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Why Stable Teams Are More Efficient in Education

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Abstract

It is known that study groups speed up learning. Recent studies have shown that stable study groups are more efficient than shifting-membership groups. In this paper, we provide a theoretical explanation for this empirical observation.

1 Formulation of a Problem

Group work, its success and its challenges. It is well known that study groups and group work enhance education.

Of course, one needs to be cautious when using student groups. There is always a serious danger that, when the class results are largely depending on a group work, a weaker student in a team will "hide" behind his team's success and not learn as much compared to learning strategies involving more individual responsibility.

Shifting-membership groups as a way to deal with group work challenges. To deal with the above problem, practicing educators use shifting-membership groups, in which groups are changing from one assignment to another.

A recent empirical result: stable teams are more efficient. The use of shifting-membership groups raised a natural question: is there a negative side effect in their use? In other words, does the constant change in student groups affect the effectiveness of group-related education?

Recent studies have shown that indeed, the use of shifting-membership groups leads to a drastic decrease in the education efficiency; see [1, 2].

Remaining question and what we do in this paper. While the papers [1, 2] list possible reasons why stable groups are more efficient, this somewhat unexpected recent empirical fact largely remains unexplained.

In this paper, we provide a simple explanation for this recently observed empirical phenomenon.

2 Analysis of the Problem

Why group work is efficient in the first place. In the usual educational environment, a student learns new things either from the instructor or from reading the corresponding textbook (and other teaching materials). In contrast, in a study group, students also learn from each other. This way, when a student learns some new material, he or she teaches others and thus, they all learn more efficiently.

Let us describe this idea in quantitative terms.

Towards a quantitative description of group learning. Let us first consider a simple model, in which all the students have the same learning rates for all parts of the material.

Usually:

- reading a book on your own and trying to understand the material is difficult, but
- once a student understood the material and explains to others, the other students understand it much faster.

Let r denote the time needed to understand one page of the material when a student studies on his/her own, and let $r' \ll r$ be the time need to understand one page when someone else – who already understood the material – explains this material.

Let L denote the overall number of pages that students need to learn as part of this particular assignment.

Under these assumptions, let us analyze:

- how much time a student needs to study the given material on his/her own and
- how much time a student needs if he or she studies in a group.

In this analysis, we will denote the number of students in a group by s.

Case when a student studies on his/her own. If each student studies on his/her own, then the student needs time

$$L \cdot r \tag{1}$$

to learn the material.

Case when students study in a group. When s students form a study team, they divide L pages into s parts of size $\frac{L}{s}$, so that each student learns his/her own part and then teaches others.

- Learning $\frac{L}{s}$ pages on one's own takes time $\frac{L}{s} \cdot r$.
- Teaching this material to other students requires time $\frac{L}{s} \cdot r'$.
- Finally, learning the remaining part $L - \frac{L}{s}$ of the material takes time
$$L \cdot r' - \frac{L}{s} \cdot r'.$$

As a result, the overall time that each student need to learn the material is equal to
$$\frac{L}{s} \cdot r + \frac{L}{s} \cdot r' + \left(L \cdot r' - \frac{L}{s} \cdot r'\right) = \frac{L}{s} \cdot r + L \cdot r'. \tag{2}$$

When is group study more efficient. By comparing the expressions (1) and (2), we conclude that the group study is more efficient if
$$\frac{L}{s} \cdot r + L \cdot r' < L \cdot r. \tag{3}$$
Dividing both sides of this inequality by L and multiplying both sides by s, we get an equivalent inequality
$$r + s \cdot r' \leq s \cdot r.$$
i.e., equivalently, $s \cdot r' < r \cdot (s - 1)$ and
$$\frac{s}{s-1} < \frac{r}{r'}. \tag{4}$$
Thus:

- for a group of $s = 2$ students, this condition is equivalent to $r > 2r'$;
- for a group of 3 students, it is equivalent to $r > 1.5 \cdot r'$;
- for a group of 4 students, it is equivalent to $r > 1.33 \cdot r'$, etc.

The ratio $\frac{r}{r'}$ is usually much larger than 2. This explains why group work is usually much more efficient than individual studies.

In reality, students' rate is somewhat different on different parts of the material. The above formula (2) for the efficiency of group learning does not depend on whether groups are stable or are recombined after each task. This independence comes from the fact that we only took into account the average learning times and thus, implicitly assumed that:

- all the students have the same learning time r, and that
- the above common learning time is the same on all parts of the material.

In reality, different students have different learning times, and, moreover, for each student i, the learning time r_{ij} may change depending on what part j of the material the student is learning.

Let is show that if we take these differences into account, we will be able to explain why stable groups are empirically more efficient.

3 Resulting Explanation

To provide the desired explanation, let us analyze how the individual differences affect the group's learning time in the case of shifting groups and in the case of stable groups.

Case of shifting-membership groups. In the case of shifting=membership groups, when groups are re-combined with every task, members of the new group do not know who is more capable to learning which part of the material. Thus, when they divide the learning material between different group members, they divide randomly. As a result, for each part j of the material, the groups' learning time is proportional to the mean learning time $\overline{r}_j \stackrel{\text{def}}{=} \frac{1}{s}\sum_{i=1}^{s} r_{ij}$. Thus, in effect, we get the original formula (2) for the time which is needed to learn all the material.

Case of stable groups. On the other hand, in a stable group, students get to learn each other's strong and weak points. As a result, when the overall material is distributed between the students, each student gets the part is which his or her learning rate is the highest.

Thus, for each piece j of the material, the group's learning time is determined not by the *mean* learning time, but by the *smallest* learning time

$$r_j^{\min} \stackrel{\text{def}}{=} \min_{i=1,\ldots,s} r_{ij}.$$

This explains why stable groups are more efficient. We have shown that:

- for the shifting groups, the overall learning time is proportional to the mean of different learning times, while
- for the stable groups, the learning time is proportional to the smallest of the individual learning times.

The smallest of s different numbers is always smaller than the mean. This explains why stable groups are more efficient.

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