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Gerardo Muela

*The University of Texas at El Paso*, [gdmuela@miners.utep.edu](mailto:gdmuela@miners.utep.edu)

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# Why Decimal System and Binary System Are the Most Widely Used: A Possible Explanation

Gerardo Muela

Department of Computer Science  
University of Texas at El Paso  
El Paso, TX 79968, USA  
gdmuela@miners.utep.edu

**Abstract.** What is so special about numbers 10 and 2 that decimal and binary systems are the most widely used? One interesting fact about 10 is that when we start with a unit interval and we want to construct an interval of half width, then this width is exactly  $5/10$ ; when we want to find a square of half area, its sides are almost exactly  $7/10$ , and when we want to construct a cube of half volume its sides are almost exactly  $8/10$ . In this paper, we show that 2, 4, and 10 are the only numbers with this property – at least among the first billion numbers. This may be a possible explanation of why decimal and binary systems are the most widely used.

## 1 Formulation of the Problem

**Problem.** What is so special about numbers 10 and 2 that decimal and binary systems are the most widely used?

This questions was raised, e.g., in [1].

**Observation.** One interesting fact about 10 is the following:

- When we start with a unit interval and we want to constrict an interval of half width, then this width is exactly  $1/2 = 5/10$ .
- When we start with a unit square and want to find a square of area  $1/2$ , its sides are  $\sqrt{1/2}$ , which is almost exactly  $7/10$ :

$$\left| \sqrt{\frac{1}{2}} - \frac{7}{10} \right| < \frac{1}{100}.$$

- When we start with a unit cube and want to find a cube of volume  $1/2$ , its sides are  $\sqrt[3]{1/2}$ , which is almost exactly  $8/10$ :

$$\left| \sqrt[3]{\frac{1}{2}} - \frac{8}{10} \right| < \frac{1}{100}.$$

So, whether we want to construct a piece of land which is (almost) exactly of half-area, or a piece of gold which is (almost) exactly of half-volume, decimal systems is very convenient.

**Are there any other numbers with this property?** Maybe here are other bases  $b$  with this property, i.e., bases  $b$  for which, for appropriate numbers  $n_1$ ,  $n_2$ , and  $n_3$ , we have

$$\left| \frac{1}{2} - \frac{n_1}{b} \right| < \frac{1}{b^2}, \quad \left| \sqrt{\frac{1}{2}} - \frac{n_2}{b} \right| < \frac{1}{b^2}, \quad \left| \sqrt[3]{\frac{1}{2}} - \frac{n_3}{b} \right| < \frac{1}{b^2}. \quad (1)$$

**What we do in this paper.** In this paper, we show that – at least among the first billion numbers  $b$  – only the numbers  $b = 2$ ,  $b = 4$ , and  $b = 10$  satisfy this property.

Base 4 is, in effect, the same as the binary system – we just group two binary digits to get one 4-ary digit, just like we get an 8-ary system when we group three binary digits or 16-based system when we group 4 binary digits together.

Thus, the above result may be a good explanation of why decimal and binary systems are the most widely used.

## 2 Analysis of the Problem

**Considering the first condition.** Let us first consider the first of the desired inequalities:  $\left| \frac{1}{2} - \frac{n_1}{b} \right| < \frac{1}{b^2}$ . When the base is even, i.e., when  $b = 2k$  for some integer  $k$ , then this property is clearly satisfied: indeed, in this case, for  $n_1 = k$ , we get  $\frac{n_1}{b} = \frac{1}{2}$  and thus,  $\left| \frac{1}{2} - \frac{k}{b} \right| = 0 < \frac{1}{b^2}$ .

On the other hand, if  $b$  is odd, i.e., if  $b = 2k + 1$  for some natural number  $k \geq 1$ , then, for  $\frac{1}{2} = \frac{k + 0.5}{2k + 1} = \frac{k + 0.5}{b}$ , the closest fractions of the type  $\frac{n_1}{b}$  are the fractions  $\frac{k}{b}$  and  $\frac{k + 1}{b}$ . For both these fractions, we have

$$\left| \frac{k + 0.5}{2k + 1} - \frac{k}{2k + 1} \right| = \left| \frac{k + 0.5}{2k + 1} - \frac{k + 1}{2k + 1} \right| = \frac{0.5}{2k + 1} = \frac{1}{2 \cdot (2k + 1)} = \frac{1}{2b}.$$

The desired inequality thus takes the form  $\frac{1}{2b} < \frac{1}{b^2}$ , which is equivalent to  $2b > b^2$  and  $2 > b$ . However, odd bases start with  $b = 3$ . So, the first condition cannot be satisfied by odd bases  $b$ .

Thus, the first condition is equivalent to requiring that the base  $b$  is an even number.

**How do we check the second condition.** If we check the second condition  $\left| \sqrt{\frac{1}{2}} - \frac{n_2}{b} \right| < \frac{1}{b^2}$  literally, then we need to consider all possible values  $n_2$  from 0

to  $b$ . However, this can be avoided if we multiply both sides of the desired inequality by  $b$  and consider the equivalent inequality  $\left| b \cdot \sqrt{\frac{1}{2}} - n_2 \right| < \frac{1}{b}$ . In this case, we can easily see that  $n_2$  is the nearest integer to the product  $b \cdot \sqrt{\frac{1}{2}}$ :

$$n_2 = \left[ b \cdot \sqrt{\frac{1}{2}} \right],$$

where  $[x]$  denotes the nearest integer to the real number  $x$ . In these terms, the desired inequality takes the form

$$\left| b \cdot \sqrt{\frac{1}{2}} - \left[ b \cdot \sqrt{\frac{1}{2}} \right] \right| < \frac{1}{b}. \quad (2)$$

This is the inequality that we will check.

**How to check the third condition.** Similarly, if we check the third condition  $\left| \sqrt[3]{\frac{1}{2}} - \frac{n_3}{b} \right| < \frac{1}{b^2}$  literally, then we need to consider all possible values  $n_3$  from 0 to  $b$ . However, this can be avoided if we multiply both sides of the desired inequality by  $b$  and consider the equivalent inequality  $\left| b \cdot \sqrt[3]{\frac{1}{2}} - n_3 \right| < \frac{1}{b}$ . In this case, we can easily see that  $n_3$  is the nearest integer to the product  $b \cdot \sqrt[3]{\frac{1}{2}}$ :

$$n_3 = \left[ b \cdot \sqrt[3]{\frac{1}{2}} \right],$$

where  $[x]$  denotes the nearest integer to the real number  $x$ . In these terms, the desired inequality takes the form

$$\left| b \cdot \sqrt[3]{\frac{1}{2}} - \left[ b \cdot \sqrt[3]{\frac{1}{2}} \right] \right| < \frac{1}{b}. \quad (3)$$

This is the inequality that we will check.

**The checking.** For each even number  $b$  from 2 to  $10^9$ , we checked whether this number satisfies both conditions (2) and (3). A simple Java program for this checking is given in the appendix.

**The result of the checking.** The result is that among all the bases  $b$  from 1 to  $10^9$ , both roots are only well approximated for  $b = 2$ ,  $b = 4$ , and  $b = 10$ . Thus, only for these three bases, the desired condition (1) is satisfied.

This may explain why decimal and binary systems are the most frequently used.

**Natural conjecture.** We have checked all the values  $b$  until  $10^9$ . This makes us conjecture that out of *all* possible natural numbers  $b \geq 2$ , only the numbers 2, 4, 10 satisfy the property (1).

## References

1. D. E. Knuth, *The Art of Computer Programming*, Addison-Wesley Professional, Boston, Massachusetts, 2011.

## A Code

```
public static void main(String [] args){
    double value;
    //Loop that iterates from 2 to 10^9
    for(int b = 2; b <= 1000000000; b += 2){
        value = Math.sqrt(0.5) * b;
        //Checks if the square root is well approximated
        if(Math.abs(value - Math.round(value)) < 1. / b){
            value = Math.cbrt(0.5) * b;
            //Checks if the cubic root is well approximated
            if(Math.abs(value - Math.round(value)) < 1. / b){
                System.out.println("Square and cubic roots "
                    + "are well approximated in base " + b);
            }
        }
    }
}
```