selection to the future-gain option.

## 10 Why We Are Often Too Optimistic or Too Pessimistic

**Phenomenon.** People tend to make (rash) conclusions based on a few facts – not taking into account that results based on a small sample are rarely statistically significant, they can usually be explained by randomness. For example (see, e.g., [17], Chapter 17):

- when we hear that several depressed kids felt better after drinking an energy drink makes us conclude that energy drinks can cure depression, even if this is not statistically confirmed;
- when we see several examples of smart women marrying below their intelligence level, we naturally make a conclusion that, on average, such women tend to marry less intelligent men, etc.

Similarly to how a few positive facts (or even a single positive fact) make us more optimistic, a few negative facts (or even a single negative fact) make us more pessimistic (e.g., thinking that “all men are scoundrels” after encountering an unfaithful boyfriend).

**Our explanation.** Similarly to the previous section, we estimate the probability based on the evidence. As a result, even a single fact changes our probability and thus, makes us, correspondingly, more optimistic or more pessimistic.

Strictly speaking, we should have waited for a sample large enough to allow us to make statistically significant conclusions. However, we often *have* to make decisions based on available information, and in such cases, estimating probability as frequency is probably the best we can do.

## 11 What If We Have No Information About Probabilities of Different Alternatives: Laplace Indeterminacy Principle

**Description of the situation.** According to the general decision making strategy, to make a decision, for each alternative, we need to know the values of the utility and the probability. In practice, however, we often do not know these values.

Let us first consider the simplest case when we have no information at all about the utilities or probabilities of different alternative. What shall we do in this situation?

**Natural idea.** If we have  $n$  alternatives, and we have no reasons to believe that some alternatives are more probable than others, then it makes sense to

assign, to each of these alternatives, the same probability  $1/n$ . This idea – first formulated explicitly by Laplace – is known as *Laplace Intederminacy Principle*; see, e.g., [16].

Similarly, if we have no information about the utilities of two different alternatives, and we have no reason to believe that one of these situations is preferable, it makes sense to assign equal utility to both situations.

It turns out that this principle explains several examples of seemingly irrational behavior.

## 12 Why People Like to Keep Options Open

**Phenomenon.** In many practical cases, people tend to select a decision that keeps several options open; see, e.g., [2, 46] and references therein.

- In some cases, it is a good idea – it prevents us from selecting a not-so-optimal option before exploring all the possibilities.
- However, in many other practical situations, this tendency leads to bad decisions: e.g., when a person who honestly wants to settle down keeps dating several people year after year instead of selecting a future partner – as a result of which, often, these potential partners give up on this person and he/she loses his chances.

**Our explanation.** If all we know is the number of options remaining about each decision, and we do not have any information about which possible options are better and which are worse, it is natural to assume that each of the possible options has the same probability to be good. In this case, the probability that a decision will lead to a good option is simply proportional to the number of options remaining after this decision.

From this viewpoint, it is reasonable to select a decision that leaves the largest amount of options open.

*Comment.* The idea of keeping as many options open as possible may be sometimes a bad strategy for human decision making, but, interestingly, this idea of maximizing freedom of choice has led to many efficient algorithms; see, e.g., [22, 24, 30, 31]. A similar idea is also actively used in computer vision, under the name of David Marr’s *Principle of Least Commitment*.

## 13 Why Power of Positive Thinking

**Phenomenon.** It is known that more optimistic people are, on average, more successful and even live longer; see, e.g., [6, 12, 13, 35]. It looks like optimism – i.e., in decision-making terms, selecting a larger value of the Hurwicz optimism-pessimism parameter  $\alpha$  – helps us make better decisions. But why?

**Our explanation.** Let us consider the simplest case, when for each action, the utility  $u$  of the result depends on a single parameter  $x$ :  $u = u(x)$ . (For



several parameters, as one can easily check, the result will be similar.) We consider the case of interval uncertainty, when we do not know the actual value  $x$  of this parameter, we only know the interval  $[\underline{x}, \bar{x}]$  of possible values of this parameter. By introducing a midpoint  $\tilde{x} \stackrel{\text{def}}{=} \frac{\underline{x} + \bar{x}}{2}$  and half-width  $\Delta \stackrel{\text{def}}{=} \frac{\bar{x} - \underline{x}}{2}$ , we can represent this interval in a symmetric form  $[\tilde{x} - \Delta, \tilde{x} + \Delta]$ . In this case, a general point  $x$  on this interval has the form  $x = \tilde{x} + \Delta x$ , where  $\Delta x$  can take any value on the interval  $[-\Delta, \Delta]$ .

We have no reason to believe that some values  $\Delta x$  from the interval  $[-\Delta, \Delta]$  are more probable and some are less probable. Thus, in line with the Laplace Indeterminacy Principle, we can assume that we have a uniform distribution on this interval. Under this distribution, what is the expected utility value?

In general, as we have mentioned earlier, we can expand the dependence  $u(x) = u(x_0 + \Delta x)$  into a power series and keep the first few terms in this expansion:

$$u(x_0 + \Delta x) = u(x_0) + c_1 \cdot \Delta x + c_2 \cdot (\Delta x)^2.$$

In the first approximation, it makes sense to keep only linear terms, i.e., assume that

$$u(x_0 + \Delta x) = u(x_0) + c_1 \cdot \Delta x.$$

In this case, as one can easily check:

- the smallest value  $\underline{u}$  of utility on this interval is equal to

$$\underline{u} = u(x_0) - |c_1| \cdot \Delta;$$

- the largest value  $\bar{u}$  of utility on this interval is equal to

$$\bar{u} = u(x_0) + |c_1| \cdot \Delta; \text{ and}$$

- the average value  $u$  of utility on this interval is equal to

$$u = u(x_0).$$

In this case,  $u = 0.5 \cdot \bar{u} + 0.5 \cdot \underline{u}$ , so the value  $\alpha = 0.5$  is the most adequate one.

In this approximation, we only took into account linear terms, and ignored quadratic and higher order terms. However, linear terms are dominant only if they are non-zero. If the linear term is 0, we need to take at least quadratic terms into account. In this case, the simplest approximation would be if we only term quadratic terms into account.

And the linear term can be zero: e.g., if we are close to the maximum or to the minimum of the utility function. If we are at the maximum, then we have

$$u(x_0 + \Delta x) = u(x_0) - |c_2| \cdot (\Delta x)^2.$$

In this case:

- the smallest value  $\underline{u}$  of utility on this interval is equal to

$$\underline{u} = u(x_0) - |c_2| \cdot \Delta^2;$$

- the largest value  $\bar{u}$  of utility on this interval is equal to

$$\bar{u} = u(x_0); \text{ and}$$

- the average value  $u$  of utility on this interval is equal to

$$u = u(x_0) - (1/3) \cdot |c_2| \cdot \Delta^2.$$

In this case,  $u = (2/3) \cdot \bar{u} + (1/3) \cdot \underline{u}$ , so the most adequate value  $\alpha$  is  $\alpha = 2/3$ . This value is optimistic in the sense that it is closer to perfect optimism ( $\alpha = 1$ ) than to perfect pessimism ( $\alpha = 0$ ). One can check that if we not exactly at the maximum, but close to the maximum, then also the optimal  $\alpha$  is an optimistic one.

Similarly, if we are at the minimum, then we have

$$u(x_0 + \Delta x) = u(x_0) + |c_2| \cdot (\Delta x)^2.$$

In this case:

- the smallest value  $\underline{u}$  of utility on this interval is equal to

$$\underline{u} = u(x_0);$$

- the largest value  $\bar{u}$  of utility on this interval is equal to

$$\bar{u} = u(x_0) + |c_2| \cdot \Delta^2; \text{ and}$$

- the average value  $u$  of utility on this interval is equal to

$$u = u(x_0) + (1/3) \cdot |c_2| \cdot \Delta^2.$$

In this case,  $u = (1/3) \cdot \bar{u} + (2/3) \cdot \underline{u}$ , so the most adequate value  $\alpha$  is  $\alpha = 1/3$ . This value is pessimistic in the sense that it is closer to perfect pessimism ( $\alpha = 0$ ) than to perfect optimism ( $\alpha = 1$ ). One can check that if we not exactly at the minimum, but close to the minimum, then also the optimal  $\alpha$  is an pessimistic one.

Based on these three cases, we can make the following conclusion:

- if we live in a reasonably happy world, close to the maximum of the utility function, then a more adequate behavior is to be optimistic; and
- if we live in an unhappy world, close to the minimum of the utility function, then a more adequate behavior is to be pessimistic.

Honestly, at present, there are many things about our world to be unhappy about, but overall, we live much better than any time before: we live longer lives, we eat better, we communicate better, we fight each other less, etc. In comparison with how people lived centuries and millennia ago, we live in a reasonably happy world. According to the above general conclusion, in this world, it makes sense to be more on the optimistic side. This explains why optimists are more successful in our world.

*Comment.* On the other hand, for people living in ancient time, when lifespan was short and dangers were everywhere, by similar logic, it was more reasonable to be pessimistic. This difference is easy to explain:

- If I walk a street and I see a shadow of some big object passing by, I am not worried: there is a probability that it is something not good, but most probably it is just a big truck.
- On the other hand, for a primitive man, a big shadow probably meant either a big predator that could eat him or just a big dangerous animal like an elephant or a rhinoceros that could kill him.

## 14 Yet Another Explanation for Nash's Bargaining Solution

**Formulation of the problem: reminder.** In a group decision making situation, an option that leads to utilities  $(u_1, \dots, u_n)$  has a potential to be accepted only if each participant will gain something by selecting this option, i.e., only if each person's utility will increase (or at least not decrease):  $u_i \geq u_i^{(0)}$ .

There are usually several options with this property. Which one should we choose?

**Idea.** We want an option which is the best. Ideally, this means that it should be better than all other options, i.e., that the probability  $p$  that the selected option is better than a randomly selected option should be equal to 1. Of course, if we understand "better" as "better for each participant", then it is not possible to select such a "best" option: indeed, usually, there is always a trade-off, we can always slightly increase one participant's gain by slightly decreasing other's gain.

Since we cannot have  $p = 1$ , a natural idea is to select the option which is the closest to this idea, i.e., for which the probability  $p$  is as large as possible. How can we estimate this probability?

We have no reason to conclude that some options are more probable than others, so, according to the Laplace Indeterminacy Principle, we consider them all equally probable. In other words, we assume that there is a uniform distribution on the set of all possible tuples  $(u_1, \dots, u_n)$ . In the  $n$ -dimensional uniform distribution, for each set, the probability that a randomly selected tuple will belong to this set is proportional to the set's volume. So, selecting the option

with the highest probability  $p$  is equivalent to selecting the option for which the volume of the set of all “worse” options is the largest possible.

For each selection  $(u_1, \dots, u_n)$ , an option  $(w_1, \dots, w_n)$  is “worse” if for each  $i$ , we have  $u_i^{(0)} \leq w_i \leq u_i$ . Thus, the set of all such options is a box

$$\left[ u_1^{(0)}, u_1 \right] \times \dots \times \left[ u_n^{(0)}, u_n \right]$$

of volume  $(u_1 - u_1^{(0)}) \dots (u_n - u_n^{(0)})$ . Maximizing this volume is exactly what Nash’s bargaining solution is about.

So indeed, the Laplace Indeterminacy Principle leads to a new explanation for the Nash’s bargaining solution.

## 15 How People Behave in Prisoner Dilemma Situations

**What is Prisoner’s Dilemma.** In this situation, each of two people needs to make (or not make) a certain decision:

- if both people make this decision, they both suffer a medium loss;
- if neither of the two makes the decision, they both suffer a small loss;
- if one makes the decision and the other doesn’t, then the one who made the decision does not suffer any loss at all, but the other one suffers a big loss.

The name comes from a model situations in which:

- two gang members are arrested for a minor crime (e.g., possession of a small amount of heroin), but
- the police suspects that they have committed a more serious one (e.g., that they were involved in serious drug trafficking).

The two suspects are kept in different cells, they cannot communicate with each other. Each of them needs to make a decision whether he confesses about the serious crime or not.

- If neither of them confesses, they will get a small jail sentence.
- If only one of them confesses to a more serious crime, then this person will get a special deal and avoid jail time altogether, while the other one will get a full term for this crime.
- If both confess, they get a medium jail term – since the judge will take into account that they collaborated with the investigators.

In this situation, it is not immediately clear which of the two possible strategies is better – to confess or not to confess.

**How do people behave in such situations.** In the simulated situations, about a half of the people decided to confess; see, e.g., [38] and references therein.

**Our explanation.** Since it is not clear which of the two strategies is better, according to the Laplace Indeterminacy Principle, it is reasonable to assign the exact same utility to both strategies and thus, select each of them with an equal probability.

*Comment.* Another application of the Laplace Indeterminacy Principle is presented in the next section.

## 16 What If We Only Know the Order?

**Description of the situation.** According to decision theory, to properly make a decision, we need to know the utilities and probabilities of different alternatives. In many practical situations, however, we do not know the utilities and/or probabilities.

In one of the previous sections, we considered the case when we have no information at all. Let us now consider the next simplest case, when we only know which alternatives have higher utility or higher probability.

In such situations, how can we make a decision?

**Analysis of the problem: simplest case of two unknown utility values.**

Let us start with the case when we have two alternatives, and we know that one of them is preferable to the other. In terms of utility, this means that we know that the utility  $u_1$  of the first alternative is larger than the utility  $u_2$  of the second alternative, but we do not have any other information about their utilities.

In general, we know that – at least in the original scale – a utility can be equal to any value between 0 and 1. So, in principle, we can have different pairs  $(u_1, u_2)$ , with only one restriction – that  $u_1 > u_2$ . In line with the general decision theory, it is reasonable to select an alternative with the higher value of expected utility.

To find the expected utility, we need to know the probability of different pairs  $(u_1, u_2)$ . In our case, we have no information about these probabilities. Thus, we have no reason to conclude that one of these pairs is more or less probable than others. So, according to the Laplace Indeterminacy Principle, we should assign equal probability to all such pairs. In other words, we should consider a uniform distribution on the set of all possible pairs  $(u_1, u_2)$  for which  $0 \leq u_2 < u_1 \leq 1$ .

It is relatively easy to compute the expected values  $E[u_i]$  of  $u_1$  and  $u_2$  over this distribution: they are  $E[u_1] = 2/3$  and  $E[u_2] = 1/3$ , with the larger value exactly twice larger than the smaller one; see, e.g., [1, 3, 7, 20, 21].

**A similar case of two unknown probability values.** As we have mentioned earlier, subjective probability of an event  $E$  can be described in terms of the utility of an alternative in which we get some gain if this event happens.

If we start with a situation in which we have two unknown probabilities about which the only thing we know is that the first probability is larger than the second one  $p_1 > p_2$ , then for the related alternatives, we get the exact same relation between their utilities  $u_1 > u_2$ .

We already know that for utilities, this implies that  $u_1 = 2u_2$ . Thus, we can conclude that the first probability is also twice larger than the second one:

$$p_1 = 2p_2.$$

**The simplest case of three alternatives.** Let us now assume that we have three alternatives, and we have some information about the order between these alternatives. The simplest case is when we only have the order between two of the three alternatives. Without losing generality, we can say that we know that the first alternative is better than the second one, but we do not know how good any of the first two alternatives are compared to the third one.

In terms of the utilities, this means that  $u_1 > u_2$  and that we know nothing about  $u_3$ . In this case, it is also reasonable:

- to consider a uniform distribution on the set of all the triples

$$(u_1, u_2, u_3) \in [0, 1]^3$$

that satisfy the above inequality, and

- to take the expected values of these three utilities as our estimates for their utilities.

In this case, the variable  $u_3$  is independent on the two others. So, for  $u_3$ , we simply get a uniform distribution on the interval  $[0, 1]$  for which the mean value is, clearly, the midpoint  $1/2$  of this interval. For the pair  $(u_1, u_2)$ , we already know the mean values: they are  $2/3$  and  $1/3$ . Thus, in this case, reasonable estimates for the utilities are  $u_1 = 2/3$ ,  $u_2 = 1/3$ , and  $u_3 = 1/2$ .

## 17 Why Twice Larger?

**General description of the phenomenon.** In many practical situations, intuitively, we know that one value should be larger than another one – be it price or probability or frequency. Interestingly, in many such situations related to human behavior, the larger value is (almost exactly) twice larger than the smaller one.

Let us present a few examples of such “twice larger” effect, both in economics and in human behavior in general.

We will then show that all these examples can be explained by the above what-if-we-only-know-the-order formulas.

**“Twice larger” effects in economics.** It is well known that in a market economy, each price is largely determined by the relation between supply and demand.

- It is difficult to manipulate demand.
- However, in a controlled experiment, we can easily manipulate supply, all the way from scarcity to abundance.

Interestingly, in both extreme situations, we observe the “twice larger” effect.

**Case of scarcity.** When people learn that some commodity has become (or will soon become) scarce, they buy more. Interesting, on average, they buy twice more [4, 18].

**Case of abundance.** Stores often create artificial abundance-type situations, when they have sales. At a sale, some items are sold at a highly discounted prices. Sometimes, the store even give them out for free. Usually, there is a limitation to how many such items a customer can purchase: e.g., only one item per customer.

The experiment considered the case of two items:

- an item of normal quality which is given free, and
- another item of somewhat better quality which is sold at a highly discounted price.

It turned out that in this experiment, the number of people who selected the free item was twice larger than the number of people who selected the discounted item [2, 45].

**How to best distribute charity?** The above example bring us closer to another social behavior in which we give away items for free: charity. Charitable organizations:

- give some funds directly to people in dire need, without asking for anything in return, while
- some other funds are given in the form of no-interest (or low-interest) loans, to help people start businesses or pay for education.

It is generally accepted that since it is more important to help people in dire need, the portion allocated to helping these people should be larger than the portion given as no-interest or low-interest loans.

Interestingly, an empirical recommendation is that  $2/3$  of the overall funds should be given to people in dire need, while  $1/3$  distributed as no-interest or low-interest loans (see, e.g., [5], Chapters 17 and 18).

**“Twice larger” effect in human behavior in general.** The “twice larger” effect is not limited to economics or charity, it can be observed in many other examples of human behavior. It turns out that this effect can be observed in many cases when we show a preference to some people.

- This preference can be due to some objective characteristics of these people, characteristics which are not related to us – e.g., their attractiveness.
- This preference can also be due to some characteristics which are related to us:
  - it could be that they are, in some aspect, similar to us,
  - it could be that they did something good for us in the past.

It turns out that in all these situations, we observe the same “twice larger” phenomenon:

**The effect of attractiveness.** We often treat attractive people better than non-attractive ones. Interestingly:

- Attractive candidates receive, on average, about twice the number of votes as unattractive ones [4, 9].
- Attractive defendants in criminal trials were twice as likely to avoid jail as unattractive ones [4, 47].
- Attractive victims were awarded, on average, twice bigger compensations by the jurors than unattractive ones [4, 25].

**The effect of similarity.** People are (naturally) more trustful to those who are similar to them in some aspect. Interestingly, when requests to fill a survey came from a person with a similar name, the number of responses grew not just somewhat, but twice [4, 14].

**The effect of reciprocity.** It is natural to expect that we benefit more those who gave us some favor. Interestingly, people’s donation to those who gave some them some favor was not just larger – it was, on average, twice larger [4, 42].

*Comment.* Two additional examples of the “twice larger” effect were given earlier, when we compared maximizing the expected amount of money with maximizing the expected utility.

**Our explanation.** In all the above cases, we have two alternatives. We know that one of them has a higher utility and/or higher probability, and we do not know much else.

In this case, according to the above what-if-we-only-know-the-order formulas, the larger utility (probability) should be twice larger than the smaller one – and this is exactly what we observe in all the above examples.



## 18 Why Decoy Effect

**What is decoy effect.** If we have two alternatives  $A$  and  $B$  between which there is no clear choice, then, in the absence of any other information, both alternative should be selected with equal probability  $1/2$ .

Suppose now that we have added a third alternative  $C$  (called a *decoy*) which is worse than  $A$  and incomparable with  $B$ . At first glance, since the alternative  $C$  is clearly worse than  $A$  and will, thus, never be selected, the actual selection is still between  $A$  and  $B$ . And since we got no new information about  $A$  and  $B$ , we should expect that a user still selects  $A$  and  $B$  with equal probability.

Interestingly, this *not* what happens: in the presence of a decoy, more people start preferring the option  $A$ .

Companies use this idea to nudge us to buy a product they want us to buy. It works even with people who are somewhat knowledgeable about economics. For example, the *Economist* journal, to entice subscribers to select a more expensive option  $A$  (\$129 for print and online versions) in comparison with a cheaper version  $B$  (\$59 for online only), added a decoy option  $C$  of \$129 for print only – and it did work; see, e.g., [2, 29, 51].

It is worth mentioning that the decoy effect is not only used for bad purposes: it can be used to nudge people to take better care of their health – e.g., by making healthier life choices and/or by encouraging them to undergo regular health screening; see, e.g., [48].

However, irrespective of whether selecting  $A$  is a good choice or a bad choice, the preferred selection of  $A$  is *not* rational – as we can easily see if we thoroughly analyze all possible consequences of each option. So how can we explain this seemingly irrational behavior?

**Our explanation.** Here, we have three alternatives:  $A$ ,  $B$ , and  $C$ . The only information that we have about these three alternatives is that  $A > C$ . This is exactly the case that we analyzed before. In this case, the above what-if-we-only-know-the-order formulas lead to  $u(A) = 2/3$  and  $u(B) = 1/2$  and thus, to

$$u(A) > u(B).$$

This explains the decoy effect.

*Comment.* Of course, this explanation works only in one case: when the only information that we use to make our decision is that  $A > C$ . If instead we thoroughly analyze the three alternatives, we will conclude that the preferences between  $A$  and  $B$  should not be affected by the presence of  $C$ , and the decoy effect will disappear. And indeed, as shown in [29], the more people reason and the less they rely on their intuition, the smaller the observed decoy effect.

## 19 Seven Plus Minus Two Principle

**Seven plus minus two principle: a brief reminder.** Our ability to store and process information is limited. So, usually, we can only handle between

5 and 9 objects in our mind, and when we are given more objects, we classify them into 5 to 9 groups; see, e.g., [34, 41]. The exact value from 5 to 9 depends on the individual:

- some people classify everything into 5 groups,
- some people classify everything into 6 groups,
- . . . , and
- some people classify everything into 9 groups.

**Possible consequences.** Suppose that we have an event that happens, on average, in  $1/9$  of all the cases. Then:

- If a person classifies everything – objects, events, etc. – into 7 groups, then this particular event will, most probably, be ignored.
- On the other hand, a person who divides everything into 9 groups will take this rare event into account.

**What we do.** Let us show that this simple law can explain several quantitative features of human decision making.

## 20 Why 10% Increase/Decrease?

**Description of the phenomenon.** In many social situations:

- a 10% increase or decrease is socially acceptable, while
- a larger increase or decrease is not socially acceptable.

Let us give two examples.

**When unemployed people accept a new job with a smaller pay?** It turns out that unemployed people are willing to take a new job if their pay decrease does not exceed 10%. When the pay decrease is larger, they will be very reluctant to agree to a new job; see, e.g., [17], p. 291

**When people cheat?** In many situations such as filing insurance claims, many otherwise honest people cheat a little bit, somewhat inflating the amount of their damage. Interestingly, this cheating usually does not exceed 10%; see, e.g., [2, 32, 33].

**Our explanation.** A natural idea is that people will be eager to select a new job if the decrease in pay is negligible – both for the worker him/herself and for all his friends and relatives.

Among friends and relatives, there are, in general, people with different value of the corresponding parameter  $n$ , from the smallest possible value 5 to the largest possible value 9.

- For some of them, the decrease is negligible if it is below  $1/5$ .
- For others, the decrease is negligible if it is below  $1/9$ .

To make sure that the decrease in pay is negligible for all the friends and relatives, this decrease must be smaller than the smallest of these fractions – i.e., smaller than  $1/9 \approx 10\%$ .

This conclusion is perfectly in line with the above observation, according to which, when people accept the new job with a decreased pay, they new salaries is usually about 10% smaller.

Similarly, a normal person may cheat as long as the increase can be viewed as negligible by him/her and all his/her friends and family. In line with the above, this means that the cheating amount should not exceed 10% – and this is indeed the usual amount of cheating increase.

## 21 Why 5:1 Good Events to Bad Events Ratio is a Threshold for Predicting Couple’s Stability

**Empirical fact.** According to [17], p. 302, it is possible to predict whether a relationship will remain stable or not by observing the couple’s behavior:

- if there are at least 5 times more events when interaction was good than events with bad interaction, then the relationship will most probably remain stable;
- on the other hand, if the ratio of bad to good interactions exceeds  $1/5$ , the couple has a high probability of breaking up.

*Comment.* Of course, 5 is not an exact threshold here, it is an approximate estimate separating stable from unstable couples.

**Our explanation.** How can we explain this empirical fact? Bad things happen, misunderstanding happen, so staying together is work. If at least one partner is willing to ignore the bad things, to work on staying together, to adjust if needed, the couple has a good chance of remaining stable.

From the viewpoint of the seven plus minus two law, for each partner, the fact that bad interactions can be ignored (in the large scheme of being) means the proportion of bad interactions should be smaller than the smallest important fraction, i.e., that the value  $1/n$ , where  $n$  – the parameter describing this partner – can be any number between  $7 - 2 = 5$  and  $7 + 2 = 9$ .

The couple remains stable if for at least one of the two partners, the proportion of bad interactions is smaller than  $1/n$ . So, the couple remains stable if this proportion is smaller than the larger  $\max(1/n_1, 1/n_2)$  of the two values  $1/n_1$  and  $1/n_2$ . Since the function  $1/n$  is decreasing, the larger of these two values corresponds to the smaller value  $n_i$ , i.e., this larger value is equal to

$$1/\min(n_1, n_2).$$

What is the average value of the corresponding minimum  $\min(n_1, n_2)$ ? All we know about each of the values  $n_i$  is that it can be any number between 5 and 9. We do not know the probability of different values between 5 and 9. So, in line with the Laplace Indeterminacy Principle, it makes sense to assume that the corresponding five values 5, 6, 7, 8, and 9 are equally probable, each of them having the same probability  $1/5$ . It is also reasonable to assume that the values  $n_1$  and  $n_2$  describing the two partners are independent, so we have 25 equally probable pairs  $(n_1, n_2)$ , with probability of each pair equal to  $1/25$ . Here:

- we have exactly one pair (9, 9) for which  $\min(n_1, n_2) = 9$ , so the probability that  $\min(n_1, n_2) = 9$  is equal to  $1/25$ ;
- we have 3 pairs (8, 8), (8, 9), and (9, 8), for which  $\min(n_1, n_2) = 8$ , so the probability that  $\min(n_1, n_2) = 8$  is equal to  $3/25$ ;
- we have 5 pairs (7, 7), (7, 8), (7, 9), (8, 7), and (9, 7) for which  $\min(n_1, n_2) = 7$ , so the probability that  $\min(n_1, n_2) = 7$  is equal to  $5/25$ ;
- we have 7 pairs (6, 6), (6, 7), (6, 8), (6, 9), (7, 6), (8, 6), and (9, 6) for which  $\min(n_1, n_2) = 6$ , so the probability that  $\min(n_1, n_2) = 6$  is equal to  $7/25$ ;
- and, finally, we have 9 pairs (5,  $n_2$ ) and ( $n_1$ , 5) for which  $\min(n_1, n_2) = 5$ , so the probability that  $\min(n_1, n_2) = 5$  is equal to  $9/25$ .

The expected value of this minimum is thus equal to

$$\frac{1}{25} \cdot 9 + \frac{3}{25} \cdot 8 + \frac{5}{25} \cdot 7 + \frac{7}{25} \cdot 6 + \frac{9}{25} \cdot 5 = \frac{9 + 24 + 35 + 42 + 45}{25} = \frac{155}{25} = 6.2.$$

Thus, on average, the couple is stable if its proportion of bad interactions is smaller than  $\frac{1}{6.2}$ . For this threshold value, the proportion of good interactions is equal to

$$1 - \frac{1}{6.2} = \frac{5.2}{6.2}$$

and thus, the ratio of good to bad interactions is

$$\frac{5.2}{6.2} : \frac{1}{6.2} = \frac{1}{5.2}.$$

So, 5.2 is the threshold for detecting whether a couple will be stable:

- if the ratio is 5.2 or larger, on average, the couple will be stable, and
- if this ratio is smaller than 5.2, the couple will, on average, break up.

Taking into account that the number 5.2 comes from approximate reasoning, this is exactly what has been empirically observed.

## 22 Conclusion

In this paper, we have shown that many examples of seemingly irrational behavior can be explained – and often explained on the quantitative level – by the fact that our rationality is bounded, i.e., that we have limited time and ability to thoroughly process all the information and make a perfectly rational decision. From this viewpoint, while we are not always perfectly rational, we are definitely perfectly *boundedly* rational.

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