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A Bad Plan Is Better Than No Plan: A Theoretical Justification of an Empirical Observation

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Abstract In his 2014 book "Zero to One", a software mogul Peter Thiel lists the lessons he learned from his business practice. Most of these lessons make intuitive sense, with one exception – his observation that "a bad plan is better than no plan" seems to be counterintuitive. In this paper, we provide a possible theoretical explanation for this somewhat counterintuitive empirical observation.

1 Formulation of the Problem

A bad plan is better than no plan: a counterintuitive empirical observation. In his 2014 book "Zero to One" [1], a software mogul Peter Thiel lists the lessons he learned from his business practice.

Most of these lessons make intuitive sense, with one exception – his observation that a bad plan is better than no plan. At first glance, this empirical observation seems to be counterintuitive.

What we do in this paper. In this paper, we provide a possible theoretical explanation for this empirical observation.

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2 How to Describe This Problem in Precise Terms?

We need to describe actions. We decide between different plans of action. There may be many parameters that describe possible actions. For example, for the economy of a country, the central bank can set different borrowing rates, the government can set different values of the minimal wage and of unemployment benefits, etc. For a company, such parameters include percentage of income that goes into research and development, percentage of income that goes into advertisement, etc.

In general, let us denote the number of such parameters by n, and the parameters themselves by $x_1,\ldots,x_n$. From this viewpoint, selecting an action means selecting the appropriate values of all these parameters – i.e., in mathematical terms, a point $x=(x_1,\ldots,x_n)$ in the corresponding n-dimensional space.

Initial state. Let $x_1^{(0)},\ldots,x_n^{(0)}$ denote the values of the parameters corresponding to the current moment of time t_0. Our goal is to select parameters at future moments of time $t_1=t_0+h$, $t_2=t_0+2h$, $\ldots$, $t_T=t_0\cdot T\cdot h$, for some time quantum h.

Changes cannot be too radical. Whether we talk about the economy of a country or of a big company, it is very difficult to make fact drastic changes, there is a large amount of inertia in these economic systems.

Therefore, we will only consider possible actions $x_1,\ldots,x_n$ which are close to the initial state, i.e., which have the form $x_i=x_i^{(0)}+\Delta x_i$ for some small changes Δx_i.

Changes $x(t_{j+1})-x(t_j)$ from one moment of time t_j to the next one t_{j+1} are even more limited. Let b be the upper bound on such changes:

$$\|x(t_{j+1})-x(t_j)\|=\sqrt{\sum_{i=1}^{n}(x_i(t_{j+1})-x_i(t_j))^2}\le b.$$

On the other hand, the very fact that we talk about changes means that we are not completely satisfied with the current situations. The more changes we undertake at each moment of time, the faster we will reach the desired state.

The size of each change is limited by the bound b. Within this limitation, the largest possible changes are changes of the largest possible size b. Thus, we assume that all the changes from one moment of time to the next one are of the same size b:

$$\|x(t_{j+1})-x(t_j)\|=\sqrt{\sum_{i=1}^{n}(x_i(t_{j+1})-x_i(t_j))^2}=b.$$

What is our objective. Since we talk about which plans are better, this assumes that we have agreed on how we gauge the effect of different plans, i.e., we have agreed on a numerical criterion y that describes, for each possible action, how good is the result of this action.

The value of this criterion depends on the action: $y=f(x_1,\ldots,x_n)$ for some function $f(x_1,\ldots,x_n)$. In some cases, we may know this function, but in general, we do not know the exact form of this function. In other words, we know what we want to optimize, but we do not necessarily know the exact consequences of each action.

Since changes are small, we can simplify the expression for the objective function. We are interested in the values $y = f(x_1,\ldots,x_n)$ of the agreed-upon objective function $f(x_1,\ldots,x_n)$ in the small vicinity of the original state $x^{(0)} = (x_1^{(0)},\ldots,x_n^{(0)})$. In other words, we are interested in the values

$$f(x_1,\ldots,x_n) = f(x_1^{(0)} + \Delta x_1,\ldots,x_n^{(0)} + \Delta x_n)$$

corresponding to small deviations Δx_i.

Since the deviations Δx_i are small, we can expand the objective function into Taylor series in Δx_i and keep only the main – linear – terms in this expansion. In other words, it makes sense to consider a linear approximation of the original objective function:

$$f(x_1,\ldots,x_n) = f(x_1^{(0)} + \Delta x_1,\ldots,x_n^{(0)} + \Delta x_n) = a_0 + \sum_{i=1}^{n} a_i \cdot \Delta x_i,$$

where $a_0 \stackrel{\text{def}}{=} f(x_1^{(0)},\ldots,x_n^{(0)})$ and

$$a_i \stackrel{\text{def}}{=} \frac{\partial f}{\partial x_i}_{|x=x^{(0)}}.$$

Maximizing this expression is equivalent to maximizing the linear part

$$\sum_{i=1}^{n} a_i \cdot \Delta x_i.$$

Thus, if we denote the deviations Δx_i by u_i, we arrive at the following problem:

- we start with the values $u^{(0)} = (u_1,\ldots,u_n) = (0,\ldots,0)$;
- at each moment of time, we change the action by a change of a given size b:

$$\|u(t_{j+1}) - u(t_j)\| = \sqrt{\sum_{i=1}^{n} (u_i(t_{j+1}) - u_i(t_j))^2} = b;$$

- we want to gradually change the values u_i so that the (unknown) objective function $\sum\limits_{i=1}^{n} a_i \cdot u_i$ gets as large as possible.

What does "no plan" mean. An intuitive understanding of what "no plan" means is that at each moment of time, we undertake a random change, uncorrelated with all the previous changes.

In other words, at each moment of time, as the change vector $u(t_{j+1}) - u(t_j)$, we select a vector of length b with a random direction. Since we have no reason to select one of the possible directions, we thus consider all the directions to be equally probable.

In other words, we assume that the change vector uniformly distributed on the sphere of radius b, and that the changes corresponding to different moments of time are independent. The resulting trajectory $u(t)$ is thus an *n-dimensional random walk* [2] (or, equivalently, n-dimensional Brownian motion).

What we mean by a plan. Intuitively, a plan means that instead of going in different directions at different moments of time, we have a *systematic* change $u(t)$.

We consider local planning, for a few cycles $t_1, t_2, \ldots$, for which the difference $\Delta t \stackrel{\text{def}}{=} t - t_0$ is small. Thus, we can expand $u(t) = u(t_0 + \Delta t)$ into Taylor series and keep only linear terms in this expansion: $u(t) = u(t_0 + \Delta t) = u(t_0) + v \cdot \Delta t$, where

$$v \stackrel{\text{def}}{=} \frac{du}{dt}_{|t=t_0}.$$

By definition of the deviation $u(t)$, we have $u(t_0) = 0$, and thus, $u(t) = v \cdot \Delta t$. So, the change between each moment of time and the next one takes the form

$$u(t_{j+1}) - u(t_j) = v \cdot (t_{j+1} - t_j) = v \cdot h.$$

In other words, in contrast to the no-plan case, when changes at different moments of time are completely uncorrelated, here the changes at different moments of time are exactly the same.

Of course, from the practical viewpoint, they cannot always be the same: if a plan is bad, and we see that the desired objective functions decreases moment by moment, we will abandon this plan and select a new one.

Of course, it does not make sense to abandon the plan after a single decrease in the value of the objective function: it is known that even the best plans take some time to turn the economy around. Let us denote by m the reasonable number of decreases after which the plan will be abandoned.

From this viewpoint, selecting a plan means selecting a single change vector of length b, and following it for m steps, after which:

- if we had m decreases in the value of the objective function, we select a different plan,
- otherwise, we continue with the original plan for all T moments of time.

What we mean by a possible bad plan. We consider the situation in which we do not know the shape of the objective function. In this case, we do not know which change vector to select, so we select a random vector $w = (w_1, \ldots, w_n)$ of size a.

If $a \cdot w \stackrel{\text{def}}{=} \sum\limits_{i=1}^{n} a_i \cdot w_i > 0$, the resulting plan will lead to a consistent improvement of an objective function, so we will have a *good plan*. Vice versa, if

$$a \cdot w = \sum_{i=1}^{n} a_i \cdot w_i < 0,$$

the resulting plan will lead to a consistent decrease of an objective function, so we will have a *bad plan*.

Resulting description of two strategies. In this paper, we compare two strategies:

- the no-plan strategy, and
- the possibly-bad-plan strategy.

In the no-plan strategy, we consider random walk with step size b.

In the possibly-bad-plan strategy, we select a random vector w of size b. If $a \cdot w < 0$, then after m moments of time, we select a new random vector, etc.

Which of the two strategies leads to better results? Both strategies rely on a random choice. So, for the same situation, the same strategy may lead to different results.

Our goal is to improve the value of the objective function. Each strategy sometimes improves this values, sometimes decreases it:

- For example, if every time we select a change vector which is improving, both strategies will improve the value of the objective function.
- On the other hand, if every time, we select a decreasing change vector, both strategies will decrease the value of the objective function.

Thus, for each of the two strategies, a reasonable performance measure is the probability that by the final time t_T, this strategy will increase the value of the objective function in comparison to its original value.

Let us compare these probabilities.

3 Comparing the Results of the No-Plan and the Possibly-Bad-Plan Strategies

Case of the no-plan strategy. Under this strategy, the vector $U \stackrel{\text{def}}{=} u(t_T)$ describing the difference between the final action and the original one is the sum of T independent change vectors, each of which has a random direction.

All original distributions are invariant with respect to rotations. Thus, for the sum of these change vectors, the distribution is still invariant with respect to rotations – and hence, the direction $e \stackrel{\text{def}}{=} \dfrac{U}{\|U\|}$ of the sum vector U is also random: uniform on the unit sphere.

This implies, in particular, that the distribution of e is the same as the distribution of the opposite vector $-e$.

The vector U leads to an improvement if $U \cdot a > 0$, i.e., equivalently, if $e \cdot a > 0$. Since e and $-e$ have the same distribution, the probability that $e \cdot a > 0$ is the same as the probability that $(-e) \cdot a > 0$, i.e., that $e \cdot a < 0$. The probability of a degenerate case $e \cdot a = 0$ is 0. Thus, with probability 1, we have two equally probable cases: improving and decreasing. Therefore, the probability of each of these cases is exactly 1/2.

So, for the no-plan strategy, the probability of improvement is 0.5.

Case of the possibly-bad-plan strategy. In this case, similarly, with probability 1/2, we select an improving change vector w, in which case the value of the objective function improves.

With the remaining probability 1/2, we select a decreasing change vector, in which case the value of the objective function starts decreasing. However, it does not necessarily always decrease: after m steps, once we see that the value of the objective function decreases, we select a new change vector. In this case, with probability 1/2, we will select an improving change vector – and with a positive probability, the resulting improvement in the remaining $T-m$ moments of time will compensate for the decrease in the first m steps.

Thus, in this case, the probability that this strategy will lead to an improvement is larger than 1/2, since:

- in addition to the probability-1/2 situations in which we select an improving change vector from the very beginning,
- we also have situations in which we first decrease and then increase,
- and the probability of such additional situations is positive.

Conclusion: the possibly-bad-plan strategy is indeed better than the no-plan strategy.

- For the no-plan-strategy, the probability of improvement is 1/2, while
- for the possibly-bad-plan strategy, this probability is larger than 1/2.

So, indeed, the possibly-bad-plan strategy is theoretically better than the no-plan strategy.

Thus, we indeed have a theoretical explanation for Thiel's empirical observation.

How better? How larger is the probability of success of the possibly-bad-plan strategy than 1/2?

To answer this question, let us consider what happens when T is sufficiently large, i.e., in mathematical terms, when T tends to infinity. In this case, the probability that the no-plan method will succeed remains the same: 1/2.

On the other hand, the probability that the possibly-bad strategy will succeed tends to 1. Indeed, if in this strategy, at some moment of time t, we select an improving change vector w, then with $T \to \infty$, the resulting increase $(T-t)\cdot(a\cdot w)$ will tend to infinity and thus, eventually overcome decreases that happened before moment t.

So, the only possibility not to improve is when we consistently select a decreasing vector w.

- The probability of selecting such a vector in the beginning is 1/2.
- The probability of selecting it again after m iterations is also 1/2, so the overall probability that we have a decrease for the first $2m$ moments of time is $(1/2)^2$.

- Similarly, the probability that for all T/m selections, we selected a decreasing vector, is equal to $(1/2)^{T/m}$.

When $T \to \infty$, this probability tends to 0 and thus, indeed, the probability that the possibly-bad-plan strategy will led to improvement tends to 1.

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