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Borderland Pedagogy Study Of High School Mathematics Teachers' Lesson Plan Development And Implementation Practices

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BORDERLAND PEDAGOGY STUDY OF HIGH SCHOOL MATHEMATICS
TEACHERS' LESSON PLAN DEVELOPMENT AND IMPLEMENTATION
PRACTICES

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Dedication

I dedicate this dissertation to all my family members: my mom (*Maria Elena Gallardo*), my father (*Jose Arturo Gallardo*), my sister (*Verónica Gallardo*) and brothers (*Martin and Hugo Gallardo*), and my mother in law who prayed for me all these years to obtain this goal. I especially dedicate this dissertation to my husband, *Moises Melchor* for all the time, day in and day out, he supported me during this stage of my life. Without his help I would not achieved this success. Thanks my love because every day you make me stronger, because you were always supporting me during this hard journey.

Finally, I dedicate this dissertation to one special member (my new baby, Sofia) of my family who has formed part of my life.

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TEACHERS' LESSON PLAN DEVELOPMENT AND IMPLEMENTATION
PRACTICES

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ROCIO E. GALLARDO

DISSERTATION

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The University of Texas at El Paso
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Abstract

The aim of the study is to examine high school mathematics teachers' lesson plan development and implementation practices used in the border region of Mexico and USA. The study also attempts to determine how a transition from Mexico (Ciudad Juarez, Chihuahua) to the U.S. (El Paso, TX) impacts high school mathematics teacher's lesson plan development practices incorporating the Borderland Pedagogy. The Borderland Pedagogy theoretical framework (Cline & Necochea, 2006; Romo & Chavez 2006; Fiume, 2005) was developed to explore educational experiences of teachers situated within border regions. The framework highlights key characteristics of Borderland Pedagogy that influence lesson plan development and implementation practices. The framework was used to design multiple case studies research to examine and understand teaching practices on both sides of the border in general, and pedagogical experiences of transitioning teachers in particular. Elbaz-Luwish (2007) and Sabar (2004) defined teacher transition as an adaptation of a teacher to a new language, culture, and new educational system. Scholars (Shimizu, 2008; Diazgranados et al., 2008; Lit and Lit, 2009) suggest that lesson plans are designed according to teachers' experiences, knowledge about the subject matter, and beliefs about teaching, and learning. The study is built on understanding that teaching on the border impose unique requirements on lesson plan development practices reflecting flexibility, cultural and linguistic diversity. The research sample included two Mexican teachers, two US teachers, and one transitioning teacher. The design of the study is operationalized based on the following data sources: (1) teacher-developed lesson plans, (2) classroom observations, and (3) structured interviews. Data was analyzed using frequency-based initial and focus coding scheme. The key observation in lesson plan development among participating Mexican and US teachers revealed complexity and uniqueness of borderland

teachers' practices in recognizing, addressing, and implementing national/ state standards and curriculum (Secretaría de Educación Pública, Texas Education Agency). Results of the study suggest that the Borderland Pedagogy could serve not only as a framework but also as an instrument to document and interpret transformative pedagogical practices of teachers teaching on the border.

Table of Contents

Acknowledgements.....	v
Abstract.....	vi
Table of Contents.....	viii
List of Tables	xii
List of Figures.....	xiii
Chapter 1: Introduction.....	1
1.1 Introduction.....	1
1.2 Statement of the Problem.....	3
1.3 Purpose of the Study	5
1.4 Significance of the Study.....	7
1.5 Origins of the Study	10
1.6 Research Questions.....	12
1.7 Organization of the Study	12
1.8 Definition of Terms.....	14
1.9 Summary.....	16
Chapter 2: Literature Review and Conceptual Framework.....	17
2.1 Introduction.....	17
2.2 High School Mathematics (HSC) Curriculum in Mexico and USA....	17
2.3 High School Mathematics Teaching Practices	22
2.4 (HSC) Mathematics Teachers Lesson Plan Development Practices....	36
2.5 Teachers Borderland Pedagogy	42
2.6 Theoretical Framework.....	44
2.7 Summary.....	52
Chapter 3: Methodology	53
3.1 Introduction.....	53
3.2 Research Design.....	53
3.3 Context of the Study	57
3.4 Gaining Access and Entry to the U.S Participants.....	68
3.5 Gaining Access and Entry to the Mexican Participants.....	70

3.6	Gaining Access and Entry to the Transitioning Participants	71
3.7	Participants.....	73
3.8	Summary	75
Chapter 4: Results		76
4.1	Introduction.....	76
4.2	Explanation of the Characteristics for Borderland Pedagogy	78
4.3	How the Findings are Reported.....	79
4.4	Case Study Description.....	80
4.5	Summary	146
Chapter 5: Discussion, Implications and Conclusion		147
5.1	Introduction.....	147
5.2	Discussion of Findings.....	148
5.3	Discussion of Findings for Research Question 1	148
5.4	Discussion of Findings for Research Question 2	163
5.5	Discussion of Findings for Research Question 3	171
5.6	Summary	181
5.7	Implications.....	181
5.8	Recommendations for Policy and/or practice	184
5.9	Limitations	184
5.10	Conclusions.....	188

References	193
Appendix A Interview Questions.....	201
Appendix B Esteban’s case study	203
Appendix C Virginia’s Case Study.....	211
Appendix D Carlo’s Case Study	224
Appendix E Angelica’s Case Study	239
Appendix F Yolanda’s Case Study	245
Appendix G Survey.....	251
Appendix H Esteban’s Lesson Plan	257
Appendix I Virginia’s Lesson Plan.....	258
Appendix J Carlo’s Lesson Plan	260
Appendix K Angelica's Lesson Plan.....	262
Appendix L Yolanda's Lesson Plan.....	263
Appendix M IRB Proposal.....	265
Appendix N Consent Form.....	270
Vita.....	279

List of Tables

Table 2.1: Comparative Analysis of Mathematics' Curriculum in Mexico and U.S-1.	20
Table 2.2: Comparative Analysis of Mathematics' Curriculum in Mexico and U.S-2.	21
Table 2.3: High School Algebra Teaching Practices in Mexico and U.S.:Comparison Chart.	29
Table 2.4: High School Geometry Teaching Practices in Mexico and U.S.:Comparison Chart. .	32
Table 2.5: High School Pre-Calculus Teaching Practices in Mexico and U.S.: Comparison Chart.....	35
Table 3.1: Participant Sampling Characteristics.....	59
Table 3.2: Observation Protocol.....	65
Table 3.3: Characteristics of the Borderland Pedagogy.	66
Table 3.4: Participants' Demographic Information.	74
Table 4.1: Characteristics' Enacted in Lesson Plan Dev. and Imp: Esteban.....	88
Table 4.2: Actions Most Frequently Enacted Under the Characteristic of Flexibility: Esteban.	90
Table 4.3: Actions Most Frequently Enacted Under the Characteristic of Passion:Esteban.	91
Table 4.4: Actions Most Frequently Enacted Under the Characteristic of Culture:Esteban.	93
Table 4.5: Actions Most Frequently Enacted Under the Characteristic of Language:Esteban. ...	94
Table 4.6: Characteristics' Enacted in Lesson Plan Dev. and Imp: Virginia.	98
Table 4.7: Actions Most Frequently Enacted Under the Characteristics of Lack of Flexibility: Virginia.....	100
Table 4.8: Actions Most Frequently Enacted Under the Characteristics of Passion:Virginia....	103
Table 4.9: Actions Most Frequently Enacted Under the Characteristic of Culture: Virginia.....	104
Table 4.10: Actions Most Frequently Enacted Under the Characteristic of Language:Virginia.	105
Table 4.11: Characteristics' Enacted in Lesson Plan Dev. and Imp: Angelica.	107
Table 4.12: Actions Most Frequently Enacted Under the Characteristics of Lack of Flexibility: Angelica.	113
Table 4.13: Actions Most Frequently Enacted Under the Characteristic of Lack of Passion: Angelica.	114
Table 4.14: Actions Most Frequently Enacted Under the Characteristics of Lack of Cultural Sensitivity: Angelica.	115
Table 4.15: Actions Most Frequently Enacted Under the Characteristics of Lack of Pluralistic Language Orientation: Angelica.	116
Table 4.16: Characteristics' Enacted in Lesson Plan Dev. and Imp: Yolanda.	117
Table 4.17: Actions Most Frequently Enacted Under the Characteristic of Lack of Flexibility: Yolanda.	123
Table 4.18: Actions Most Frequently Enacted Under the Characteristic of Lack of Passion: Yolanda.	126
Table 4.19: Actions Most Frequently Enacted Under the Characteristic of Lack Cultural Sensitivity: Yolanda.....	127
Table 4.20: Actions Most Frequently Enacted Under the Characteristic of Lack of Pluralistic Language Orientation: Yolanda.	129
Table 4.21: Characteristics Enacted in Lesson Plan Dev. and Imp: Carlos.....	130
Table 4.22: Actions Most Frequently Enacted Under the Characteristic of Flexibility: Carlos.	138
Table 4.23: Actions Most Frequently Enacted Under the Characteristic of Passion: Carlos. ...	141
Table 4.24: Actions Most Frequently Enacted Under the Characteristic of Culture: Carlos.	142
Table 4.25: Actions Most Frequently Enacted Under the Characteristic of Language: Carlos..	143

Table 5.1: Frequency table on the Characteristics of the Borderland Pedagogy: Esteban and Virginia.	150
Table 5.2: Frequency table on the Characteristics of the Borderland Pedagogy: Yolanda and Angelica.	165
Table 5.3: Frequency table on the Characteristics of the for Borderland Pedagogy: Carlos.	172
Table 5.4: Key Considerations.	191

List of Figures

Figure 4.1: Esteban's classroom arrangement: Front wall.....	81
Figure 4.2: Esteban's classroom arrangement: Graphing calculator section.....	82
Figure 4.3 Esteban's lesson plan fragment-1.....	84
Figure 4.4: Esteban's lesson plan fragment-2.....	85
Figure 4.5 Esteban's assignment to students	86
Figure 4.6: Casandra trip: Graphical representation.....	90
Figure 4.7 Mathematics Concepts in both languages-1.....	96
Figure 4.8: Mathematics Concepts in both languages-2.....	96
Figure 4.9 Mathematics problem taught in classroom by Virginia.....	97
Figure 4.10: Virginia's worksheet on segment.....	97
Figure 4.11: Virginia's angles term lesson plan	98
Figure 4.12: Geometry activity implemented by Virginia to Teach Geometry	101
Figure 4.13: Handout used by Virginia during Angles Lesson	103
Figure 4.14 Parabola concept representation.....	105
Figure 4.15: Angelica's classroom description.....	110
Figure 4.16: Angelica's assignment about product of a binomial with a common term.....	111
Figure 4.17: Formation of student group to implement factorization	112
Figure 4.18: Yolanda's lesson plan: Notable's products	120
Figure 4.19: Yolanda's lesson plan development: Conjugate binomial.....	120
Figure 4.20: Yolanda's lesson plan: Product of a binomial with a common term.....	121
Figure 4.21: Yolanda's homework assignment conjugate binomial.....	121
Figure 4.22: Yolanda's homework assignment: Product of a binomial with a common term ..	122
Figure 4.23: Exercises developed by Yolanda.....	125
Figure 4.24: Carlo's classroom description arrangement: On side view wall.....	133
Figure 4.25: Carlo's classroom description arrangement: Front wall.....	133
Figure 4.26: Inverse function lesson plan-1.....	135
Figure 4.27: Inverse function lesson plan-2.....	135
Figure 4.28: Inverse function lesson plan-3.....	136
Figure 4.29: Comparison functions lesson plan.....	136
Figure 4.30: Lesson plan about function definition.....	141

Chapter 1: Introduction

1.1 INTRODUCTION

The aim of this study was to use a Borderland Pedagogy framework to examine how a transition from Mexico (Ciudad Juarez, Chihuahua) to the U.S. (El Paso, Texas) impacts the lesson plan development and implementation practices of high school mathematics teachers. Elbaz-Luwisch (2007), defines the term *transition* as the "...complex process of shaping an identity moving between two places" (p. 389). This author points out that a transition occurs when people move to live in another country, where they have to learn a new culture, and language. Sabar (2004) also mentions that teachers' transitions could be considered a development of "...teaching self, in the context of concrete details of biography, schools settings, relationships and educational systems within which teachers work" (p. 150). Previous studies (Duchesne and Stitou, 2010) also maintain that teachers who migrate to another country were influenced by ethnicity and culture in education, in the classroom and in the academic community. Duchesne and Stitou (2010) state that:

"Many immigrant teachers come from a culture in which education is associated with the wearing of a uniform, lecture- style teaching, memorization as the principal learning strategy, and knowledge assessment through standardized testing. These teachers must change both their practice and their educational beliefs to meet the expectations of the educational system. They may also have to change their expectations about the amount of work that is demanded of them outside the classroom, particularly relating to lesson creation and planning and the use of evaluation methods based on learning processes". (p. 50).

Scholars (Duchesne and Stitou, 2010; Bryan and McLaughlin, 2005) assert that teachers who migrate from one country to another were faced with challenges such as culture, different educational systems, and language.

As mentioned before, this study focuses on using Borderland Pedagogy as a framework for investigating how a transition from Mexico (Ciudad Juarez, Chihuahua) to the U.S (El Paso TX) impacts high school mathematics teachers' lesson plan development and implementation practices. Some authors identify lesson plan development as an important tool in teaching practice (Beyer, Davis, Forbes and Stevens, 2012). Stigler and Stevenson (2004) and Li., Chen, and Kulm (2009) highlight lesson planning as an essential activity to achieve effective classroom instruction.

In Mexico, the Secretaría de Educación Pública (SEP) (2012) established that teaching of mathematics plays an important role in problem solving as an element of motivation and the learning environment for students. Thus, for students to be attracted to the study of mathematics, the content should have a sense of meaning relevant to their experience. That is, content should be related to situations in students' environment or field of knowledge that accessible based on their level of personal maturity and cognitive development (SEP, 2004). SEP addresses several aspects of the surrounding environment, (e.g., economic, social, environmental) as well as different fields of knowledge, which promote the development of critical thinking and committed performance from students. Gooya, (2007); Daher, AlQasemi, Al-Qasemi, (2010) have identified the teaching of mathematics as a science that was described by properties and multiple connections with another areas where it is applied. Daher, Al-Qasemi, and Al-Qasemi (2010) state that there are some factors that "should be considered when attending to understand teachers' conceptions of teaching mathematics to understand teachers'

practices” (p. 141). They identify these factors as: teachers’ knowledge, teachers’ practices, and teachers’ beliefs.

The above information provides an overview of key factors that have been identified as major influences in how aspects of Borderland Pedagogy are incorporated into high school mathematics teachers’ lesson plan development and implementation.

1.2 STATEMENT OF THE PROBLEM

This study used Borderland Pedagogy to look closely at how a transition from Mexico (Ciudad Juarez, Chihuahua) to the U.S. (El Paso, Texas) impacts the lesson plan development and implementation practices of high school mathematics teachers. This study compared the lesson plan development practices of Mexican and U.S mathematics teachers to provide an understanding of the process of teachers’ transition. Elbaz-Luwisch (2007) argues that “teachers who have made a transition from one cultural setting to another were likely to have developed an awareness of teaching and schooling in the new culture that other teachers may not have” (p. 387).

Some authors have identified lesson plan development practices as the teaching-learning methods that mathematics teachers utilize to teach the subject (Shimizu, 2008; Li and Li, 2009). For example, Li., Chen and Kulm (2009) state that “lesson planning practices can be taken as the link that connects what was given in the curriculum guides and textbooks with what was enacted in the classroom” (p. 718). Lesson plan development practices contain a set of goals, sources of information, and criteria for judging the effectiveness of planning. Shimizu (2008) indicated that through “...the practices of lesson plan development teachers share and maintain the scripts for teaching mathematics” (p. 948).

Li et al. (2009) stated that “in a decentralized education system such as the U.S, in lesson planning development, teachers had much more freedom in selecting and organizing textbook content for teaching” (p. 720). Also, Shimizu (2008) affirmed that the “autonomy in textbook content selection and organization can presumably place more responsibility on the U.S teachers in preparing and organizing content for teaching” (p. 945). However, in Mexico (including Ciudad Juarez) the centralized system provided by the Secretaría de Educación Pública (SEP) establishes the materials that teachers will be using to teach mathematics at the high school level. Saiz (2002) states that lesson plan development practices in Mexico are standardized in such a way that mathematics teachers must to follow the state provided lesson plans in order to teach mathematics. On the other hand, Li et al (2009) asserts that lesson plan development in the U.S. is influenced by textbooks and other written materials.

Exploring the lesson plan development practices in Mexico and U.S. helps educational researchers have a deeper understanding of the teacher borderland transition phenomena. According to Cline and Necochea (2006) and Elbaz-Luwisch (2007) the borderland transition can be understood as how a teacher adapts to a new culture, language and educational system when crossing an international border.. Educational researchers have tried to examine factors such as teachers’ mathematics knowledge and teachers’ view of effective teaching (Li et al, 2009). Lesson plan development practices are conceptualized as a part of teachers’ general pedagogical knowledge. Li et al (2009) argue that this “conceptualization suggests a pedagogical perspective to examine and understand lesson planning in an international context” (p. 718). Shulman (1986) emphasizes that pedagogical knowledge is an important aspect of successful teaching. Teachers are critical players in considering in any pedagogical model. For instance,

research developed by Beyer, Davis, Forbes, and Stevens (2012) affirms that “lesson plans played a central role in guiding teachers’ practice” (p. 798).

Furthermore, in this study I examined teachers’ transition with regard adaptation and to changes in their teaching strategy and methods in a new work setting.

This case study drew upon and contributes to the theoretical knowledge and research literature related to lesson plan development practices. This case study identifies the Characteristics of the Borderland Pedagogy most frequently used by the mathematics teachers to develop and implement mathematics teaching. As Artigue and Wislow (2010) state, “the aim of the case study was to identify and explain the differences between the phenomenon in two or more contexts” (p. 4). Similar approaches have been used to compare lesson plan development practices between U.S. and Chinese teachers’ reference. I used a cross case study approach to compare Mexican and U.S. teachers’ practices in order to know how a transition from Mexico (Ciudad Juarez, Chihuahua) to the U.S. (El Paso, Texas) impacted high school teachers’ lesson plan development practices in the context of teaching mathematics at the high school level.

1.3 PURPOSE OF THE STUDY

The primary purpose of this study was to use a Borderland Pedagogy framework to document and analyze how a transition from Mexico (Ciudad Juarez, Chihuahua) to the U.S. (El Paso, Texas) impacts the lesson plan development and implementation practices of high school mathematics teachers. Studies conducted by Elbaz-Luwisch (2007) and Gastein et al (1997) found that teachers who migrated to another country made adaptations to their new work settings and faced issues of teacher burnout.

This study explored how teaching practices are part of a complex process of lesson plan development for mathematics classes in different countries and within different educational

systems. The purpose of this study was to know how this process was influenced by different settings. These influences included standards and policy issues related to No Child Left Behind (NCLB), national, state, or local assessment programs, textbooks, and/or the mathematics content being taught (Regis, 2008).

The second purpose was to identify how the mathematics teachers incorporated elements of Borderland Pedagogy into the development and implementation of their lesson plans according to standards in Mexico and U.S. For example, in Mexico the SEP (2012) established a theme target that “student must solve theoretical and practical problems related to angles, by identifying ranking” (p. 9). On the other hand, the Texas Education Agency (2012) stated that by teaching mathematics, students should develop an understanding of “.....number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry; measurement; and probability and statistics are essential foundations for all work in high school mathematics” (p. 7). Additionally, in Mexico and U.S there is a different and implementation of their lesson plan according to SEP and Texas Education Agency.

Mathematics lesson plan development is significant because this subject is linked to many other topics in math that are essential to other fields (Gooya, 2007).

Additionally, the purpose of this study was to determine how a transition from Ciudad Juarez to the U.S. (El Paso, Texas) impacted high school teachers’ incorporation of Border Pedagogy into their high school level mathematics development and implementation practices. Researchers (Gustein et al. 1997; Elbaz-Luwisch, 2004; and Seah, 2002) have stressed that immigrant teachers bring with them their respective cultural baggage such as attitudes, experiences as mathematics teachers. Additionally, each immigrant teacher brings with them different their own attitudes, as well, their own experiences as mathematics teachers.

1.4 SIGNIFICANCE OF THE STUDY

The significance of this study consists of several themes. The importance of conducting this study was to emphasize and analyze the immigrant teachers' practices related to lesson plan development. It was found that teachers' practices were linked directly with challenges in their daily professional lives and to their general well-being, as well as, their beliefs and practices influenced student motivation and achievement. Li, et. al (2009) states that "...a curriculum perspective would also allow us to gain a better understanding of teachers' lesson planning practices in a specific system context. The nature of different system contexts likely imposed different expectations for teachers to structure content for teaching" (p. 729). Over time, education has faced a number of challenges, the main challenge being to meet and conduct an effective teaching-learning process in order to better meet the needs of the society, which is why lesson plan development is an essential tool in educational contexts. Exploring the teachers' transitions in lesson plan development practices is important since teaching practices are related to effective classroom learning and student outcomes (Good and Grouws, 1989). It was important to examine whether teachers are designing well thought-out and high quality lesson plans and to identify if teachers are "...building a solid base for classroom implementation" (Li et al., 2009, p. 717). This study substantiated that teachers were influenced in their lesson plan development by authorities, such as the Secretaría de Educación Pública (Saiz, 2002) and Texas Education Agency, assessment practices at the national, state, or district level (Lin, Baker, & Betebenner, 2002; Travers, 1986; National Council of Teachers of Mathematics, 1989, 2000, 2006, and textbooks and accompanying activities (Good & Grouws, 1989; Remillard, 2000; Tarr, Chavez, Appova, & Regis, 2005; Venezky, 1992). In addition to considering these influences, this study examined the factors that influenced teachers' incorporation of Borderland Pedagogy

during the development and implementation of their lesson plans. It was important to know if teachers were developing lesson plans effectively. A lesson plan is developed effectively by creating and designing a curriculum creatively and through consideration of the different backgrounds of the students (Cline & Necochea, 2006).

Moreover, recent information reports that minority populations of mathematics teachers in several states, including California, Florida and Texas (U.S. Bureau of the Census, 2000) are members of an immigrant teachers' population. This population faces challenges such as language and culture. To address these challenges, it is necessary that immigrant teachers adapt to these new conditions in order to be successful the classroom. Supporting the presence of challenges, Elbaz-Luwisch (2007) and Sabar (2004) revealed that immigrant teachers need to be adapted to a new language, culture, and new educational system.

Migration news (2012) reports that Mexican teachers immigrate often to the U.S. in order to find better economic opportunities. These economic opportunities permit financial remittances back to Mexican families (due to drastic differences in teacher salaries in Mexico and the U. S.) to improve their families' economic status. Transitioning teachers who immigrate to the U.S. and get a teaching job, obtain legal documentation (e.g., U.S passport, or Green Card) to work in the United States. Because of their diverse international backgrounds, transitioning teachers who immigrate to the U.S. have a unique opportunity to help students develop the civic skills that will enable them to operate in and contribute to the modern globalized world. These skills may be the most powerful tools that transitioning teachers can teach students to use. For example, transitioning teachers can help students see the world in new ways through both indigenous and innovative lenses that expand their horizons beyond the often ethno, language, culture, and gender-centric viewpoints they may encounter among family and friends, in their previous

schooling. Most importantly, transitioning teachers have the personal background knowledge to help students understand the immigrant experience and the drama of border life.

Recent studies for Programme for International Student Assessment (2011) , states that every year there is a significant percent of Hispanic students' population increase in the U.S. Additionally; it is important that transitioning teachers' population increase too. As more students and teachers immigrate from Mexico to the U.S., it becomes apparent that teachers' practices in the U.S., both transitioning and native, need to be evaluated using the context of characteristics of Borderland Pedagogy to determine recommendations for changes in policy and practice that will facilitate learning and address the gaps identified in PISA data. As Cline & Necochea, (2006); Romo & Chavez, (2006); Fiume, (2005); Garza, (2007) point out that to be effective educators in the borderland area, transitioning teachers should have certain characteristics: *(1) should be bilingual, (2) should know both borderland cultures, (3) should have an open mind to understand cultural diversity, (4) should be passionate about their profession, and (5) should to be creative.*

Studies conducted by Petrovic's (2000) revealed that the standards between U.S. and Mexico in the area of mathematics appear to share many standards, especially under the topics of number sense/ algebraic functions, measurement/geometry, statistics/data/probability and mathematical reasoning. As well, Romo and Chavez (2013) mentioned that a comparison between these standards revealed that "both countries include content knowledge as well as pedagogy in the teacher lesson plan development. The cross-cultural similarities in content knowledge coursework would appear to be a positive factor in helping to alleviate the transition of normalistas into U.S. schools and vice versa" (p.250). Consequently, it is important that the theoretical framework of Borderland Pedagogy is implemented in the lesson plan development

and implementation practices by the mathematics teachers according to the standards in U.S. and Mexico.

Moreover, Romo and Chavez (2013) and Alexander (2001) state that border pedagogy in teacher education is a topic that merits further research because of its evident potential to develop educators that root their practice in the paradox of the border and make a difference for diverse students in borderland schools. Correspondingly, border pedagogy promotes the skills of critical thinking (interrogating power, meaning, and identity), addressing the elements of mathematical knowing, applying and reasoning that are measured in studies such as PISA. It encourages tolerance, ethical sophistication and openness. Ultimately, border pedagogy works to de-colonize and revitalize learning and teaching to promote liberty and justice for all.

In the next section present the important definition considered in the study. In this important include it to facilitate the reading of the research study.

1.5 ORIGINS OF THE STUDY

On the one hand, I was drawn to the lesson plan development practices used in teaching mathematics because I have experience as a mathematics teacher. I have had the opportunity to work as a mathematics teacher at the high school and college levels in Mexico for four years.

Through my experience as math teacher, I discovered that mathematics was a difficult subject for students at the high school and college levels, especially for students taking classes in the subjects of math for Engineering, Architecture, and other fields. Through my teaching experience, I learned that high school and college students have difficulties when they learn mathematics concepts through certain teaching strategies. Students learned “mathematics” in a superficial manner, when the only support from the teacher was to propose activities established by the textbook, forbidding work with peers. This prevented students from overcoming their

difficulties and lost the opportunity to relate math to its context. Studies conducted in Mexico (Chapa & Gutierrez, 1992) affirmed that “the teaching of mathematics in the curriculum has been noted as a primary cause of students’ poor performance, both in what topics are addressed and how they are communicated” (p. 52). The study of teaching mathematics was one of the most controversial issues debated for some mathematics educators (Fehr, 1973; Chazan, D, 1993). This is one of the reasons why I decided to conduct a study focusing on teaching mathematics at the high school level.

This study helped me understand teachers’ transitions related to lesson plan development practices used in teaching mathematics on both sides of the U.S.-Mexico border. As well, help me to understand how the mathematics teachers on both sides of the borders implement the Characteristics of Borderland Pedagogy. As a mathematics teacher, I intuited that high school students visualize “mathematics” as an academic subject that is too abstract and cannot be applied to real life situations. Through these experiences over the years, I faced some obstacles that helped me reflect on teacher practices, materials used in the classroom, activities implemented in the classroom, and lesson plan development.

The idea for this study began to emerge when I was mathematics teacher at the high school level in Mexico. In my mathematics experiences in Ciudad Juarez, I learned that lesson plan development practices were provided step by step without changing the order, without bringing innovation to one’s own proposed activities, through metered content per month, and that this leads to lessons that that work linearly and systematically; as a result of that, I saw that students who did not learn according to the lesson plan steps fall behind in the classroom. My experience is somehow connected to what Rivera (2007) refers to as the teaching of mathematics

appears to be a problem in the mathematics learning. This issue referred to the use of materials, books, and teaching strategies that are not the most appropriate.

1.6 RESEARCH QUESTIONS

The study responds to the following research questions.

(1) What elements of Borderland Pedagogy are evident in the lesson plan development and implementation practices used by high school mathematics teachers in El Paso, Texas?

(2) What elements of Borderland Pedagogy are evident in the lesson plan development and implementation practices used by Mexican high school mathematics teachers in Ciudad Juarez, Chihuahua?

(3) How does a transition from Mexico (Ciudad Juarez, Chihuahua) to the U.S. (El Paso, Texas) impact high school mathematics teachers' incorporation of Borderland Pedagogy during the development and implementation of their lesson plans?

1.7 ORGANIZATION OF THE STUDY

In this study, I explored what Borderland Pedagogy lesson plan development practices were used by high school mathematics teachers in El Paso, Texas and Ciudad Juarez, Chihuahua. I studied a group of five mathematics teachers in terms of how they implemented different strategies, methods and practices to develop and teach their lesson plan for mathematics. It was important to know and learn about these factors in order to identify whether teachers are “designing well thought-out and high quality lesson plans and to identify if teachers are building a solid base for classroom implementation” (Li et al, 2009, p. 717). In this study, I also investigated how the teachers integrate their knowledge of mathematics into their teaching practices. For example, mathematics teachers develop

lesson plan based on their personal experiences math educators. Additionally, in this study, I explored the key factors that impact the how Borderland Pedagogy is reflected in lesson plan implementation practices.

The five chapters of this dissertation are designed with the purpose of supporting the main arguments in answer to the above three research questions. Chapter Two is a literature review about teachers' lesson plan development and implementation practices. Chapter Two also contains an explanation about the theoretical framework used within this research project. In Chapter Three, I provide extensive details about the methods of data collection and analysis, which relied heavily on semi-structured interviews and classroom observation. Thus, in Chapter Three, an explanation is provided as to the methodology for this study and its importance. Chapter Four contains a description of the main findings that emerged from the three data sources for the study (observations, interviews, and lesson plan). In Chapter Five I present a discussion of the implications of the study in light of research, theory, and teachers' lesson plan development practices.

1.8 DEFINITION OF TERMS

Borderland Pedagogy: The art and science of teaching that affirms the notion of difference as a central organizing principle of a common effort to enhance the quality of public life by linking the classroom to democratic society. Border pedagogy posits pedagogical processes, in part, as a form of border crossing where existing social boundaries can be challenged and redefined. Thus, it presumes the need to create conditions that enable students to become border crossers in order to develop an understanding of others in their own terms so that knowledge can be constructed in light of such understandings (Giroux. 1992).

Cultural Sensitivity: Teacher has to understand the dynamics of how the communities in the border area work and the needs they have; teacher needs to know and learn about different cultures; enjoys and accepts the diversity of cultures, languages, and regions; teacher in the border region need to have contact with people of different socioeconomic classes, with people from distinct cultures and languages, and preferably to have an experience personally living in another country to better understand cultural values different from his or her own.

Flexibility: Respects students, without caring about their race, religion, or socioeconomic status; designs curriculum creatively to incorporate the students' backgrounds and ways of knowing the world; are adept at working with parents who may define their role differently and are unaware of the expectations that schools and educational system hold for them.

Language: Teacher encourages students to speak multiple languages; teacher needs to understand the acquisition of languages and the language used by these communities; teacher needs to have knowledge of Spanish and English.

Lesson plan development: Refers a teacher's detailed description of the course of instruction for one class. A daily lesson plan is developed by a teacher to guide class instruction. Details will

vary depending on the preference of the teacher, subject being covered, and the need and/or curiosity of students. There may be requirements mandated by the school system regarding the plan

Mexican Teachers: Refers to person who studied for a degree in Mexico. Also, they are Mexican citizenship.

Passion: Positive attitude; has passion, understanding for each individual student, or ability to help all students at different levels.

SEP: Secretaria de Educación Publica

Transition: “...complex process of shaping an identity moving between two places” (Elbaz-Luwisch, 2007, p. 389). Places is defined as “a given location that is not only specific, describable and distinct from other locations, but that holds meaning, that matters to the persons who inhabit it” (Elbaz-Luwisch, 2004, p. 388).

TEKS: Texas Essential Knowledge and Skills

U.S. Teachers: Refers to person who studied their bachelor degree in the United States. Also they have American Citizenship. Finally, they are certified as mathematics teachers.

1.9 SUMMARY

In the introduction, several important aspects of the study were discussed. Most importantly, we presented the aim of this study. The need for this study was determined by how a transition from Mexico (Ciudad Juarez, Chihuahua) to the U.S. (El Paso, TX) impacts high school teachers' lesson plan development and implementation practices incorporating Borderland Pedagogy. The definition of teacher transition was introduced based on the work of Elbaz-Luwish, (2007 and Sabar (2004). The critical aspect to conduct this study was to compare the lesson plan development practices from Mexican and U.S. mathematics teachers to provide an understanding of the process of teachers' transition. Thus, the purpose of the study was to explore how the teaching practices were part of a process of the lesson plan development to teach mathematics at high school level. Lesson plan development and teaching practices are compared between Mexico and U.S. educational system. This comparison was considered important since each educational system have their own lesson plan development and teaching practices. In the purpose we also stated the importance of the Borderland Pedagogy framework. In order to achieve this purpose we developed the following research questions: (1) What are the lesson plan development and implementation practices incorporating the Borderland Pedagogy used by U.S. high school mathematics teachers in El Paso, TX? (2) What are the lesson plan development and implementation practices incorporating the Borderland Pedagogy used by Mexican high school mathematics teachers in Ciudad Juarez, Chihuahua? (3) How does a transition from Mexico (Ciudad Juarez, Chihuahua) to the U.S. (El Paso, TX) impacts high school mathematics teacher's lesson plan development and implementation practices incorporating the Borderland Pedagogy? The five chapters of this dissertation were developed with the purpose to find the main arguments to give respond to the three research questions. Additionally, definitions of terms were provided.

Chapter 2: Literature Review and Theoretical Framework

2.1 INTRODUCTION

This chapter will discuss key bodies of literature that shape and inform current research relevant to the present study. As a basis for using Borderland Pedagogy as a framework for investigation and analysis, this chapter reviews literature about the impact of teachers' transitions (Elbaz-Luwisch, 2007; Sabar, 2004) of lesson plan development practices (Saiz 2002; Remillard, 2005; Buckley, 2010; Sun, Kulm, & Capraro, 2009; Li, Chen, & Kulm, 2009; Shimizu, 2008; Secretaría de Educación Pública (SEP), 2012; & Texas Essential Knowledge and Skills [TEKS], 2012) and on mathematics teaching practices at high school level. Specifically, this literature review will focus on the following topics: (1) High school mathematics curricula in Mexico and the United States (U.S.); (2) high school mathematics teaching practices in Mexico and the U.S. (e.g., Saiz, 2002, SEP, 2010, 2012; TEKS, 2012); (3) an analysis of high school mathematics teachers' lesson plan development practice in Mexico and the U.S. (SEP, 2010, 2012; & TEKS, 2012); and (4) an overview of theoretical frameworks (Giroux, 1992; and 1994; Artigue & Wislow 2010; Cline & Necochea, 2006; Romo & Chavez, 2006; Fiume, 2005) appropriate for further examination of the impact of teachers' transitions on lesson plan development practices.

2.2 HIGH SCHOOL MATHEMATICS CURRICULUM IN MEXICO AND U.S.

According to Buckley (2010), in formal schooling, each educational institution distributes the course curriculum to the students. Buckley (2010) and Saiz (2002) state that curricula in Mexico and the U.S. are prepared based on standards of the educational institutions approving the courses. A curriculum provided by an educational institution has the general adopted syllabus as the base for mathematics classes.

Serrano (1990) describes certain characteristics of curricula in Mexico that may differ from those of U.S. curricula. Specifically, in Mexico, curricula are structures. As Popkewitz (2001), asserts, "...the curriculum in Mexico had a very important role in the historicity of knowledge that inscribes rules and standards by which we reason about the world" (p.98). In Mexico, curricula are designed and organized so that the shape, order, continuity and consistency reflect not only the intended content but the actual ways of teaching in schools.

For Sun, Kulm, and Capraro (2009) and Remillard (2005), curriculum or lesson planning is regarded as part of the experience of teaching in an educational setting, and includes national/state standards, textbook, lesson plan, classroom instruction. Sun, Kulm, and Capraro (2009) assert that textbooks are one of the major curriculum resources, not only for facilitating lesson planning, but also for implementing classroom instruction. Research conducted by McCutcheon (1980) and Remillard (2005) affirmed that there was a relationship between textbooks and lesson planning. For example, when teachers plan their lessons, they read and use the textbooks in varied and substantial ways. This research found that the relationship between textbooks and teachers depended on how teachers interpret and react to students in the context of the classroom setting. The study found that individual teachers chose different criteria upon which to base their lesson plans. Some teachers may rely heavily on textbooks while others did not, since each teacher has different beliefs, knowledge, and experiences with various mathematics topics or pedagogy. Additionally, several researchers explored factors that influence how teachers use textbooks in combination with curriculum materials; however, there was not substantial information that would permit a close examination of how teachers use textbooks and curriculum materials to teach specific mathematics topics (Remillard, 2005).

Conversely, Sun et al. (2009) stated that the U.S. Curriculum and Evaluation Standards for School Mathematics (1989) called for alignment of high quality curricula with national standards (p. 20). The notion of standards-based curriculum has been an important topic in the mathematics education community. Sun and fellow researchers (2009) pointed out that some of the characteristics of a standard-based curriculum have been identified by the National Science Foundation (NSF), which has sponsored projects aimed at developing high school curriculum materials. Trafton, Reys, and Wasman (2001) proposed that a standards-based curriculum contained six characteristics: comprehensibility, coherence, development of ideas in depth, promotion of sense-making, engagement of students, and motivation for learning (p.60). However, for Buckley (2010), the standards-based curriculum in the U.S. contains the following characteristics: (1) engaging students; (2) promoting students' participation; (3) promoting interaction with peers. These characteristics can help the teacher know if the instruction or content is relevant so that students can participate through the activities and in response to the teacher's questions.

Research conducted by Beyer and Davis (2011) reported that the mathematics curriculum in the U.S. is positively correlated to mathematics achievement while the number of new topics presented at the eighth-grade level is negatively correlated to mathematics achievement. Regardless of mathematics skills before high school, taking algebra in the middle school is strongly related to achievement gains in high school. Furthermore, the math curriculum must provide students with opportunities to learn math at an early age.

**Table 2.1: Comparative Analysis of Mathematics' Curriculum in Mexico
and U.S.-1**

Mexico High School Mathematics Curriculum

The table below describes the topics and the general objectives established by the Secretaría de Educación Pública (SEP) in Mexico. In Mexico, these topics are taught by semester at the high school level. SEP designs the curriculum so that math teachers can know how to plan the components of each semester. These planning components include: (1) block distribution, (2) expected student performance at the end of each block, (3) learning objectives, (4) skills to develop, (5) teaching materials that teachers should use. SEP elaborates the math curriculum with the objective that math teachers can use to improve their teaching.

Texas High School Mathematics Curriculum

On the other side of the borderland, the Texas Education Agency established Texas Essential Knowledge and Skills (TEKS) standards to design high school mathematics curriculum. The TEKS for each high school mathematics course include basic understandings as well as knowledge and skills, which are subdivided by mathematical process and content standards. According to TEKS (2012), "... the basic understandings of number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry; measurement; and probability and statistics are essential foundations for all work in high school mathematics" (p. 1).

Topics	Mexico	U.S./Texas
Factorization	Math I	Algebra I
Alg. transformations II (Rational expressions)	Math I	Algebra I
Linear equation	Math I	Algebra I
Quadratics equation	Math I	Algebra I & II
Angles and triangles	Math II	Geometry
Congruence triangles	Math II	Geometry
Properties of polygons	Math II	Geometry
Circumference	Math II	Geometry
Trigonometric ratios	Math II	Geometry
Trigonometric functions	Math II	Pre-calculus
Elementary statistic	Math II	Pre-calculus
Criterion of similarity and Pythagorean Theorem	Math II	Geometry
Circle equation	Math III	Geometry
Parabola equation	Math III	Algebra II
Ellipse equation	Math III	Pre-calculus
Polar coordinate system	Differential calculus	Pre-calculus
Functions	Math IV	Algebra II
Inverse functions	Math IV	Pre-calculus
Polynomial functions	Math IV	Algebra I
Rational functions	Math IV	Algebra II
Exponential and logarithm functions	Math IV	Algebra II
Periodic functions	Math IV	Advanced quantitative reasoning
Limits, rate of changes	Differential calculus	Pre-calculus
Maximum and minimum problems	Differential calculus	Pre-calculus
Anti-derivative	Integral calculus	Pre-calculus
Area under a curve and Definite Integral	Integral calculus	Pre-calculus
Complex numbers	Middle school	Algebra II
Translation, rotation and reflection	Middle school	Geometry
Deductive reasoning	Geometry	Geometry
Proof of congruence	Math III	Geometry
3 Hyperbola equation.	Math III	Geometry
Parametric functions	Differential Calculus	Pre-calculus

Table 2.2: Comparative Analysis of Mathematics' Curriculum in Mexico and U.S.-2

The analysis of Mexico and U.S/ Texas borderland math curricula found that some topics implemented in high school mathematics curricula are different, as shown in the comparative tables above. Through a comparative curriculum analysis from Mexico and U.S./Texas, it was found that not all math topics are included in both countries' curricula. Also, this analysis revealed that the topics that are taught in both countries are not always taught in the same grade order.

México			U.S./ Texas		
Mathematics High School Curriculum			Mathematics High School Curriculum		
Semester	Topics	General Objectives	Course	Topics	General Objectives
1	Arithmetic and Algebraic problems; real numbers; sums and successions of numbers; algebraic transformation; linear equation; quadratic equation; congruence of triangles; similar triangles and the Pythagorean theorem; properties of polygons; circumference; trigonometric functions.	The student will learn to solve: Algebraic expressions in the context of positive, ratios, proportions and proportional variation as simple case of linear relationship between two variables; problems on sequences and series (arithmetic and geometric) of numbers, operations with polynomials in one variable; systems of equations 1x1, 2x2 and 3x3; quadratic equations in one variable .	Algebra I	Linear functions, equations, and inequalities; quadratic; exponential functions; number and algebraic methods	The student will understand: The function represents a dependence of one quantity; how algebra can be used to express generalizations; to simplify algebraic expressions and solve equations and inequalities in problem situations; the meaning of the slope; that the graphs of quadratic functions are affected by the parameters of the function and can interpret and describe the effects of changes in the parameters of quadratic functions.
2	Circumference definition; trigonometric ratios to solve triangles; trigonometric functions; sine and cosine laws; elementary statistics; concepts of probability	The student will learn to solve: And apply the properties of the elements associated with a circumference as radius, diameter, secants and tangents; exercises of perimeter and area of the circle; trigonometric functions; the meaning of population and sample, and central tendency.	Algebra II	Inverse functions; systems of equations and inequalities; quadratic and square root functions; exponential and logarithmic functions; Cubic, cube root, absolute value and rational functions, and algebraic methods.	The student will understand: The importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions; the connection between algebraic and geometric representations of functions; the relationship between the geometric and algebraic descriptions of conic sections; root functions; that quadratic functions can be represented in different ways and translates among their various representations; and formulates equations and inequalities based on quadratic functions, based on rational functions, and based on exponential and logarithmic functions,
3	Properties of line segments and polygons; elements of a line as geometric place; elements and the equations of a circle; the parable, and the ellipse equation. And hyperbola.	The student will learn to solve and understand: The analytical possibilities for metric calculations and line segments, polygons, geometrical properties of the line and its analytical possibilities; geometric properties, as well as properties of the parabola; and the characteristics of ellipses and hyperbolas.	Geometry	Transformational geometry; congruence; similarity, proof, and trigonometry; two-dimensional and three-dimensional figures ; circles; probability.	The student will understand: The structure of, and relationships within, an axiomatic system; and analyzes geometric relationships in order to make and verify conjectures; apply the logical reasoning to justify and prove mathematical statements; and uses a variety of representations to describe geometric relationships; and analyzes the relationship between three-dimensional geometric figures; and extends measurement concepts to find perimeter, area, and volume in problem situations; and applies the concept of congruence to justify properties; and applies the concepts of similarity
4	Exponential and logarithmic and periodic functions	The student will learn to solve and understand: Exponential and logarithms functions, as well as the periodic functions.	Pre-calculus	Functions; geometric reasoning; algebraic reasoning.	The student will understand: The characteristics of functions, and translates among verbal, numerical, graphical, and symbolic representations of functions, including polynomial, rational, power (including radical), exponential, logarithmic, trigonometric, and piecewise-defined functions; and use sequences and series as well as tools and technology to represent, analyze, and solve real-life problems; and use conic sections, their properties, and parametric representations, as well as tools and technology, to model physical situations.
5	Limits; rate of changes; maximum and minimum problems.	The student will learn to solve and understand: The historical background of the branch of mathematics and how its birth has contributed to the great progress of humanity.	Mathematical models with applications.	Mathematical modeling in personal finance, in science and engineering, in fine arts, in social sciences.	The student will understand to: Solve both routine and non-routine problems; graphical and numerical techniques to study patterns and analyze data; develops and implements a plan for collecting and analyzing data (qualitative and quantitative) in order to make decisions; and use probability models to describe everyday situations.
6	Apply the differential definition in the sciences, social, natural and administrative; Anti-derivative; area under the curve; definite integral.	The student will learn to solve and understand: Differential concepts; antiderivative functions; how to interpret the area under the curve using the sums of Riemann and solids of revolution , relating both methods to Newton's laws problems and population growth.	Advanced Quantitative Reasoning	Numeric reasoning; algebraic reasoning (expressions, equations, and generalized relationships); probabilistic and statistical reasoning.	The student will understand to: Develops and applies skills used in college and careers, including reasoning, planning, and communication, to make decisions and solve problems in applied situations involving numerical reasoning, probability, statistical analysis, finance, mathematical selection, and modeling with algebra, geometry, trigonometry, and discrete mathematics.

2.3 HIGH SCHOOL MATHEMATICS TEACHING PRACTICES

In Mexico and the U.S., high school mathematics teachers need to develop their practices and instruction so that students can develop reasoning and critical thinking. Critical thinking in mathematics involves reasoning in a logical manner in which students begin to formulate and test conjectures, make sense of things, and form and justify judgments, inferences, and conclusions (Mubark, 2011). Rivera (2007) asserts that the new role of the teacher established by the education reform in Mexico is to transition from practices where the teacher transmits knowledge, to innovative practices actively engaging students to use prior knowledge to solve problems. Studies conducted by Battista (1999) stated that it was important to increase the focus on improving student's achievement in mathematics through teaching practice. Teaching practices serve as a platform to value and promote mathematics classroom instruction excellence (Li & Li, 2009).

For Abdelfattah, Shumrani, and Hilal (2012), teaching practice refers to principles related to mathematics teaching and learning. These authors recognize that teachers were the most important factor in student achievement. Teachers need to know and understand the mathematics they are teaching as well as participate in on-going professional development to enhance knowledge of content and pedagogy. As well, teachers need to discover and use concrete materials for teaching practice to improve mathematics achievement (Maat & Zakaria, 2010). Studies conducted by Rivera (2007), showed that Mexican schools demonstrate weak knowledge of teaching mathematics. Rivera also said that teacher knowledge is scarce and, therefore, problem solving approaches were hardly carried out in their teaching practice.

To incorporate teaching practices effectively, teachers need to routinely reflect and collaborate on instructional practices, student progress, in addition to knowing and understanding

the mathematics they were teaching at a deep enough level to be able to explain and apply that understanding in a variety of formats (Abdelfattah et al., 2012; Li & Li, 2009). To have an effective pedagogy, teachers need to reinforce collaborative teaching practices (Walker, 2008). Collaborative learning has been singled out as the most promising technique to improve the students' development.

Granados, Núñez, Gallego, and Gonzalez (2008) noted that the teaching practice was not defined solely by what happens in the classroom. The spectrum of teaching practice was much broader, as its major task was not only performing teaching but transcending it. Granados et al. (2008) conducted a study to understand general concepts of creativity in teaching using a sample of six teachers from a school in the city of Barranquilla. The qualitative study was developed within an interpretative explanatory design and it not only accounted for the implicit theories on creativity, but it also allowed for the understanding of the reason why implicit theories exist. The authors defined implicit theories as a system of cultural knowledge and personal experiences used in daily life.

In addition, some of the perspectives of effective teaching practice can take place once curricular objectives clearly specify the particular knowledge, skills and values. Shimizu (2008) states that, "...effective teaching was one that produces demonstrable results in terms of the cognitive and affective development of the college students" (p. 944). By building and using effective teaching practices, students can achieve significant learning.

In contrast, another requirement of mathematics teachers in high school is that they can develop the instruction such that students can understand and can respond to teacher questions. For example, teachers can develop instructional methods to create learning environments and to specify the nature of the activity in which the teacher and learner were involved during the

lesson. While particular methods were often associated with certain strategies, some methods may be found within a variety of strategies. Instructional strategies determine the approach a teacher may take to achieve learning objectives (Johnson, 2000; and Li, 2007). Teachers who provide effective instructions can promote a reflective practice and discussion between teachers and students.

Johnson (2000) reported that one of the characteristics of effective mathematics instructions in high school was effective assessment. Effective assessment of mathematics learning must be performance-based, using multiple strategies and employ more open-ended assessment tasks than have been used in the past. Effective assessment practices are essential to support mathematics instruction that produces improved student performance (Rice, 1999). Trafton, Reys, and Wasman (2001) state that effective assessment in a standards-based environment requires that students be judged in terms of mathematical literacy, understanding of concepts and procedures, and the application of mathematical knowledge in problem-solving situations. Since most traditional assessment strategies are not designed for these purposes, new assessment models must be developed and their related assessments allow teachers to plan and adjust instruction accordingly.

In a study conducted by Li (2007) it was identified that there are other important characteristics that are believed to contribute to the quality of mathematics instruction. Lesson instructions commonly contain a teacher's analysis of textbook content, instructional objectives, consideration and design of teaching methods and procedure, and the teacher's consideration about students and their learning (Li et al., 2009, Li & Li, 2009., Shimizu, 2008). As well, Walker (2008) asserts that the need for effective instruction in the mathematics classroom is further documented in a study conducted by the U.S Department of Education, wherein it was

declared that there were significant disconnects between the high school curriculum and the expectations of the first year of college. The study suggested the need to increase the level of challenging academic content in high school. Trafton et al. (2001) and Johnson (2000) state that the need to increase the level of challenging high school curriculum is even more critical for poor and minority students as they are less likely than higher socioeconomic white students to attend college.

The quality of mathematics classroom instruction has not been a major concern of policy makers. For example, research by Hiebert et al. (2003) and Abdelfattah et al. (2012) used a classroom video approach as part of the Trends in International Mathematics and Science Study (TIMSS). These studies showed that a teaching gap has been revealed between U.S. mathematics classrooms and classrooms in other educational systems (Hiebert et al. 2003; Abdelfattah, et al., 2012). To Li and Li (2009), high school instruction may be designed with the objective to clarify what students should learn at each high school level. For example, Sun et al. (2009) found that standards-based instructions in high school mathematics are designed to provide a curriculum framework, concepts and knowledge that were to be mastered.

2.3.1 Analysis of High School Teaching Practices in Mexico

In Mexico, teachers' practices have had many purposes throughout its history. The disciplinary inquiry approach has resulted from concerns about the teaching of mathematics as teaching takes place in schools. Those concerns refer to teachers' skills and the practices they engage in when teaching their subject matter. Aguilar and Cruz (2003) affirm that in Mexico teachers' practices are characterized "... as the set of activities performed cyclically from a class planning, selection of teaching methods and learning strategies, problem solving" (p. 665).

The Secretaria de Educación Publica (SEP) (2012) defines teachers' practices as a set of strategies and actions used by the teacher in the teaching-learning process. The teachers' practice was a "...central variable of educational quality" (Greybeck., Gomez, M., Mendoza, 2004, p. 245). Teaching strategies generally depend on the personal and professional characteristics of the teacher. The ways teachers apply teaching strategies may vary even within the same courses, depending on their ideas and interests, aptitudes, institutional conditions in which it operates, and especially, the characteristics of the students. According to Cabrero, et al. (2008), teaching practice in Mexico is conceived of as the action that develops the teacher in the classroom, especially regarding the process of teaching, and is distinguished from global institutional practice and the social practice of teaching. Cabrero. and Enriquez (2010) report that teachers' practices are a dynamic, reflective process, comprising the events in the interaction between teacher and students.

Additionally, numerous studies (Saiz, 2002; Aguilar & Cruz, 2003; Garcia, Azcárate, & Moreno, 2006), reveal that the education system in Mexico has evolved at a constant method, in other words it refers to a traditional method to teach mathematics. These authors state that the study of teachers' practices inside the classroom is made with different intentions, for example to analyze aspects related to teacher training, teaching methods used by teachers in the classroom, learning strategies, and educational beliefs and thoughts.

2.3.2 Analysis of High School Mathematics Teaching Practices in U.S.

Studies focused on mathematics education (Saiz, 2002), mention that further research is required to explore the practices of teachers with respect to class planning and thought behind their practice. Several studies (Li et al., 2009; Chen, Brown, Hattie, & Millward, 2012; Menlo, Marich, Collet, Evers, Fernandez, & Ferris, 1990), state that most of the research on teachers'

practices have been linked to learning outcomes and student development. Cabrera and Nora (1994) mention that the practices of teachers in the development of choice plan were part of a complex process (e.g., classroom experiences). These experiences take place during teaching practices in order to succeed in developing the lesson plan.

Li et al. (2009) state that teachers in the U.S. were the most frequent users of practices in lesson plan development, compared to teachers in other countries (e.g, Mexico, Singapore) and saw their practices as more important for learning and for their own development and job satisfaction than do other teachers.

Scholars (Chen, Brown, Hattie, & Millward, 2012) assert that master teachers in the U.S. develop lesson plans and their practices in order to develop analytical critical thinking in the students. As Blomeke (2008) mentioned, the practices of teachers in the U.S. could be placed in the center of the teacher and the student, and the teacher was the key person in student learning (e.g., knowledge transmitted through an imitation, or a process of memorizing).

2.3.3 Comparing High School Algebra Teaching Practices in Mexico and the U.S.

Table 2.3 represents a comparative analysis of teaching practices of algebra in Mexico and the U.S., according to Chavarria, Carrillo, and Sanhueza, (2010); teaching practice of algebra, in general, and an algebraic expression, in particular, is not a simple task. These authors point out that it is not easy for students to see that the equal sign does not always mean equal. Also, it is not easy to understand that some letters are not always unknowns. According to the SEP (2012), teaching practices of algebra “...consist of knowledge of the action and the conditions that must be done” (p.3). This involves providing the student with the necessary knowledge system on the algebraic object that is in the algebraic actions to be performed and the order in which they must be implemented; such action initiates voluntary recall. During this

stage the student performs the action by just learning how to do the process to solve the problem. After completion, the student receives feedback from the teacher. In SEP (2012) it is stated, the teacher first suggests examples, gradually increasing complexity, from which the students will practice both the modeling and the solution to these examples. After that, the student receives feedback on the successes achieved in both the error correction model and its established solution. According to Chavarria, Carrillo, and Sanhueza, (2010) and the SEP (2012), algebra in general is a discipline in the service of other sciences and teacher dominant position proposed actions and procedures to follow during algebra class, ideas that do not show significant differences by teaching approach.

Furthermore, according to the Texas Essential Knowledge and Skills (TEKS) (2013) and Vallepedaz (1998), the teaching practices of algebra can be performed using technology. The TEKS (2013) do not mention what kind of mathematical software may be used, but suggest that the teaching practices to teach linear equation can be taught and resolved with the use of technology. Chavarria, Carrillo, and Sanhueza (2010), Vallepedaz (1998), and the TEKS (2013) concur that the development of technology for the practice of teaching algebra may be helpful to finding properties and modeling and to eliminating excessive symbolic manipulation. Furthermore, the SEP (2012) affirms that the development of technology for teaching practices is essential for the teaching of algebra at the high school level. More importantly, these authors maintain that through this mathematical analysis process, using technology or not, the student is responsible for their own learning. Table 2.3 presents a comparative analysis of high school algebra teaching practices in Mexico and the U.S.

Table 2.3: High School Algebra Teaching Practices in Mexico and the U.S.: Comparison Chart.

Mexico High School (Preparatoria)	United States: Texas High School
<p>According to SEP (2012), teaching practices for algebra include the following:</p> <ul style="list-style-type: none"> *Inquire about students' prior knowledge and skills regarding learning objects considered in block IV. *Suggest examples, whose complexity increases gradually, for which the student will practice both the modeling and the solution. *Provide feedback to the group on the successes achieved and on error correction in both the establishment model and its solution. *Motivate the class group to participate in peer assessment and self-assessment of attitudes as a measure of the learning obtained during the development of the activities. *Organize teams of three to five students and assign the task of researching available information on these topics: Natural numbers, Rational Numbers, Irrational Numbers, Real Numbers and Complex Numbers. *Propose the development of an educational game whose team members where most 5 shows the different real numbers, showing ingenuity quality. *Suggest problems that involve rates, ratios and proportions. *Show the way the calculator will serve as a tool for obtaining the result of the sum of a sequence or to find any term. * Explain the algebraic transformations (for basic operations and factoring) used in the solution of a problem and justify its use. *Suggest situations in which represents and transforms in algebraic language and trinomials rational expressions. *Explain how linear equations are solved with both unknown whole and fractional numbers. *Organize teams of 3 members to investigate what related to the characteristics and properties of a system of simultaneous equations in two unknowns. *Present systems of simultaneous equations numerical methods, algebraic and graphics. *Algebraic and graphics Organize teams of 3 members to investigate what related to the characteristics and properties of a system of simultaneous equations in three unknowns. *Present systems of simultaneous equations numerical methods, algebraic and graphics, as well, how graphing quadratic functions converting the general form of the standard. 	<p>According to TEKS (2013), teaching practices for algebra include the following:</p> <ul style="list-style-type: none"> *Present mathematical process standards using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations. *Applies the mathematical process standards using graphs of linear functions, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations. *Analyze mathematical process standards to solve, with and without technology, linear equations and evaluate the reasonableness of their solutions. *Model mathematical process standards when using properties of quadratic functions to write and represent in multiple ways, with and without technology, quadratic equations. *Apply mathematical process standards using graphs of quadratic functions and their related transformations to represent in multiple ways and determine, with and without technology, the solutions to equations. *Show mathematical process standards to solve, with and without technology, quadratic equations and evaluate the reasonableness of their solutions. *Applies the mathematical process standards when using properties of exponential functions and their related transformations to write, graph, and represent in multiple ways exponential equations and evaluate, with and without technology, the reasonableness of their solutions *Present algebraic methods to rewrite in equivalent forms and perform operations on polynomial expressions. *Apply mathematical processes to understand that exponential and logarithmic functions can be used to model situations and solve problems. *Present cube root, absolute value and rational functions, equations, and inequalities can be used to model situations, solve problems, and make predictions. *Present mathematical processes to simplify and perform operations on expressions and to solve equations. *Analyze data, select appropriate models, write corresponding functions, and make predictions.

2.3.4 Comparing High School Geometry Teaching Practices in Mexico and the U.S.

Table 2.4 represents a comparative analysis of geometry teaching practices in Mexico and the U.S. In Mexico, geometry is taught from the earliest levels of education emphasizing the construction of spatial thinking, which is an important component in the development of mathematical thinking. Geometry allows for numerical calculations through images, mental calculation, and estimation in problem solution finding. The teaching practices of geometry in Mexico include teacher creation of a variety of activities, intended figures, and knowledge of the properties that enables developing reasoning to solve problems and justify solutions. The SEP (2012) states:

“The teacher must apply geometric problems with applications to everyday life through the use of techniques, concepts and methods of geometry, which benefits from the deduction of graphic performance figures formed by lines in the plane”. (p. 8)

“El maestro debe aplicar problemas geométricos con aplicaciones a la vida cotidiana a través del uso de técnicas, conceptos y métodos de la geometría, que se beneficia de la deducción de comportamiento gráfico de figuras formadas por líneas en el plano” (p.8).

Additionally, teaching practices developed in Mexico are designed through the elaboration of activities established by students. Such teaching practices in Mexico are based on activities and presentations developed by the mathematics teacher.

Comparatively, the Texas Education Agency (2013) highlighted that teaching practices of geometry in the United States were designed in ways to help students apply geometry concepts in everyday life. The TEKS (2013) stipulate teaching practices of geometric problems involving situations relevant or related to the community, asking students to solve problems where one might similarly apply the criteria. This means that students can use knowledge acquired in the

mathematics lesson applied to real life. Within the teaching practices of geometry in the United States, teachers create activities wherein the student must apply mathematical problems to real life in addition, to incorporating an analysis of the problems they represent. Also, students may represent geometric shapes in two dimensions and three dimensions and may be able to use manipulatives and technology to formulate geometric representations. Table 2.4 represents the comparative analysis of high school geometry practices in Mexico and the U.S.

Table 2.4: High School Geometry Teaching Practices in Mexico and the U.S.: Comparison Chart.

Mexico High School (Preparatoria)	United States: Texas High School
<p>According to the SEP (2012), teaching practices for geometry include the following:</p> <ul style="list-style-type: none"> *Present to students the classification of angles triangles. *Request a collage where students display different angles and triangles. *Present students with the criteria of congruence: L, L, L (Side, Angle, Side), L, A, L (Side, Angle, Side), A, L, A (Angle, Side, Angle). *Formulate problems involving situations relevant in your community and ask the students solving exercises and / or problems where you apply similar criteria. *Show the students how exercises are solved using proportionality. If possible, the students visit a regional archaeological area and measure the height of pyramids or visit your city and measure the tallest buildings, considering the shadow cast by the sun. *Present and demonstrate to students the Pythagorean theorem. *Ask the students to investigate the concept and elements associated with a circumference. *Ask students to apply the properties of circumference in solving exercises and / or theoretical or practical applications, which will be developed by the teachers and whose characteristics should relate to real problems that occur in their community. *Explain to students how to calculate the perimeter and area of a circle. *Provide exercises to students that require them to convert angles from degrees to radians and vice versa. *Ask the students to make a concept map of direct and reciprocal trigonometric ratios of acute angles. *Ask the students to make a table for calculating the values of the trigonometric functions for 300, 450, 600 and their multiple implications. *Present students with the trigonometric functions of the Cartesian plane. *Construct graphs of the behavior of the trigonometric functions sine, cosine and tangent, and expose the class to electronic presentation, forming teams of five. <p>Explain to students the laws of sines and cosines.</p>	<p>According to the TEKS (2012), teaching practices for geometry include the following::</p> <ul style="list-style-type: none"> *Present the connections between algebra and geometry and use the one- and two-dimensional coordinate systems to verify geometric conjectures. *Describe rigid transformations (translation, reflection, and rotation) and non-rigid transformations (dilations that preserve similarity and reductions and enlargements that do not preserve similarity). *Present the deductive [inductive] reasoning to understand geometric relationships. *Analyze constructions to validate conjectures about geometric figures. *Apply the process skills with deductive reasoning to prove and apply theorems by using a variety of methods such as coordinate, transformational, and axiomatic and formats such as two-column, paragraph, and flow chart. *Apply the process skills in applying similarity to solve problems. *Apply relationships in right triangles. *Present the characteristics and dimensional changes of two- and three-dimensional figures. *Apply the formulas to determine measures of two- and three-dimensional figures. Apply theorems and equations about circles. *Present probability in real-world situations and how to apply independence and dependence of events.

2.3.5 Comparing High School Pre-Calculus Teaching Practices in Mexico and the U.S.

According to the SEP (2012), in Mexico, pre-calculus is not considered a separate subject. It is not considered a subject since Calculus I and II are taught in the last two semesters of high school; in Mexico, the equivalents of Calculus I and II are referred to as Differential Calculus and Integral Calculus. In Mexico, the content of the U.S. Pre-Calculus and Calculus 1 courses is attended to within Differential Calculus. Thus, since in Mexico pre-calculus is not considered a subject, some similarities in the teaching practices described by the SEP and the TEKS were found between the U.S. subject of pre-calculus and the subject of differential calculus in Mexico. First of all, in Mexico the teaching practices of differential calculus are presented through activities where teachers consider situations that occur in real, physical contexts. For example, according to the SEP (2012), one of the teaching practices focuses on the objective to provide different readings of the work done by Newton and Leibniz, and highlight the importance of their work in solving mathematical models applied in everyday situations. Additional teaching practices include having students design a blog on the internet or make a multimedia presentation and integrate a brief comment on the historical background of differential calculus and its applications in solving environmental problems. Another practice is to ask students to make a list of equipment, such as the figures and observable bodies in their immediate environment, and brainstorm by exposing the group element chosen, the mathematical model and its graphical interpretation. Additionally, the mathematics teacher may coordinate the group to team-build a box without a cover, making symmetrical bends in the edges of the sheet or gluing sand or some other material that allows comparison and explanation of volumes. Some practices for teaching pre-calculus concepts are conducted through the use of technology. For example, the SEP (2012) suggests the use of technology (e.g. geogebra, graph,

matlab) for teaching students to understand and graphically represent calculus topics like rate of change, speed and acceleration.

On the other hand, according to the TEKS (2013), in the U.S. the pre-calculus is considered a subject at the high school level. The teaching practices related to pre-calculus are to describe and analyze the attributes of functions. Students make connections between multiple representations of functions and algebraically construct new functions, as well as model and make connections between algebraic and geometric relations. Finally, they apply appropriate techniques, tools, and formulas to calculate measures in mathematical and real-world problems. Based on a comparative analysis, it appears that unlike the SEP (2012), the TEKS (2013) do not describe the development of teaching practices that include the use of technology. Table 2.5, below, represents an analysis of high school pre-calculus teaching practices in Mexico and the U.S.

Table 2.5: High School Pre-calculus Teaching Practices in Mexico and the U.S.: Comparison Chart.

Mexico High School (Preparatoria)	United States: Texas High School
<p>According to the SEP (2012), teaching practices for Pre-Calculus include the following:</p> <ul style="list-style-type: none"> *Provide different readings of the work done by Newton and Leibniz, and highlight its importance in solving mathematical models applied in everyday situations. *Designing a blog on the Internet or make a presentation multimedia and integrate a brief comment on the historical background of differential calculus and its applications in solving environmental problems. *Ask students to make a list of equipment the figures and observable bodies in their immediate environment, brainstorm by exposing the group element chosen, the mathematical model and its graphical interpretation. *Coordinate the group to team-build a box without a cover, making symmetrical bends in the edges of the sheet. You can use glue to add sand or some other material that allows for the comparison and explanation of volumes as first approach of highs and lows. *Form teams and explain the changes undergone in the landscape, crop production in the household goods, articles electronic, among others, and as the calculation contributed to change. *Promote the use of available software for graphs, such as Geogebra, Derive, Graph Math. *Develop practical experience in which the movement rectilinear uniformly accelerated vertical shooting, parabolic, circular motion freefall and to calculate the instantaneous velocity, acceleration and the average velocity. *Explain how to solve problems and represent them graphically, indicating which is the reason for change, instantaneous speed and acceleration; simulate movement of objects using software (derived, geogebra, graph, matlab). . *Guide the search for information on the Internet, to address mathematical models and graphical displays. *Organize a roundtable group and provide three questions to reflect on the importance of the study of calculus and its relation to everyday life. 	<p>According to the TEKS (2013), teaching practices for Pre-Calculus include the following:</p> <ul style="list-style-type: none"> *Describe and analyze the attributes of functions. The student makes connections between multiple representations of functions and algebraically constructs new functions. *Model and make connections between algebraic and geometric relations. *Apply appropriate techniques, tools, and formulas to calculate measures in mathematical and real-world problems. *Describe patterns, formulate models, and solve equations and inequalities using properties, procedures, or algorithms.

2.4 HIGH SCHOOL MATHEMATICS TEACHERS' LESSON PLAN DEVELOPMENT PRACTICES

To fully examine and explore lesson plan development, a deeper understanding about the factors that influence it should be considered. Li, Chen, and Kulm (2009) point out that some educational research has tried to explore how teachers' mathematics knowledge and their views of effective teaching impacts lesson planning development. For Li, et al. (2009), a teacher's mathematics knowledge is one of the most important parts in lesson plan development, since teachers need to understand subject matter deeply and flexibly to develop a successful lesson plan. For example, understanding the subject provides a foundation for pedagogical content knowledge that enables teachers to make ideas accessible to others (Shulman, 1986). Studies conducted by Beyer and Davis (2012) argue that "...teachers often engage in curricular planning by critiquing and adapting existing curriculum materials to contextualized lessons and competence for their deficiencies" (p. 132). They discuss how teachers' interactions with curriculum materials were mediated by their knowledge and beliefs about the subject matter, teaching, and learning. Moreover, students should be an integral part of a teacher's consideration in lesson planning. Also, Li, et al. (2009) discuss the fact that a teacher's view of effective teaching influences lesson plan development. For Walker (2008), teachers viewed effective teaching as that which makes the most significant impact on their students' lives.

Li, et al. (2009) argue that the values of lesson planning and developing high-quality lessons are still contested in teachers' views and practices. Peng (2007) maintains that lesson plan development commonly contains a teacher's analysis of textbook content, instructional objectives, consideration and design of teaching methods and procedure, and the teacher's consideration about students and their learning. Lesson plan development promoted teachers'

reflective practice and discussion in which it was used as an important activity in many schools that helped improve teachers' mathematics knowledge and classroom instruction (Li & Li, 2009; Li, Y. 2007 & Shimizu, 2008).

According to Stigler, and Stevenson, (2004), in some cases, the development of lesson plans are a complex three-part document which includes an introduction to the lesson plan, information about the unit, and information about the lesson. Shimizu (2008) points out that lesson plan development can be described as generalized views on a sequence of classroom events in the lesson by focusing on the treatment of a specific topic in mathematics. According to Shimizu (2008), teachers tend to be explicit in describing their instructional behaviors in relation to the sequence of events they describe. Furthermore, lesson plan development is a window through which we can see how teachers conceive of the structure of lessons in relation to their concrete instructional activities (Li & Li, 2009; Shimizu, 2008).

O'Donnell and Taylor (2006) state that if teachers can design a high quality lesson plan, they can build a solid base for classroom implementation. Quality instruction is, therefore, more likely to occur. Furthermore, Li et al. (2009) state that "...a better understanding of teachers' lesson planning would require further specifications of different system contexts and their nature" (p. 718).

A study conducted by Shimizu (2008) indicates that German and the U.S. lesson plans tend to have two phases: an initial acquisition phase followed by an application phase. During the acquisition phase, the teacher demonstrates and/or explains how to solve a sample problem. During the application phase, students practice solving examples on their own while the teacher helps individual students who were experiencing difficulty. In Japanese lessons, on the other hand, the teacher first posed a complex, thought-provoking problem for the students to solve.

Then, various students presented their ideas or solutions to the class. After the teacher summarizes the class's conclusions, students work on similar problems (Shimizu, 2008, p. 941). Blomeke, et al. (2008), in particular, indicate that high-achieving education systems focus much more on student cognition in classroom instruction, in contrast to teaching performance in Germany and the United States.

In Mexico, according to the SEP (2010), lesson plans consist of four phases. The first phase focuses on troubleshooting (theoretical or practical) conducted by the teacher. The second phase consists of individual and team work. The third phase consists of cooperative work. The fourth phase is constructed of group techniques used by the students to solve problems.

Research conducted by Stigler and Stevenson (2004) argues that development of the lesson plan should focus on discussing and solving mathematically challenging problems. Also, these authors found that it is important to engage students in solving problems with multiple solutions and justifications. For example, the SEP (2004) states that development of the lesson plan must focus on raising theoretical and practical problems, organizing student teams to solve problems, providing assessment tools, and feedback on students' errors or misconceptions.

Barab and Luehmann (2003) suggest that to devise a lesson plan, teachers need to develop the ability to adapt curriculum materials. They maintain that teachers need to analyze and adapt even high-quality, reform-oriented curriculum materials to better support students' learning. Studies conducted by Davis, Beyer, Forbes & Stevens (2012) state that adaptations can include insertions, deletions, or substitutions, and may be based on aspects of the teachers' contexts, their students' needs and strengths, and their learning goals, knowledge, beliefs, identities, and orientations. For instance, Thomson et al. (2008) found that Canadian teachers adapt curriculum materials to increase relevance to students and, occasionally, to gain insight

into student thinking. A teacher's ability to employ personal resources as well as resources embedded in the materials themselves to make productive changes to curriculum materials was referred to as the teacher's pedagogical design capacity (Sun, et al., 2009). Shimizu (2008) conducted a study wherein pre-service and in-service teachers independently developed a lesson plan for teaching a mathematics topic. The study focused on the teachers' conception of what a mathematics lesson comprises. The analysis revealed both similarities and differences between lesson plans produced by in-service and pre-service teachers. In particular, it was found that in-service teachers tended to retain the description of the problem to be posed and the anticipation of student responses in their lesson plans, while they abandoned other elements that they were trained to write when they were pre-service teachers. The results revealed that pre-service teachers wrote much more detailed lesson plans than those produced by in-service teachers in terms of the reference to goals and evaluation, number of steps/phases, and anticipated students' responses to the problem.

Additionally, Lewis (2009) discussed that lesson development was a professional learning approach in which teachers work in teams to formulate goals for students' learning and long-term development. Teachers conduct the "...lesson in a classroom, with one team member teaching and others gathering evidence on student learning and development; and discuss the evidence gathered during the lesson, using it to improve the lesson, the unit, and instruction more generally" (Lewis, 2009, p. 96).

2.4.1 Analysis of High School Teacher's Lesson Plan Development Practices in Mexico

In Mexico, the school education has been guided by the Secretaría de Educación Pública (SEP). Educational programs in Mexico focus on two important aspects: the goals they pursue as part of the formation of the personality of every Mexican and areas that together provide

experiences and activities with the objective to achieve student learning (SEP, 2010, SEP, 2012, Saiz, (2002). The Secretaria de Educación Pública is responsible to establish the lesson plans for all educational levels. The SEP (2012) points out that the lesson plan is shown as part of a list of contents for a course in which it should be taught at the high school level. In Mexico, the lesson plan is characterized as a set of instructions that teachers use to teach a course at a certain educational level. Lesson plan development helps the teachers to facilitate their educational activities in the classroom.

In Mexico, at the high school level, lesson planning development focuses on practices to strengthen students' skills to solve problems, make decisions, develop their creativity productively, and to proactively interact (SEP, 2012, p.4). In Mexico, the SEP prepares the course curriculum and distributes it as a brochure to students to describe the various features involved in studying a particular course.

On the other hand, lesson plan development practices in Mexico are considered actions and tools for teachers (SEP, 2010; 2012; Coll, 1992). Also, lesson plan development practices in Mexico are organized by subjects and disciplines. The areas of knowledge are the different subjects that are established at each educational level. Through the design of activities for the classroom, teachers attempt to provide students with tools to construct mathematics models (Díaz & Hernández, 2002).

The SEP (2012) asserts that mathematics plays an important role in problem solving as an element of motivation and in the learning environment for students. Thus, for students to be attracted to the study of mathematics, the content should have a sense and meaning relevant to their experience, that is, related to situations in their environment or field of knowledge accessible to their level of personal maturity and cognitive development (SEP, 2004). Also the

SEP addresses aspects of the surrounding environment (e.g., economic, social, environmental, demographics) and different fields of knowledge, which promote the development of critical thinking and a commitment in terms of students' performance.

2.4.2 Analysis of High School Mathematics Teacher's Lesson Plan Development Practices in the U.S.

Li et al. (2009) state that in the U.S. the lesson plan development practices "...serve as a link that connected what was provided in the curriculum guides and textbooks with the content to be taught in the classroom" (p. 717).

In the United States, the lesson plan development practices contain (Peng, 2007) the analysis of the textbooks that teachers use to develop their lesson plan practices. Several scholars (Li & Li, 2009; Li, 2007), maintain that lesson plan development helps teachers' "reflective practice and discussion in which it is used as an important activity in many schools that help improve teachers' mathematics knowledge and classroom instruction" (p. 721).

According to Stigler and Stevenson, (2004), in some cases, the development of lesson plans practices consist of an introduction to the curriculum, information on the thematic unit and lesson objectives. Shimizu (2008) states that lesson plan development practice "...can describe generalized views on a sequence of classroom events in the lesson by focusing on the treatment of a specific topic in mathematics" (p. 942).

Additionally Lewis, (2009) Li, et al. (2009) and Shimizu (2008), have written that lesson development is a professional learning approach in which teachers work together to formulate goals for students' learning. As Blomeke (2008) argues, "...lesson plans are likely treated differently in various education systems, such as frames of reference teachers' lesson planning

and thinking in the process of constructing curriculum for classroom instruction that were still largely unknown to outsiders” (p. 755).

2.5 TEACHERS’ BORDERLAND TRANSITION

Several studies conducted by Elbaz-Luwisch (2007), mentions that teachers who migrate from one country to another are faced with certain challenges that they must face when teaching in a new school system. For instance, this author states that "...teachers who have made a transition from one setting to another culture were likely to have developed an awareness of teaching and schooling in the new culture that other teachers may not have" (p. 387).

Several studies (Necochea, & Cline, 2006; Fiume, P, 2005, Romo & Chavez, 2006; Elbaz-Luwisch, 2007) mention that the teacher plays an important role in education at any level. These teachers may be different from the corresponding dominant assumptions, attitudes, beliefs and values (Cline & Necochea, 2006). For example, Rodriguez (2002) and Saiz (2002) state that Mexican teachers bring with them their own beliefs, attitudes and educational practices.

Several studies (Cline & Necochea, 2006) state that "...the cultural confluence of the two countries impacts all the daily life aspects” (p. 269); for this reason it is important that educators understand and know the language, the expectations of society, and its provisions as they may affected students. The teachers who migrate to another country bring with them their respective cultural baggage of assumptions, attitudes, beliefs, and values (Cline &Necochea, 2006; Romo & Chavez, 2006). Therefore, it is important that mathematics teachers who immigrate (e.g. from Mexico to the U.S.) must know the education system and should meet the needs of border communities and culture.

For this reason, teaching in a new educational system is not the only challenge for migrant teachers; challenges also include dealing with a new culture and language. As Cline and Necochea (2006) state:

“Teacher dispositions, attitudes, and motivation play important roles in educating students so that they are highly successful in school. This is particularly true for students in borderland areas who frequently need to negotiate two cultures, two languages, and two worlds”. (p. 268)

Studies (Cline & Necochea, 2006; Romo & Chavez, 2006) discussed that teachers should have the right disposition (p. 268) to educate transnational students. Perkins, Jay and Tishman (1993) define dispositions as “...people’s tendencies to put their capabilities into action” (p. 70). Cline and Necochea (2006) define dispositions as “...open-mindedness and flexibility; passion for borderland education; ongoing professional development; culturally sensitive; and pluralistic language orientation” (p. 271). Additionally, borderland teachers must have an open mind and respect their students. The diversity in the border presents significant challenges for the educational system. These challenges flow within the lives of the people living in the border. As Fiume (2005) declares, “...the borders that diversity influences and the categories they circumscribe, permeate the lives of people. Social borders present significant challenges for an educational system that brings together many different cultural perspectives for participation in a common dialogue” (p. 50).

Borderland teachers should not discriminate against students on the basis of religion, race, and socioeconomic status. As Cline and Necochea (2006) state:

“An effective teacher for the border region used his or her knowledge to work with students of different cultures. He or she realizes the need to value the different customs of

each child and support the children in class, and knows how to appreciate these differences. The differences between us were an opportunity to learn about a much bigger world". (p. 273)

In addition, several studies (Cline & Necochea, 2006; Romo & Chavez, 2006; Fiume, 2005) comment that borderland teachers must understand each student. Understanding each student means "...knowing each student by his family, past experiences and language skills and level of education" (Cline & Necochea, 2006, p.275).

Teachers in the borderland must understand the community and the needs of the student and community has. In other words, the teacher needs to understand and learn the language of the community, as well as the social economic status, which will help the teacher effectively communicate with students so they can succeed. In addition, teachers in the borderland need to learn about different cultures and customs so they can assess each student. As Cline and Necochea (2006) affirm, teachers in the borderland "...learned about different cultures and customs that make it possible to value each student for what he or she brings to the classroom" (p. 277).

2.6 THEORETICAL FRAMEWORK

In this section, I discuss an overview of theoretical frameworks (Giroux, 1992; and 1994; Artigue & Wislow 2010, Cline & Necochea, 2006; Romo & Chavez, 2006; Fiume, 2005) appropriate for further examination of the impact of teachers' transitions on lesson plan development practices. The theoretical framework is introduced for (1) Borderland pedagogy theory and (2) Lesson plan development practices. These major components of the theoretical framework helped me to analyze and examine the impact of high school level mathematics teachers' transition on lesson plan development practices to. The proposed study used qualitative

and quantitative approaches and had its theoretical perspective and its approach to the gathering of data, the types of data that constitute feasible areas of investigation, and the appropriate types of analyses for these data.

2.6.1 Borderland Pedagogy

Scholars (Cline & Necochea, 2006; Romo & Chavez, 2006, Fiume, 2005; Giroux, 1994 and 1992), mention that teachers, who teach in the border, should know the culture, language, community and educational system. For example, Cline and Necochea (2006) affirm that teachers in the border region "...need to understand and not discriminate the students by their race, socio-economic status and religion" (p. 272). Additionally, as Romo and Chavez (2006) and Cline and Necochea (2006) point out, it is necessary that teachers who immigrate to another country be open minded and be culturally sensitive.

Giroux (1992) defines border pedagogy as "...the need to create conditions that enable people to become border crossers in order to develop an understanding of others in their own terms so that knowledge can be constructed in light of such understandings" (p. 60). That is, the concept of border pedagogy suggests that teachers exist within the local social, political and cultural contexts. Giroux (1992) states that border pedagogy offers "...teachers the opportunity to express their feelings about race from the point of view of the positions of the subjects experiencing as establishing their own identity" (p. 24).

For Giroux (1994) through border pedagogy, teachers have a theoretical control over the ways in which difference is constructed, as this can take several representations and practices that "...name, legitimate, marginalize and exclude the voices of subordinate groups" (Giroux, 1994, p.53). Giroux (1994) affirms that:

“This will allow the teacher theoretical control work so educational content relevant to the achievement of educational goals already outlined, but the actual achievement of these purposes requires that the teacher who becomes a border crossing to legitimize the difference as a basic to understand the limits of self-knowledge” (p.53).

At the heart of the notion of border pedagogy are a number of important educational questions about the role that teachers can take to make a commitment to the fight against racism in classrooms, schools, communities and society in general. The concept of border pedagogy also helps teachers locate within social, political and cultural factors that define and mediate complex ways how the function of intellectuals to practice certain forms of moral and social regulation.

Additionally, scholars (e.g., Cline & Necochea, 2006; Romo & Chavez, 2006), indicate that there are some important characteristics of teachers who will be effective. These characteristics are considered essential to the teachers who will be successful in the border region. In the following paragraphs I describe what the characteristics of effective teachers on the borderland are.

2.6.2 Characteristics of Effective Teachers on the Borderland

Scholars (Cline & Necochea, 2006; Romo & Chavez, 2006; Fiume, 2005; Garza, 2007) point out those teachers who teach in the border area should have certain characteristics to be effective teachers on the borderland. First, teachers should be bilingual. Second, teachers should know both cultures. Third, teachers should have an open mind to understand the cultural diversity. Next, teachers should be passionate about their profession. Finally, teachers need to be creative. In the following paragraphs, I will explain each of these characteristics of effective teachers on the borderland.

Research by Cline and Necochea (2006) and Romo and Chavez (2006) suggests that if teachers know and speak both languages in a border region, they can communicate with students and parents and therefore, be bilingual teachers and help students achieve better academic performance. As Cline & Necochea (2006) state, “A teacher in the border should speak two languages to help the parents and the students with instructions. The teacher does not need to be bilingual, but needs to have enough fluency to give confidence to the parent” (p. 279).

So, while language can be helpful for instruction in border classrooms, it is also an important aspect of borderland teaching in other ways. In addition to encouraging students in their academic performance, it also helps to have a better relationship with parents. According to Cline and Necochea (2006), having a better relationship with parents refers to when there is communication and trust between parents and teachers with regards to monitoring the student's academic performance. Teachers who migrate from one country to another have additional challenges with the language and they should give importance to the native language of the country where they teach. Therefore it is important for teachers to pay attention to the language of the country wherein they are teaching.

Second, to be effective teachers on the borderland, teachers need to know both cultures. It is important for teachers to know the culture because by knowing the culture, teachers can have tolerance and respect for students. It is important to be familiarized with both cultures since it is part of the common heritage of mankind. The culture manifests itself in the diversity of language, religious beliefs, art, in music, in the social structure. Scholars, such as Cline and Necochea (2006) support the importance of familiarity with culture for teaching, stating that:

“An effective teacher for the border region uses his or her knowledge to work with students of different cultures. He or she realizes the need to value the different customs of

each child and support the children in class, and knows how to appreciate these differences. The differences between us are an opportunity to learn about a much bigger world”. (p. 273)

Additionally, Cline and Necochea (2006) assert that teachers should have an open mind in order to be tolerant of cultural diversity, being that it can help to have better relationships and stronger social networks with students and parents.

Finally, teachers need to be creative. Teachers need to provide students with the skills necessary to face new situations successfully. Creativity is one of the tools in accomplishing this task. For teachers to succeed in the task of helping students to develop their creativity, they too must also be creative, and have personality traits such as being clear on the importance of the educational mission. Teachers should strive to develop creativity because through it, they can develop a sense of responsibility, dedication, and the tenacity to transmit the enthusiasm of learning to their students. Scholars (Cline & Necochea, 2006) state that “...effective educators in the borderlands creatively look for ways to help students access the core curriculum and learn the skills necessary for success” (p.272).

2.6.3 Lesson Plan

Scholars (Spooner, Baker, Harris, Delzell & Browder, 2007; Beyer & Davis, 2012; Flannery, 1995; Seamon, 1999) highlight lesson plan design as one of the most important challenges facing education. Beyer and Davis (2012) declare that lesson plan design can be viewed as the core or center of education, and educational processes that occur and should occur in an institution. Some scholars (Spooner et al., 2007) mention that lesson plan design at the high school level refers to organizing the contents of various subject areas (e.g., mathematics, physics, chemistry, and literature). In the lesson plan, design is necessary to establish goals, develop

learning activities, and to evaluate results of the lesson plan design. Beyer and Davis (1995) state that lesson plan design is built as:

“...a framework of thinking about the curriculum so that the curriculum mentality offer a more comprehensive and complete than that (theory or practice). Such a scheme of thought requires incorporation of ideas, trends and political and ethical perspectives, cultural, scientific, academic and pedagogical techniques, claiming systematic reflection both as an organization and classification, in order not to fall into exhibitions and decisions without sense”. (p.340)

Therefore, lesson plan design presupposes knowledge and ability to plan the sequence of processes, stages, phases and articulates and gives operations and organization of the curriculum components (elements and processes).

Some scholars (Seamon, 1999; Spooner et al., 2007) point out that an effective lesson plan design should contain a set or sets of components that provide a framework for planning and execution of instructions. For example, lesson plan design should contain the objective of the lesson introduction and the basics of the subject content, providing practical guidance for instruction. As Spooner et al. (2007) highlight, an effective lesson plan design should contain/include the following steps:

- a) The lesson plan design will include content that avoids stereotypes of race, gender, disability.
- b) The lesson plan design will incorporate aspects of diversity in all subjects and curriculum areas, and in all phases of school work.
- c) The lesson plan design will ensure the use of inclusive language.
- d) The lesson plan design will ensure multilingual knowledge of society.

e) The lesson plan design will consider the baggage of past experiences of all girls and all boys.

f) The lesson plan design will provide access to all students. As Lim et al. (2010) state “an effective lesson plan design should contain and include these steps to have success in the teaching and learning of mathematics” (p.595).

Additionally, in the study “Teachers Disposition for Effective Education in the Borderlands,” Cline and Necochea (2006) state that lesson plan designs are elaborate in order to “...create flexible programs, learning schedules, and curriculum. Borderland teachers didn’t try to follow a program lockstep, but instead evaluate what is working with the students and plan accordingly to the students’ needs” (p. 272).

2.6.4 Lesson Plan Design Aimed at Developing Critical Thinking

A number of studies (Alwehaibi, 2012; Barnett & Francis, 2012; Choy & Oo, 2012), mention that lesson plans designed to teach mathematics at the high school level are created in order to develop critical thinking in the students. Other researchers (Choy & Oo, 2012; Korkmaz, 2012) comment that if the design of a lesson plan is intended to help students develop critical thinking, this means the students would develop a mental ability that allows them to interpret complex ideas, and evaluate function arguments based on this type of evidence. Also, it helps them to distinguish between something that is reasonable and something that is not. As Korkmaz, (2012) and Camichael and Farrell (2012) point out, developing critical thinking skills in students helps them to explore and question the origins of some mathematics concepts (e.g, demonstration of the equation of the circle, hyperbola, trigonometric functions). As Korkmaz (2012) affirms, “...digging in the proofs of geometric concepts students seeking settlement options and answer all the time because it is investigating in order to form an opinion unique and authentic” (p. 185).

Therefore, lesson plan design to foster the development of critical thinking in the students should be undertaken from teaching all areas. In developing critical thinking, students are able to transform everyday reality and focus on improving social conditions. Lesson plan design engages students in critical thinking that helps them to be more creative and properly handle environmental problems. As Spooner et al. (2012) asserts:

“Teachers design lesson plans as a guide in their planning, critiquing and adapting them to address reform-based goals and practices and specific contextual needs. To become well-started beginners in planning lessons, novice teachers need opportunities to develop their pedagogical design capacity-that is, their ability to use personal and curricular resources in designing instruction for students”. (p. 120)

The lesson plan must be created with the purpose to meet the content expectations in order that students become recipients of knowing.

2.7 SUMMARY

This chapter provided important and essential information about the literature review and theoretical framework. Also, this chapter included a review of the high school teaching practices in Mexico and the U.S. Furthermore, this chapter included a comparative descriptive analysis of teaching practices in Mexico and the U.S. with regards to teaching the subjects of algebra, geometry and pre-calculus. To complement this review, a comparative description was also presented about Mexico and the U.S. curriculum. The second part of this chapter provided an extensive review of the theoretical framework implemented in the study.

Chapter 3: Methodology

3.1 INTRODUCTION

This chapter describes the methodological design that was used to conduct this study. In the first section, the research design is explained, including justification for selecting a mixed methods and the importance of the vertical case study approach. In the subsequent section, the research questions that guide this study are presented, followed by an exploration of the context of the study. The description of the context of the study includes the setting where the research took place and an explanation of the characteristics used to select the participants. Data collection and data analysis are elaborated upon, including the criteria of and the process for recruitment of participants. Also provided is a description of accessing to and recruiting each participant. In addition, the analysis process is described. Finally, a summary of these sections is provided at the end of the chapter.

3.2 RESEARCH DESIGN

First of all, there were several considerations impacting the process of selecting a mixed methods quan-QUAL approach. According to Tashakkori and Teddlie (2010), the advantage of conducting a mixed methods design "...was that they allow the researcher to maintain the complexity of the phenomena within the research project" (p. 350). Moreover, these authors mentioned that quantitative methodology allows the researcher to analyze the data in a scientific manner, or more specifically in numerical form. A quantitative approach requires a relationship between elements of the research problem whose nature is representable by a model line. This means that there was clarity between the elements that make up the research problem, it was possible to define, limit and know exactly where the problem starts, in which direction it goes and what kind of impact has between its elements.

Second, there are some important considerations to conduct a qualitative approach. For example, according to Merriam (2002), qualitative studies are commonly used to understand any phenomenon more deeply. Yin (1994) describes qualitative studies as a research process whereby the researcher explores a social or human phenomenon. Also, Merriam (2002), qualitative studies are used to gain new knowledge about known issues or to gain more in-depth knowledge that can be difficult to acquire or deal with when using a quantitative approach. Additionally, qualitative studies require more time and effort to further the research efforts.

According to Denzin & Lincoln (2003), in qualitative studies, data analysis involves revising descriptions and meanings. Qualitative data analysis is exciting because themes and concepts are found tucked between the data collected. Throughout the progress of analysis of data, these themes and concepts are woven into a broader explanation of theoretical or practical importance, which then guides the final report (Rubin and Rubin, 1995). This analysis must be systematic and follow a sequence (Alvarez-Gayou, 2005).

Finally, the importance of conducting a case study as methodology is supported by multiple scholars. Gibbert, Ruigrok, & Wicki (2008), Meyer (2001) and Gerring (2004) stated that conducting a case study can help the researcher to interpret the data as well meaning that the data may become subjective. Case studies are research methodology that focus on and gather in depth information about a specific group, person, community and event. For example, my single case study included three groups of participants (Mexican, U.S. and Transitioning mathematics teachers) as well as, it was important to analyze the data through vertical case studies. Vertical case studies promised a number of advantages for this research study. Vavrus and Bartlett, (2006) assert that through a vertical case study there will be a comparative knowledge claims among each case study. In other words, there will be a cross case comparison

about the Characteristics of Borderland Pedagogy incorporated by Mexican, Transitioning and U.S mathematics teachers during their lesson plan development and implementation practices. Vertical case studies should be grounded in settings like community, schools and institutions. Another advantage of the vertical case study approach is that it "...recognizes the decentering of the nation-state from its privileged position as the fundamental entity in comparative research to one of several important units of analysis" (Vavrus and Bartlett, 2006, p. 99). This was critical in this study to identify the characteristics of Borderland Pedagogy exhibited by each of the participants and facilitate comparative analysis.

In this study, qualitative data were obtained through lesson plan collections, three classroom observations per teacher, and one semi-structured interview with each participant. Finally, the participants completed a survey asking about their demographic information and language profile. These data were used to help the researcher to build an understanding of the studied phenomenon (Creswell, 2012). Also in this study, each participant engaged in one interview, taking no more than 30 minutes each. Each interview was captured utilizing recorded and transcribed audio. Subsequent to the interviews having been transcribed, the researcher analyzed the transcriptions line-by-line and categorized by topic, assigning initial codes to the data. With additional, focused data analysis, specific codes were assigned and patterns that emerged from the data were identified; that is, the data analysis process was developed in more detail subsequent to the data collection and initial data analysis phase.

As mentioned before, in this study, data were obtained through lesson plan collection, three classroom observations and one semi-structured interview. The 15 lesson plans were collected after each observation of each participant; there were 15 observations in total. The classroom observations consisted of observing strategies and methods of lesson plan design and

implementation in the classroom used by participants when they are teaching high school mathematics. Each classroom observation took at least 45 minutes or more with each participant. The strategies and methods were applied according to their experiences as a mathematics teacher. Each of these data sources played an important role in generating new insights implicit in the data. This study attempted to use a Borderland Pedagogy framework to understand and determine how a transition from Mexico (Ciudad Juarez) to the U.S. (El Paso, TX) impacts high school mathematics teachers' lesson plan development and implementation practices. The lesson plan designs were used to know what strategies or methods were employed by the participants to teach mathematics, and what characteristics of Borderland Pedagogy focused on discovering new theory from data. The Borderland Pedagogy approach was a good fit for this study and produced rich data.

Following data collection, transcription and organization, the next task was to interpret the data (Alvarez-Gayou, 2005). The challenge was to simplify and make sense of the complexity contained in the field notes (Patton, 2002). For this reason, it was necessary to use a coding process that allowed for development of a manageable classification or coding system (Patton, 2002), as it is considered "... the heart and soul of the whole text analysis" (Ryan & Bernard, 2003, p. 274). Coding encouraged the researcher to make judgments about the meaning of contiguous blocks of text and eliminated the chaos and confusion that would occur without a classification system. This implied an intellectual and mechanical work that allowed encoding data, finding patterns, labeling issues and developing category systems. This included analyses of the core content of the lesson plan collections, interviews, and classroom observations in order to determine what was significant, and to recognize patterns in the qualitative data, while transforming these patterns into meaningful categories and themes (Patton, 2002).

3.3 CONTEXT OF THE STUDY

Recent studies demonstrate that the population of Hispanic teachers in the United States comprises over 80% of the U.S. English Language Learner population (Capraro, Capraro, Yetkiner, Rangel-Chavez, & Lewis, 2010). For example, according to the U.S. Department of Education (2010) the teacher population in Texas was 4,752,148, where 47.9% (2,276,278) are comprised of Hispanic teachers. These statistics confirmed that the population of Hispanic teachers represents almost half of the entire population of Texas teachers. According to the Texas Department of Education Report (2010), Hispanic teachers' performance in mathematics and science vary significantly not only from White teachers, but also from those who were classified as English proficient.

3.3.1 Sample Design

This mixed methods study used a purposive sampling design. According to Gay, Mills, and Airasian (2009), the justification for purposeful sampling is the notion that sample selection should be based on researchers' knowledge and experience of the participants to be sampled, using clear criteria to guide the process. Purposeful sampling strategies are frequently used in qualitative studies or portions of studies, with the objective to include information-rich cases that might be appropriate in terms of shedding light on a phenomenon of interest (Jones et al., 2006).

3.3.2 Sampling Criteria

The purposeful sampling criteria were developed in order to find six participants who could provide a deeper understanding about their lesson plan development and implementation practices incorporating Borderland Pedagogy. These practices could have been implemented and developed by U.S., Mexican and transitioning high school mathematics teachers. Specific sampling included:

- (1) Two of the participants were required to be teachers of mathematics at a high school level in the United States (El Paso, TX),
- (2) Two of the participants were required to be teachers of mathematics at a high school level in Mexico (Ciudad Juarez, Chihuahua),
- (3) Two participants were required to have experience teaching mathematics at a high school level at Mexico (Ciudad Juarez, Chihuahua) and have recently transitioned to El Paso, TX, where they continued working as mathematics teachers.

The sample for this study was Mexican and Mexican-American high school mathematics teachers. This study was conducted in Ciudad Juarez, Chihuahua and El Paso, TX. Two high schools were public schools and one was an alternative school. The alternative high school and one of the public high schools were located at El Paso, TX.

3.3.3 Participant Recruitment

The participant recruitment process was comprised of three steps:

Step One: Meetings with Dr. Mourat Tchoshanov, Professor, University of Texas at El Paso to identify the potential U.S. and transitioning participants. The objective of the meetings was to determine if identified students enrolled in classes taught by Dr. Mourat Tchoshanov met the criteria established for the selection of the U.S.A. and transitioning participants.

Step Two: Meet with identified potential participants to explain the purpose of the study.

Step Three: Meet with the principal of the high school at Ciudad Juarez to explain the purpose of the study and the objective of identifying two Mexican participants. Table 3.1 represents the sampling characteristics that were utilized for the participants' recruitment.

Table 3.1: Participant Sampling Characteristics

	First Language	Bachelor Degree obtained	Teacher Certification	Teaching Location	Instructional Grade Level
Teacher A	English	U.S	Yes	El Paso, TX.	HS
Teacher B	English	U.S	Yes	El Paso, TX.	HS
Teacher C	Spanish	Mexico	No	Ciudad Juarez	HS
Teacher D	Spanish	Mexico	No	Ciudad Juarez	HS
Transitioning-Teacher E	Spanish	Mexico/U.S	Yes	El Paso, TX.	HS

3.3.4 Informed Consent

Of the potential participant pool, five viable participants were identified and were invited to participate in the research project two Mexican teachers, two US teachers, and one Transitioning teacher. Subsequent to extending an invitation, information about the purpose of the study was provided and an overview of human subject participation, including privacy, confidentiality, time commitments and other topics was covered, as well as the information found in the consent form (See Appendix D). The purpose of the research project having been explained and agreement to participate obtained, participants were asked to complete volunteering in the research project and that they had the right to withdraw with no penalty or repercussions at any time.

3.3.5 Data Collection

The research was designed to acquire a deeper understanding of lesson plan development and implementation practices incorporating aspects of Borderland Pedagogy, as used by U.S., Mexican, and transitioning high school mathematics teachers. The data collection was achieved

through lesson plan collection, three classroom observations per participant, and one semi-structured interview.

Lesson Plans Collection

Lesson plans were one of the main sources of qualitative data collection and representative of a vital component of the teaching-learning process. Collection of lesson plans allowed for responding to the three research questions focusing on lesson plan development and implementation influenced by the teachers' unique experiences. Utilizing characteristics of Borderland Pedagogy as the theoretical framework, lesson plan development and implementation were linked to the teachers' unique experiential backgrounds. Finally, lesson plans were collected to answer the question of how the mathematics teachers communicated to learners what students were to learn and how students would be assessed. It is asserted by the researcher that lesson plans help instructors organize content, materials, time, and instructional strategies, and provide structure in the classroom. Lesson plans were created employing an activity to introduce a concept or skill (e.g., to introduce new vocabulary by asking learners to work in groups to identify words related to taking medications) and then introduction of information through a variety of modalities using visuals, description, explanation, and written text. Lesson plans were collected, as provided by the teachers at the end of each classroom observation during which the lesson plan had been implemented, for each observation time frame. Two of the five participants used the same lesson plan during the three observations.

Classroom Observations

Two primary data sources in qualitative research include: observations and interviews. To conduct an observation, qualitative researchers write field notes or record everything they can observe at the scene. According to Emerson, Fretz, and Shaw (1995), writing field note

descriptions involves active processes of interpretation and sense making. In the following paragraph, a description about how many days and hours required for the classroom observations for each participant is included.

For each participant, there were three classroom observations. The observations took approximately 50 minutes or less per participant and were conducted consecutively over three days; in total, 15 classroom observations were conducted. The classroom observations provided a venue to validate the information obtained from interviews and documents, as well as a means to seek explanation of the lesson plan development connected with the methods and strategies that mathematics teachers used to develop the lesson plan. The main focus of the classroom observations was to research how mathematics teachers in Mexico and the U.S. develop and implement lesson plans incorporating Borderland Pedagogy. As well, these studies examined and identify what strategies and methods participants used to design implement and their lesson. Furthermore, the classroom observations facilitated understanding of the teachers' perceptions about what strategies and methods were the most appropriate to develop their lesson plans.

In addition, the researcher was positioned as an observer rather than a participant observer. By "being there" (Wolcott, 1999), the researcher did not intend to participate but just observe and make sense in order to find the logic underlying the behaviors of people's verbal and non-verbal behaviors (Marshall & Rossman, 1999). Thus, the researcher was a participant as observer by being present in the classroom during lesson plan implementation (Wolcott, 1999). The researcher observed all participating mathematics teachers and their classes from the start to end of a unit. As mentioned previously, each class period observed encompassed approximately 50 minutes or less for each participant. After each classroom observation, the researcher took pictures of the classroom to describe the classroom environment. It helped as another source in

the analysis part. Also, the researcher wrote field notes in detail about the main things that the researcher observed into the classroom throughout the lesson implemented by the mathematics teacher, taking in consideration the characteristics of Borderland Pedagogy.

Observation Protocol

The observation protocol (See Appendix O) was developed according to the characteristics of Borderland Pedagogy proposed by Cline and Necochea (2006). This observation protocol was developed in order to clearly identify what characteristics of Borderland Pedagogy were incorporated into the development and implementation practices of high school level mathematics teachers. An extensive explanation about the observation protocol is described in the data analysis section, including description of each of the characteristics.

Interview Questions

Supporting a qualitative approach, another main data source was the interview. A semi-structured interview was defined as "... an interview whose purpose is to obtain descriptions of the life world of interviewee with respect to interpreting the meaning of the described phenomena" (Kvale, 1996, p. 6), and was the type of interview used to gather data for the purposes of this study. Oakley (1981) argued that "... some of the aspects in interviewing are considered legitimate or illegitimate from the viewpoint of inclusion in research" (p.31). Kvale (1996) further defined the interview as "... a conversation that had a structure and a purpose" (p. 6). The interview employed for the purposes of this study was considered to be a significant instrument for data collection. Through transcription of the interview, the researcher was able to analyze data and answer the three research questions of this study. Siedman (2006) supports transcribing the interview as one of the processes of analyzing and interpreting the material.

Concurring, Denzin and Lincoln (2005) state that for the purpose of the study, the participants can be interviewed to obtain validating data and provide answers to the research questions.

In this study, one semi-structured interview was conducted with each participant subsequent to the three classroom observations. In the single semi-structured interview, data was gathered about participant teachers' lesson plan development practices with regard to incorporating Borderland Pedagogy (See Appendix A).

3.3.6 Data Analysis

Lesson Plan Analysis

Based on the framework of Border Pedagogy, this mixed methods research was designed to ascertain how mathematics teachers incorporated the characteristics of Borderland Pedagogy into lesson plan development and implementation of high school mathematics. The method of acquiring and analyzing the lesson plans consisted of two steps:

Step One: The researcher met with each mathematics teacher in the respective high schools where the mathematics teachers teach and the researcher gave a brief explanation about the objective of collecting the lesson plans.

Step Two: In this step, the researcher began with one of the most important phases of the research: Data analysis. The researcher determined what the most convenient or adequate data for the purpose of the research was. The researcher determined which data was appropriate for the study in terms of addressing the research questions. For this reason, the researcher met with each mathematics teacher in the high schools to collect the lesson plans. In order to analyze the lesson plans, the researcher considered the characteristics stated for Borderland Pedagogy, as per Cline and Necochea (2006).

Classroom Observation Protocol Analysis

During the observation, field notes were taken on the content of the lessons, observed behaviors and interactions that occurred during each lesson observed. Observations were guided by the Borderland Pedagogy observation protocol as applied to lesson plan development and implementation practices used when teaching mathematics. The observation protocol contained the characteristics of Borderland Pedagogy, as well as descriptive data such as time of the class, and pseudonym of the participant who was observed. The observation protocol contained negative and positive aspects of the characteristics of Borderland Pedagogy (see table 3.3 and 3.4). Negative aspects were developed to identify variance from positive aspects described by Cline and Necochea (2006). Informal conversations with each teacher during and after each of these lessons were included. These discussions helped clarify the intent of the teacher for various parts of the lesson, along with the teacher comments regarding their beliefs about them. The following table represents the observation protocol used by the researcher to conduct classroom observations.

Table 3.2: Observation Protocol





Date: _____	Time Start: _____	End: _____	Teacher: _____
Observer(s): _____		# of students: _____	
Open-mindedness and Flexibility	Lack of Flexibility 	Open-mindedness and Flexibility	
Passion for Borderland Education	Lack of Passion 	Accept diversity of cultures	
Cultural Sensitivity	Lack of Cultural Sensitivity 	Cultural Sensitivity	
Pluralistic language Orientation	Lack of Pluralistic Language Orientation 	Pluralistic Language Orientation	

Table 3.3 Characteristics of Borderland Pedagogy

Characteristics	Descriptions	Coding Examples	
		Presence	Absence
Flexibility/Lack of Flexibility	Respect students, without caring about their race, religion, or socioeconomic status	Enjoy the diversity of cultures.	Identifies environment as monoculture
	Designs curriculum creatively to incorporate the students' backgrounds and ways of knowing the world.	Use of technology, hands-on activities and foldables to implement geometry class.	Practice and practice. Don't consider important the use of technology to implement algebra class.
Passion/Lack of Passion	Positive attitude.	Enjoy explaining the geometry concepts.	Didn't enjoy explaining the case of factorization.
	Has passion or understanding for each individual student.	Different method or strategy to explain the geometry concepts.	Didn't change the method or strategy to explain the different case of factorization.
	Ability to help all students at different levels.	Content knowledge or content language to develop and explain their lesson plan.	Lack of content knowledge or lack of content language.
Cultural Sensitivity	Teacher does understand the dynamics of how the communities in the border area work and the needs they have.	Speak both languages with the students, since they are considered as ELLs.	Speaks only one language
	Teacher does need to know and learn about different cultures.	necessary to know about diversity of cultures, since they live in border area.	Math teachers not considered necessary to know about diversity of cultures, since they live in Ciudad Juarez.
	Enjoy and accept the diversity of cultures, languages, and regions.	Necessary to know about diversity of cultures. implemented the instructions in both languages.	Not necessary to know about diversity of cultures. As well, math teachers implemented the instructions in English language or they didn't speak both languages with students.
	Teacher in the border region does need to have contact with people of different socioeconomic classes, with people from distinct cultures and languages, and preferably to have personal experience living in another country to better understand cultural values different from his or her own.	Diversity of cultures. Different students' needs.	All students are of the same socioeconomic status. There are not students who come to Juarez to study the high school.
Pluralistic Language Orientation	Teacher does encourage students to speak multiple languages.	Bilingual- Instructions.	Mono-language Instructions.
	Teacher does need to understand the acquisition of languages and the language used by these communities.	English and Spanish Language spoken in the community.	Mono-language in the community.
	Teacher does need to have knowledge of Spanish and English.	Speak both languages with the students.	Not speak English language with the students.

Interview Analysis

Data analysis was an issue that was considered in the process of designing this study. Qualitative interviews and transcripts produced enough material to be categorized in order to interpret the analysis. According to Kvale (1996), methods were identified for the analysis and interpretation of qualitative interviews. In order to analyze the interviews, meaning coding was used as the most appropriate approach. Meaning coding was utilized with the purpose of providing and identifying specific issues that were important to the participant teachers within the context of lesson plan implementation. The goal of spontaneous conversation was to reveal the perceptions of participant teachers. Semi-structured interviews were conducted subsequent to the third classroom observation. The format of the interview included questions related to the teachers' lesson plan development for that specific day and implementation practices incorporating Borderland Pedagogy. This approach provided an opportunity to examine the actual beliefs of teachers reflecting on their own teaching practices when developing and implementing their lesson plan.

Interview transcripts and observation notes were analyzed as evidence of the teachers' incorporation of Borderland Pedagogy into their lesson plan development and implementation practices. The data were analyzed by questioning each of the teachers and recording their responses through transcripts and field notes, thus providing evidence of the beliefs of each teacher about the implementation of the lesson plan.

In order to get the essence of the experience, the data was analyzed and compared. The analysis plan followed recommendations cited in Creswell (2007), including the following:

- Analyses of the transcripts of the documents to become familiar with the information of the study.

- Readings of the transcripts of the documents several times to develop familiarization with the information.
- Identification of meaningful sentences in order to determine teachers' transitions from developing their lesson plans and practices to teaching mathematics at the high school level.
- Meaningful sentences grouped in different ways following a coding-structure to identify teaching practices used when teaching mathematics classes.
- Integration of results into a comprehensive, in-depth description of the phenomenon.
- Finally, reviewing and validating the results utilizing participants' observations and researcher field notes.

3.4 GAINING ACCESS AND ENTRY TO THE U.S. PARTICIPANTS: DEVELOPING A REPORT

This study was based on five case studies of selected volunteer participants. The name of each participant was identified by pseudonym to preserve anonymity. Case study was used as a methodology to develop a better idea about the contexts of people (Glesne, 2006). Case studies allowed for interpretation and analyses of data that cannot be analyzed by quantitative methods. For example, through case studies "the researcher becomes an observer" (Yin, 1994, p. 50). . Case studies helped the researcher to question and answer the "how" and "why" (Merriam, 2002, p. 31) of the research problem. In the following paragraph is a description of how access to the participants was gained.

The purposeful sampling was an important process in order for comparative purposes in these case studies. For example, Bernard (2002) states that through case study methods the

researcher learns how to choose participants efficiently and wisely, and to choose the level of analysis necessary to answer the purpose of the study.

The process used to select each of the participants is described in the following paragraphs. The first volunteer participant who was selected and who agreed to participate in the study was Esteban (pseudonym). Esteban was identified as a potential participant through meetings with Dr. Tchoshanov and precipitated by the fact that in spring 2013, Dr. Tchoshanov and Esteban had been jointly working on a research project. Following introduction of the study given by Dr. Tchoshanov, Esteban agreed to an initial meeting where the purpose of the study would be explained. With Esteban's permission, Dr. Tchoshanov provided the researcher with Esteban's contact information. The initial meeting was held at the (College of Education) at University of Texas at El Paso (UTEP), and the purpose of this study was explained and Esteban expressed his interest in participating in the study and was willing to make time in his schedule to participate in the data collection phase of the study. A timetable of observations, the interview, and lesson plan collection was agreed to. In extended conversation with Esteban, it was mentioned that another participant was needed. He mentioned that he knew another person who met the requirements for the study in that this other person is another mathematics teacher who had worked their entire life as a mathematics teacher in El Paso, TX.

At the high school where Esteban was a mathematics teacher, introduction to the second potential participant, Virginia (pseudonym) were made. Following a description of the purpose of the study, Virginia demonstrated an interest in the study, volunteered and became the second high school mathematics teacher participant selected. Esteban and Virginia revised the initial timetable and agreed on the most convenient days and times to conduct the data collection.

3.4.1 Esteban and Virginia's High School Description

Esteban and Virginia teach in a high school located in El Paso, TX. This high school offers a variety of programs in the healthcare field for their students. For example, academic courses on Pharmacology and Emergency Medical Technology (EMT) are offered. During their freshmen year, students take a semester long medical terminology course. During the sophomore year, a health science technology class is offered. Additionally, students learn legal procedures required in the medical field. A variety of nursing skills are taught and students are able to participate in clinical rotations throughout the city in their junior year. During the junior year, 20 students are accepted into Licensed Vocational Nursing (LVN) program and 10 are accepted to a dental assistant program. The remainders of students are admitted to clinical rotation where they gain experience working in a hospital, rotating to different medical specialty areas. During the senior year, more programs are available, including practicums in EMT, pharmacology, physical therapy, and veterinarian technician.

3.5 GAINING ACCESS AND ENTRY TO THE MEXICAN PARTICIPANTS: DEVELOPING A REPORT

After identifying two mathematics teachers in El Paso, TX, a search in Ciudad Juarez was initiated to identify two participants from Mexico. The selection criteria for the two Mexican participants were the following: the participants must be mathematics teachers teaching mathematics at the high school level. The second criterion required that the two mathematics teachers must teach in Ciudad Juarez, Chihuahua.

During this selection process, several problems were encountered. Previously acquired permission to conduct the study was revoked by the principal and the coordinator of the high school in Ciudad Juarez. Another high school with similar characteristics had to be located; this

second high school was geographically distant from the originally identified high school and required additional commuting time. Fortunately, the researcher was able to get an immediate meeting with the principal to discuss the study. Permission to conduct research at this high school was obtained contingent upon the principal's receipt of an official letter from UTEP describing the purpose of the study. This letter was obtained from Dr. Tchoshanov and sent by email to the principal.

During the next visit to the high school, permission to conduct my study at this high school was finalized. Introductions to the mathematics coordinator were provided with the intent of identifying two more participants. Although these participants taught different mathematics subjects than the two U.S. participants, they fulfilled the requirement of the sample that is the requirement these participants worked as high school mathematics teachers in Ciudad Juarez. Yolanda and Angelica (pseudonyms) were introduced to the purpose of the study, in which they agreed to participate. Yolanda and Angelica established a convenient timetable to allow for observations and collection of lesson plans, as well as time for the interviews.

3.5.1 Angelica and Yolanda's High School Description

The characteristics of the high school in Ciudad Juarez were that of a public school status linked to the national education system of Mexico; additionally, in Ciudad Juarez, Secretaria de Educación Publica (SEP) provided the standardized curriculum for the high school level. SEP (2012) is responsible for organizing and controlling educational services in each city in Mexico.

3.6 GAINING ACCESS AND ENTRY TO THE TRANSITINING PARTICIPANT: DEVELOPING A REPORT

The process of finding a participant who met the criterion as a transitioning teacher was more difficult, since the sample data was limited. The criterion for selecting the transitioning

teacher participant was identifying a mathematics teacher who had taught high school mathematics in Ciudad Juarez before coming to El Paso, TX where the teacher continued to teach high school mathematics. With the assistance of Dr. Tchoshanov, Carlos (pseudonym), a graduate student who was taking a class with Dr. Tchoshanov in Spring 2013, was approached and it was determined he worked as a mathematics teacher in Ciudad Juarez at the high school level. He was teaching “pre-calculus” in that semester. Carlos met the most important requirement of working as a mathematics teacher in Ciudad Juarez who had come to El Paso, TX to work as a mathematics teacher. Based on this critical factor and with his agreement, Carlos became the transitioning participant. He displayed an enthusiastic attitude towards the study and a convenient timetable was agreed upon to observe, collect lesson plans, and conduct an interview.

3.6.1 Carlos’ High School Description

Carlos teaches in an alternative high school located in El Paso, TX, which offers grades 7- 12 for students who have disciplinary issues. The number of the students ranges from 10 to 15 students per class.

As previously mentioned, each participant represents a case study. The data collection for each case study was through lesson plan collection, observation, and interview. The observation process took three days for each participant. For each day of observation, lesson plans were collected. Subsequent to observations, an interview was conducted with each participant. Before the interview, the participants filled out a survey. The survey was developed using (1) demographic information, and a (2) language and communication profile (Appendix G).

3.7 THE PARTICIPANTS

Five teacher participants or case studies were the focus of the study. The five participants were working as fulltime mathematics teachers at the high school level. Each of the five participants taught different courses and subjects. Table 3.5 provides a brief description of the demographic information of each participant, as well as their language profile. The demographic information was elaborated through a survey of eight questions. This survey was divided in three significant parts. The first section asked about the participant's demographic information. The second portion was a language and communication profile and finally, the third segment was a non-numerical rating indicating how often the mathematics teachers use their first and second language with other teachers and their principal to develop their lesson plans. The non-numerical scale was about how frequently the participants use their first and second language. The pseudonyms, Carlos, Virginia, Esteban, Angelica, and Yolanda, were selected by the researcher and the dissertation chair. Before each interview, each participant filled out a survey with the information listed above. Each participant had 5 to 10 minutes to fill out the information. When the participants finished filling out the survey, the researcher conducted the interview with each participant.

Table 3.4: Participants' Demographic Information

Table 3.4 Participants' Demographic Information								
Pseudonym	Gender	Ethnicity/ Race	Where attended the University	Current residence	Level of education	Years of experience teaching mathematics	Grade level	Course/ subject taught
Esteban	Male	Hispanic	El Paso, TX	U.S	Master's degree	10	HS	Geometry
Virginia	Female	Hispanic	El Paso, TX	U.S	Bachelor's degree	13	HS	Geometry
Carlos	Male	Hispanic	El Paso, TX	U.S	Master's degree	10	HS	Pre- calculus
Yolanda	Female	Mexican	Cd. Juarez	Cd. Juarez	Master's Degree	7	HS	Algebra
Angelica	Female	Mexican	Cd. Juarez	Cd. Juarez	Bachelor's degree	1	HS	Algebra

3.8 SUMMARY

This chapter provided a description about the methodological design that was appropriate to conduct this study. At the beginning of the chapter, the research design was explained as well as the rationale for selecting a mixed methods quan-QUAL approach. The characteristics of the participants were provided and the process of recruitment of participants and criteria was described. Included was an explanation of how the data were collected, as well as how the three data sources were analyzed. There was an explanation about the importance of the characteristics of Borderland Pedagogy and lesson plan design as a theoretical framework. Lastly, the chapter presented an explanation of the data analysis process, as well as a description of recruitment of, gaining access to, and securing agreement from the teacher participants.

Chapter 4: Results

4.1 INTRODUCTION

The aim of this study was to determine how a transition from Mexico (Ciudad Juarez, Chihuahua) to the U.S. (El Paso, Texas) impacts high school teachers' incorporation of Borderland Pedagogy within their lesson plan development practices for the teaching of mathematics at the high school level. Elbaz-Luwisch (2007) defined transition as the "... complex process of shaping an identity moving between two places" (p. 389).

The primary research questions for this study were:

1. What Borderland Pedagogy lesson plan development and implementation practices are used by U.S. high school mathematics teachers in El Paso, Texas?
2. What Borderland Pedagogy lesson plan development and implementation practices are used by Mexican high school mathematics teachers in Ciudad Juarez, Chihuahua?
3. How does a transition from Mexico (Ciudad Juarez, Chihuahua) to the U.S. (El Paso, Texas) impact high school mathematics teachers' incorporation of Borderland Pedagogy during the development and implementation of their lesson plans?

This study was implemented using a mixed methodological approach because the purpose of the study was to interpret and understand practices at a particular point in time and in a particular context. For example, it was important to develop a good relationship with the participants and a qualitative approach (Denzin and Lincoln, 2003) allowed for that. Additionally, while qualitative studies may take additional time and effort, they provide in-depth analyses. The qualitative data analysis process was exciting because themes and concepts were found tucked between the data collected. This study also included a small quantitative

component in order to better respond to the three research questions. Researchers such as Tashakkori and Teddlie (2010) have affirmed that there are major components in a qual-QUANT sequential mixed method design. For example, there are paradigms where the major part of the study is qualitative and the smaller part is quantitative.

It was decided that the study should be primarily qualitative because this type of research provides a mechanism to interpret and/or give meaning to the phenomena under study. The quantitative component contains frequency tables of every main code in each case study. The quantitative part contains a frequency table to understand which participants had more actions reflecting the characteristics of Borderland Pedagogy when lesson plans were developed and when they were implemented with mathematical teaching practices at the high school level.

In the first section of this chapter, a description of the table Characteristics for Borderland Pedagogy is included. Reyes (2005) defined Borderland Pedagogy as "...a set of multifaceted, complex, and interactive factors; educational policies; curriculum; instructional practices; and a knowledge base that educators need to consider to increase the academic achievement of diverse students in the border region" (p. 149). The table presented contains the descriptions of each characteristic of Borderland Pedagogy (Cline & Necochea, 2006). These characteristics were used to analyze each data source that contributed to the findings. In the second section of this chapter, there is an explanation about the process of gaining access and entry to the participants, as well as a description of the process of developing rapport with the participants. The description of each participant contained demographic information, language, and a communication profile. Finally, there is an explanation about how the findings were reported, that is as case studies. Each case study contained a description of the demographic information of each participant, in addition to classroom descriptions. Visual images of the data sources, which

were taken during the teacher observations, are provided. The discussion contains descriptions of the lesson plan development, the lesson plan implementation, and the characteristics of Borderland Pedagogy found for each participant. Each of these findings contains direct quotes from participants, including a frequency table. The direct quotes emerged from the data, and were developed using the table of the Characteristics of Borderland Pedagogy as a point of reference.

4.2 CHARACTERISTICS OF THE BORDERLAND PEDAGOGY

Cline and Necochea (2006) developed Border Pedagogy in order to explore education as situated within the unique U.S /Mexico border region. Cline and Necochea asserted that the "... educational systems in the U.S and Mexico were so distinct and disconnected from one another that they may as well be in two separate worlds" (p. 269). These authors affirmed that Border Pedagogy was developed in response to the need for a better "... school system and structures, that are more equitable and just, that understand the extreme complexity of the borderland experience and implement instructional practices that are more teacher driven and contextually based" (p. 46). Similarly, Romo and Chavez (2006) affirmed that "Border Pedagogy encourages tolerance, ethical sophistication, and openness" (p. 143). Garza (2007) stated that Border Pedagogy was a transformative experience that could benefit students in education courses. Thus, the characteristics of Borderland Pedagogy were a central tool in the study, contributing to the theoretical framework. These characteristics were used to develop the main codes in order to report the data and conduct the analysis. The table (3.3) presented in chapter three describes the characteristics for Borderland Pedagogy, which were used to report these study findings.

4.3 HOW THE FINDINGS ARE REPORTED

The reports of the findings of this study were organized based on the following manner and will be discussed through each case study. According to Merriam (2002), case studies should be reported individually because it can help the researcher find evidence or circumstances of each phenomenon. Reporting each case study can be achieved individually by obtaining relevant and necessary information to answer each research question thoroughly and not leave out any important aspect at hand that gives validity to the research, both internally and externally, in order for the study to truly reflect and explain the phenomenon, situation or aspect studied. Each case study in the research can achieve different objectives: to provide a description, or to provide interpretations of the findings (Merriam, 1988, Yin, 1994).

In this study, the findings are presented through three case studies. The first case study represents U.S (Esteban and Virginia) mathematics teachers; the second case study is represented by Mexican (Yolanda and Angelica) mathematics teachers; the last case is the study of a transitioning (Carlos) mathematics teacher. Each case study was developed describing the demographic information of the participants. Also, there was a description of the participants' language and communication profiles. Finally, there was a classroom description for each participant and a description of lesson plan development and implementation, as well as a description of the characteristics of Borderland Pedagogy found for each participant.

The four characteristics of Borderland Pedagogy were used to address the research questions of this mixed methods study: '*flexibility*', '*passion*', '*culture*' and '*language*'. As well the opposite characteristics that were: '*Lack of Flexibility*', '*Lack of Passion*', '*Lack of Cultural Sensitivity*' and '*Lack of Pluralistic Language Orientation*'. Each of the four main characteristics contained their own defining descriptions according to Cline and Necochea (2006).

4.4 CASE STUDY DESCRIPTIONS

The following section presents the three case studies. The three case studies include findings substantiated by direct quotes from participants and a frequency tabulation based on the coding scheme identifying absence or presence of Borderland Pedagogy characteristics.

4.4.1 U.S. Mathematics Teachers: Esteban's and Virginia's Case Study

Esteban: Demographic Information

Esteban was between the ranges of 30-40 years old and self-identified himself as Hispanic or Latino. He earned a Bachelor's Degree in Accounting and received a Master's degree in mathematics education. He was also the only participant who has continued his studies in a Ph.D. program in order to specialize in Mathematics Education. He was a mathematics teacher at the high school level with ten years of experience. He taught Algebra and Geometry to 9th and 10th graders in El Paso, Texas.

Esteban: Language and Communication Profile

Esteban indicated in the survey that his native language was Spanish. His second language was English; he does not speak any other languages. He mentioned that the languages used with his students in and outside class are Spanish and English. Finally, he said that he very frequently uses both languages when he meets with other teachers to develop the lesson plan.

Esteban: Classroom Description

At the time of the study, Esteban's classroom was large and the walls were painted white. The classroom looked like a new classroom. It was observed that students did not eat, drink, or smoke in class. In front of the classroom, there was also a large white board, so the students and Esteban could do exercises and check the answers together. Also included was a white board where Esteban could do operations manually or write the objectives of the lesson. The classroom

appeared comfortable with adequate space where Esteban could walk around the classroom to check student work. In the classroom, there was a computer connected to the internet where Esteban showed the examples and the lesson on a big screen; additionally, Esteban had a personal computer assigned to him. The students had small tables so that each student could work individually. All furniture and technology looked new in the classroom. Esteban's classroom was equipped with sophisticated technology like graphing calculators and a smart board. Each student had access to a calculator to work on the lesson implemented by Esteban.

One of the perceived disadvantages was that this classroom included posters on the walls solely related to mathematics and there were no bilingual or Spanish language posters (as were observed in Virginia's classroom described later). The following visual images show some examples of the technology that Esteban uses to develop and implement his lessons.

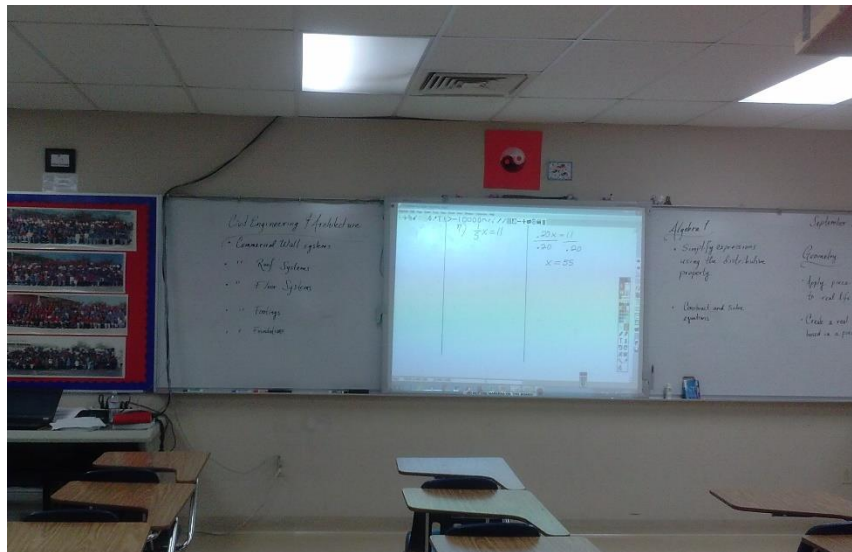


Figure 4.1: Esteban's classroom arrangement: front wall

Figure 4.1 represents the smart board, where Esteban teaches concepts in his geometry class. The smart board was one of the teaching tools used by Esteban to explain the Casandra problem. The Casandra problem consisted of constructing a PIECE-WISE function and analyzing the rates of change and area underneath the curve. Utilizing the smart board, Esteban

interacted and resolved the function behavior problem in front of the group. This tool was consistently used during the three days that Esteban was observed.



Figure 4.2: Esteban's classroom arrangement: graphing calculator station

Figure 4.2 shows the graphing calculators used by the students. Each of the students had access to one calculator to graph the function behavior. Through the use of the calculator, students can understand and learn the function behavior or how to graph a function.

Esteban: Lesson Plan Development

Esteban developed a lesson plan about rate of change and distance using piece-wise functions in Geometry. This lesson plan was developed for students who are taking a Pre-AP Geometry class at the high school level. Esteban used the same lesson plan during the three days that he was observed. The purpose of this lesson plan was to engage students while they constructed and applied piece-wise functions to real life application scenarios. For example, Esteban developed the following problem with the purpose of student understanding and the learning of why in each interval the function is increasing or decreasing.

Cassandra decided to hike 3 miles in 2 hrs. The path gets a little steeper and she walks at a rate of 3 miles in 2 hours (hrs) or 1 mile per hour (mph). She stopped for a total of 3 hrs

to eat some lunch. Casandra had to go to the restroom, so she starts jogging at a rate of 4 miles per hour. Casandra sees her friend and slows down to 1 mph and reaches her starting point. (Observation, September 12, 2013, Appendix B).

Esteban developed the lesson plan with the intent that students use technology to construct piece-wise functions and analyze the rates of change and area underneath the curve. Throughout this lesson plan, the students described the meaning of the piecewise function at particular intervals. For instance, they could determine when the function increases and decreases the fastest, when it was constant, and when it was increasing the slowest. Finally, the lesson plan was developed with an activity as homework for the students. This assignment consisted of students having to construct their own piece-wise function on a blank template of a graph and table, and having students answer reflection questions about the intervals where the function increased, decreased or was constant (Figure 4.3).

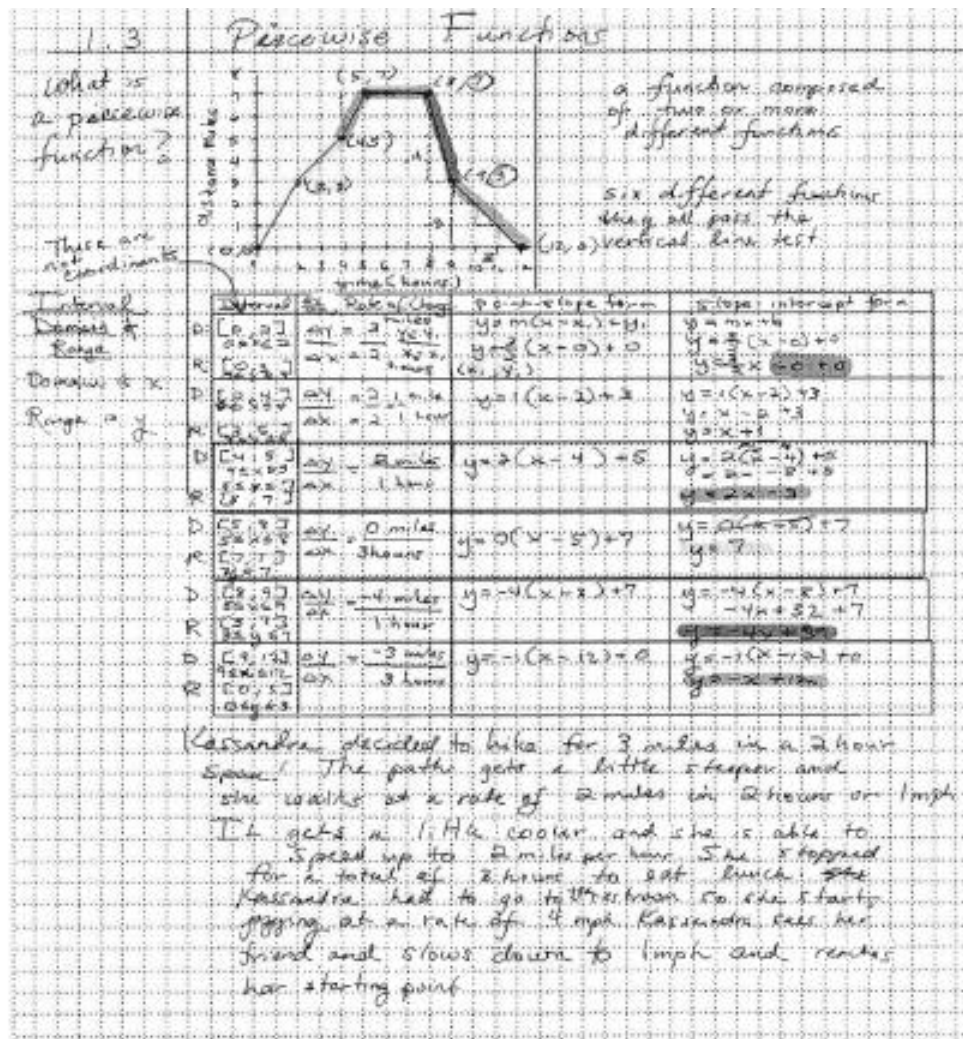


Figure 4.3: Esteban's lesson plan fragment -1

Cassandra's trip was a problem that represented the rates of change and area underneath the curve. This lesson plan provided real life context, as well as questions to engage the students in understanding the function behavior of each particular interval. As illustrated in Figure 4.4, through these questions students learned when the function increased the fastest, decreased the fastest, or when it was constant.

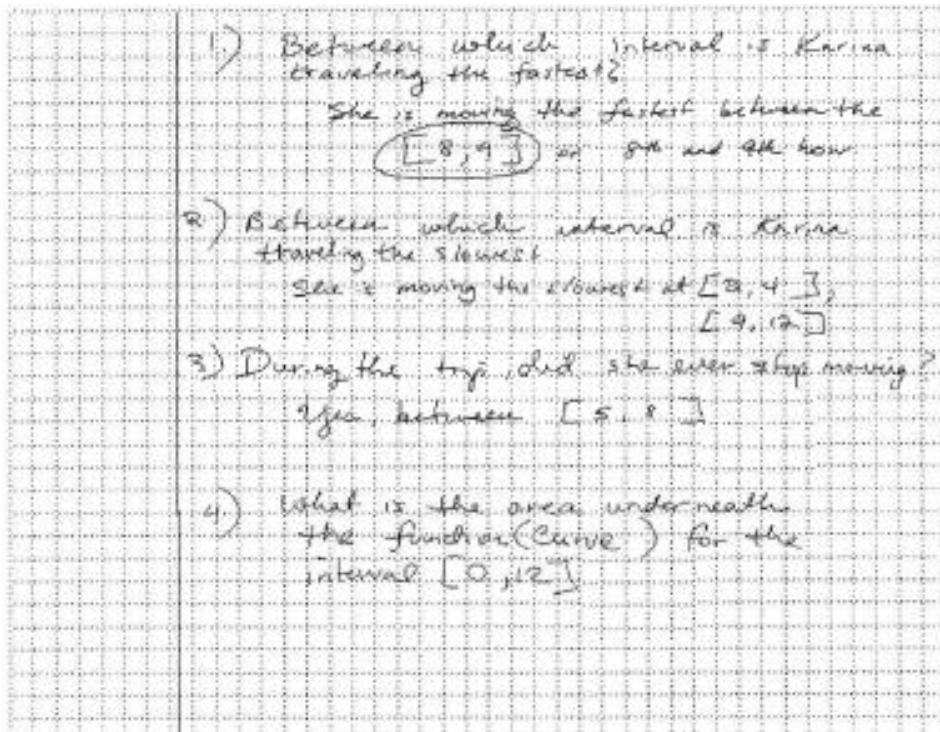
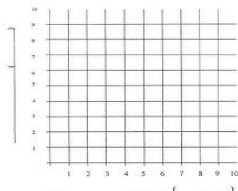


Figure 4.4: Esteban's lesson plan fragment -2

Figure 4.4 (above) is a copy of the homework template that Esteban gave to students to create their own story like the problem about Casandra's trip that was taught in the class. This assignment consisted of creating a story with 8 intervals. Each student created a different story. These intervals were represented by the graph behavior developed by each student. The graph was drawn in the chart shown above, as well as the elemental operations to find the point of each interval. Under the graph, students had to include their own story.

Name _____ Date _____ Period _____

Create a piece-wise function and write a story that corresponds to the graph. (Must have minimum of 6 functions)



Interval	Avg. Rate of Change	Equation (Point-Slope)	Equation (Slope-Intercept)

Figure 4.5: Esteban’s homework assignment given to students

Esteban: Lesson Plan Implementation

Esteban implemented the lesson plan according to the TEKS P(1)A-E (2012), which state that students can define functions, describe their characteristics and translate verbal, numerical, graphical, and symbolic representations of functions. This includes polynomial, rational, exponential, logarithm, trigonometry, and piecewise defined functions. The engagement portion of the lesson plan design integrated questions about functions to determine prior student levels of knowledge about this topic. Facilitated by this initial discussion between teachers and students, Esteban further engaged the students by providing an introduction to the topic of the lesson. Esteban implemented the same lesson plan during the three days of observation. He implemented the lesson plan while simultaneously embedding the use of technology, such as the white board and TI-Nspire calculators.

For Esteban the use of technology was a main tool in the implementation of this lesson plan. For example, he shared that

“The use of technology has led to substantial changes in the way students learn mathematics. The use of technology can be used to provide conditions where students can

identify review and communicate mathematically different ideas” (Interview, October 15, 2013, Appendix A).

The use of technology was an important tool because technology enabled students to solve the function problems visually and creatively. They could visualize using the TI-Nspire calculators connected to the white board when functions decreased and increased. This tool provided a venue for student discussion debating the reasons why a function increased and decreased in each interval . It was observed that through of the use of technology, students showed an interest and were engaged in learning this geometry topic, as was evidenced by their participation during the lesson.

Characteristics of Borderland Pedagogy Found within Esteban’s Lesson Plan Development and Implementation

In the lesson plan developed and implemented by Esteban, he made use of some of the characteristics of Borderland Pedagogy. The characteristics found in Esteban’s teaching were: *Flexibility, Passion, Culture, and Language*. These characteristics were found in the lesson plan collection, observations, and interview. Table 4.1 displays the characteristics and descriptions. The reports of these characteristics are discussed in the same order as they are presented in table 4.1.

Table 4.1: Characteristics Enacted in Lesson Plan Development and Implementation: Esteban

Characteristics	Description
Flexibility	Respects students, without caring about their race, religion, or socioeconomic status. Designs curriculum creatively to incorporate the students' backgrounds and ways of knowing the world.
Passion	Ability to help all students at different levels
Culture	Teacher in the border region needs to have contact with people of different socioeconomic classes, with people from distinct cultures and languages, and preferably to have an experience personally living in another country to better understand cultural values different from his or her own.
Language	Teacher needs to have knowledge of Spanish and English.

Flexibility: Respects students, without caring about their race, religion, or socioeconomic status.

During the three days that Esteban was observed, he never demonstrated disrespect toward the students. Reprimands were gentle and respectful. Observations and field notes included the following journal entry:

Esteban never insulted the students, nor shouted at them when they did not understand the concept of piece-wise function. On the contrary, whenever students needed help, Esteban approached them to assist them with their questions. As well, he never discriminated against students who asked questions in Spanish” (Observations conducted Sep 11-Sep 13, 2013, Appendix B).

Esteban showed the importance of respecting student opinions and suggestions about the topic of piece-wise function. He acknowledged that each student had different solutions and opinions

about how to interpret the function's behavior. Throughout the expression of these different solutions and opinions, Esteban demonstrated courtesy and respect for his students.

Flexibility: Design curriculum creatively to incorporate the students' backgrounds and ways of knowing the world.

In a conversation outside the classroom with Esteban, he explained that lesson plans should be developed and implemented in order to stimulate the interest of each student. He stated:

I develop the lesson plan in order that students can reflect on the subject being taught. For example, I let them create their own problems. This is a method I like to do a lot, I like to use this a lot between the students that create your own situation to see what they are thinking mathematically (Interview, October 15, 2013, Appendix A).

In the observation conducted (September 13, 2013 Appendix B), Esteban showed the creativity to design the lesson plan by embedding the use of technology. Through the use of technology, the students learned the rates of change and area underneath the curve. For example, the following functions (Figure 4.6) represented the behavior of Cassandra's problem (mentioned in Esteban's lesson plan development). Through this mathematics problem, Esteban represented the piece-wise function by the formula ($y = mx + b$), the point-slope form and the graphic representation. The graph represented the function behavior of Casandra's trip. Utilizing this graph the students learned in what intervals Casandra was walking faster, or when she was able to speed up additional miles per hour. The graph represented when Cassandra decided to stop for 3 hours to take her lunch, and when she started jogging at a rate of 4 miles per hour. Finally, it represented when Casandra saw her friend and she slowed down to 1 mile per hour and when she reached her starting point.

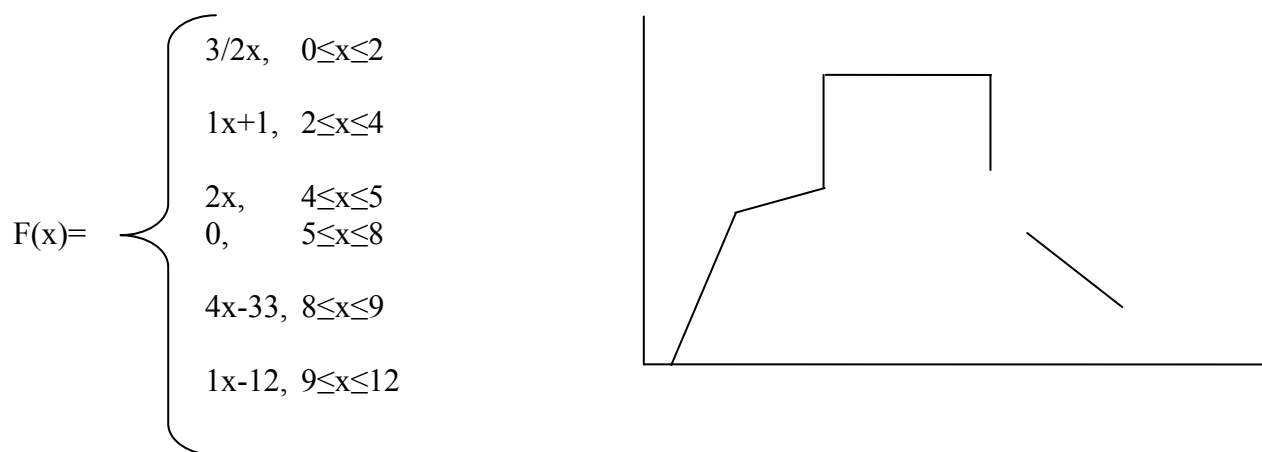


Figure 4.6: Casandra trip: graphical representation

The following Table 4.2 shows the actions most frequently enacted under the flexibility demonstrated by Esteban in the three data sources.

Table 4.2: Actions Most Frequently Enacted Under the Characteristic of Flexibility: Esteban

Characteristic	Description	Data Sources	Frequency
	Respects students, without caring about their race, religion, or socioeconomic status.	Observation	3
Flexibility	Designs curriculum creatively to incorporate the students' backgrounds and ways of knowing the world.	Lesson Plan	14
		Observations	11
		Interview	7

Passion: Ability to help all students at different levels

In the observation conducted (September 11, 2013 Appendix B), Esteban was instrumental in helping students with questions about the use of the TI-Nspire calculator. For example, in the use of technology some students did not understand how to insert the intervals into the calculator to graph the function. Through these students' questions, Esteban demonstrated mastery knowledge of the ability to use the TI-Nspire:

“Please go to the function T1 and give a click in insert function” (Observation conducted September 11, 2013, Appendix B).

Some of the students did not understand why in the interval $[2,3]$ the function is increasing . He explained:

“In this interval the function is increasing since the slope is considered as positive, please calculate the slope and explain to me what is the sign of the slope” (Observation conducted September 11, 2013, Appendix B). During the three days of observation, Esteban demonstrated mastery knowledge about the function behavior as well as about the use of technology.

The following Table 4.3 shows the actions most frequently enacted under the characteristic of passion demonstrated by Esteban.

Table 4.3: Actions Most Frequently Enacted Under the Characteristic of Passion: Esteban

Characteristic	Description	Data Sources	Frequency
Passion	Ability to help all students at different levels.	Observations	14

During the three days of observation, Esteban showed the ability to help all students to better understand the concept of function behavior through the use of technology and through his explanations.

Culture: Teachers in the border region need to have contact with people of different socioeconomic classes, with people from distinct cultures and languages, and preferably to have an experience personally living in another country to better understand cultural values different from his or her own

Esteban expressed during the interview that the border area is formed by a diversity of cultures, especially by the Latino culture. He emphasized that culture is an important part of people's lives. He supported this point of view by stating

If we understand the diversity of the cultures, we can understand the different ideologies, and racial and social division. Studying for a master's degree has helped me to learn and know about the diversity of cultures (Interview October 15, 2013, Appendix A).

Esteban expressed that taking classes at UTEP helped him to know the difference between cultures that are represented in his classroom. Knowledge of varying cultures is important for teachers to establish teaching relationships conducive to learning. He understood the importance of culture since he is a teachers and he needs to know about it. Table 4.4 presents the actions most frequently enacted under the Characteristic of Culture demonstrated by Esteban.

Table 4.4: Actions Most Frequently Enacted Under the Characteristic of Culture: Esteban

Characteristic	Description	Data Sources	Frequency
Culture	Teachers in the border region need to have contact with people of different socioeconomic classes, with people from distinct cultures and languages, and preferably to have an experience personally living in another country to better understand cultural values different from his or her own.	Observations	2

Language: Borderland Teachers need to have knowledge of Spanish and English

In the interview conducted with Esteban, he considered the importance of using both languages in his Pre-AP Geometry class:

...students can speak English or Spanish into the classroom, if I observe that they have problems with the English language, I give them confidence to. Students talk in Spanish. Also, if some terms and definitions are not understood, I translate to Spanish with the objective that they can better understand the concept. (Interview, October 15, 2013, Appendix A).

He considered the importance of understanding both languages in his class. However, in the lesson plan development he did not consider it necessary to develop the lesson in both languages, even though he knew that some of the students have problems with the English language.

Esteban did not see it as problem if the students did not understand some mathematics concepts in English, since he has the ability to explain the concepts in both languages.

During the three days of observation, Esteban demonstrated proficiency in speaking English and Spanish with the students. The first day of observation, Esteban began the instruction in the English language, but then transferred to Spanish. He said,

“No esperen aprender a utilizar la calculadora en un día”; “ it is a new tool”. (Observation September 11, 2013, Appendix B).

Table 4.5 displays the actions most frequently enacted under the characteristic of language demonstrated by Esteban.

Table 4.5: Actions Most Frequently Enacted Under the Characteristic of Language: Esteban

Characteristic	Description	Data Sources	Frequency
Language	Teachers need to have knowledge of Spanish and English.	Observations	4
		Interview	2

In summary, based on the description of development and implementation of the lesson plan to teach Piece-Wise functions and the embedded use of technology as an important tool in the teaching of this mathematics topic, the main characteristics of Border Pedagogy that were identified in the case study of Esteban were flexibility, passion, culture and language. Each of these characteristics was represented in separate frequency tables.

Virginia: Demographic Information

Virginia was between 30-40 years old. She identified as Hispanic or Latino. She received a Bachelor’s Degree in Computer Science from University of Texas at El Paso (UTEP). Virginia, having taught for thirteen years as a mathematics teacher, was the participant with the most years of experience at the high school level. She teaches Geometry and Algebra II to 9th and to 12th graders in El Paso, Texas.

Virginia: Language and Communication Profile

Virginia, when completing the survey, said that her native language was Spanish and her second language was English. She mentioned that the languages used by her students inside and outside her classes are both Spanish and English. Finally, she mentioned that she seldom uses her native language with other mathematics teachers when developing lesson plans.

Virginia: Classroom Description

Virginia's classroom arrangement was very similar to Esteban's classroom. In front of the classroom, there was a white board where Virginia explained the exercises to the students. Also, in front of the classroom, Virginia had a computer connected to the internet in order to search for examples about geometry concepts for the students. Beside the electronic smart board and the computer, the classroom contained a white board where Virginia wrote the objectives of the lesson, as well as the TEKS and the material that the students need for the lesson. Virginia could walk around the classroom without difficulty to check the student exercises. Each student had her/his own space to work individually. If they needed to form groups, there was enough space for Virginia to check the assignments of the group. Virginia had a personalized computer where she could grade the students' assignments. Finally, I observed that on the walls, Virginia had posters of mathematics concepts in both languages (Spanish and English). These posters contained geometry and algebra concept definitions to facilitate student understanding in both languages.

One of the disadvantages of Virginia's classroom was that there were no graphing calculators where the students could work together or individually on their assignments. Virginia preferred to use didactic materials, like rulers and compasses to develop and explain her lessons as she mentioned in the interview. Figures 4.7 and 4. 8 are visual images of the posters in

Spanish and English conveying content-specific mathematics concepts or vocabulary. Some of the concepts were represented by figures, such as different representations of parabolas.

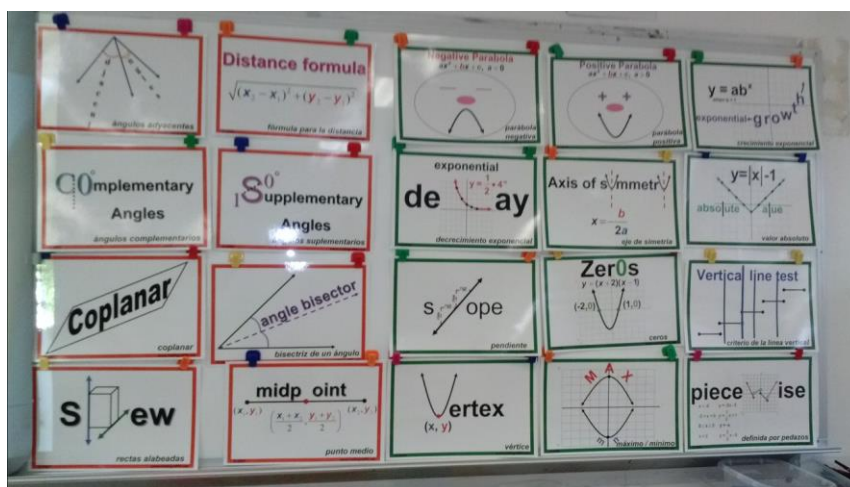


Figure 4.7: Mathematics concepts in English and Spanish languages-1

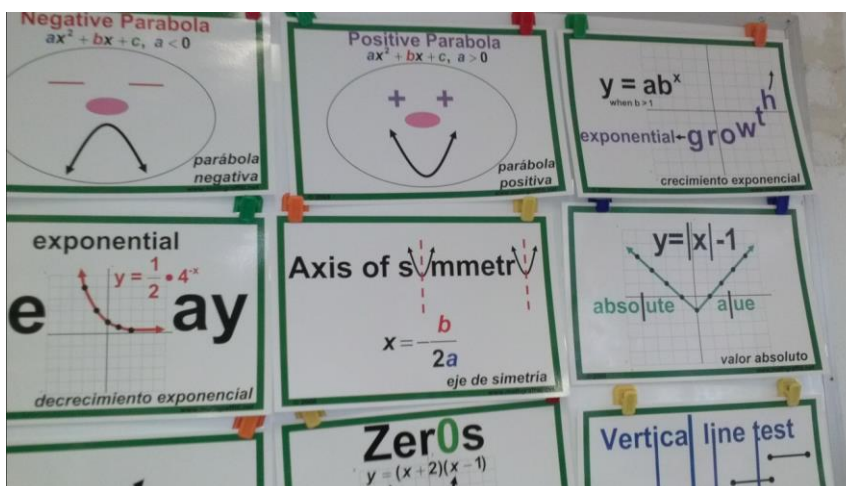


Figure 4.8: Mathematics concepts in English and Spanish languages-2

Virginia: Lesson Plan Development

Virginia developed different lesson plans as compared to Esteban, although they both taught the same content, Geometry. Esteban's lesson plans were geared toward Pre-AP courses and Virginia's students were not taking high school Pre-AP mathematics courses. Virginia developed lesson plans about segments, angles, theorems, and postulates.

The lesson plan on segments was the first lesson plan Virginia developed with the purpose of teaching students how to find the segment through the solution of the equation. In this exercise, Virginia explained why $NP=PQ$. Through this exercise the students solved the algebra equation to find the x value.

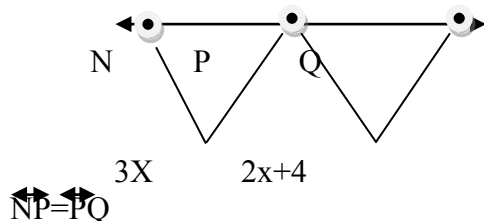


Fig. 4.9 Math problem taught in classroom

Figure 4.10 represents the worksheet developed by Virginia with the focus on finding the segment through the solution of equations. This worksheet was provided in class to each student so that they could solve the problems during the class.





Segments	Name	Date	Period
SHOW ALL WORK IN THE SPACE PROVIDED.			
1. Use a compass and straightedge to construct segment XY congruent to segment UV .		2. Use a compass and straight edge to construct the midpoint of segment UV .	
4. Write the equation you would use to find a . Find WX and XY .		4. $WX =$ _____ $XY =$ _____	
5. Write the equation you would use to find y . Solve. Find ST and SU .		5. $ST =$ _____ $SU =$ _____	
6. P is the midpoint of NQ . Write the equation you would use to find x . Solve. Find NP , PQ and NQ .		6. $x =$ _____ $NP =$ _____ $PQ =$ _____ $NQ =$ _____	
For #6-#9 H is between I and J. For all problems below, draw and label a diagram to fit each problem.			
6. $HI = 3.9$ and $HJ = 6.2$. Find IJ .		6. $IJ =$ _____	
7. $JI = 25$ and $IH = 13$. Find HJ .		7. $HJ =$ _____	

Figure 4.10: Virginia's worksheet on segment

The second lesson plan developed by Virginia focused on angles, theorems and postulates. It was developed with the purpose of teaching students to investigate patterns and make conjectures about geometric relationships, including angles formed by parallel lines and bisected by a transversal; triangle congruence; special segments of triangles; diagonals of quadrilaterals; interior and exterior angles of polygons; and special segments and angles of circles. These problems are solved choosing from a variety of tools such as a compass and straightedge, paper folding, and dynamic geometric software.

This lesson plan had the purpose of teaching students to construct congruent segments, congruent angles, a segment bisector, an angle bisector, perpendicular lines, a perpendicular bisector of a line segment, and a line parallel to a given line through a point not on a line by using a compass and a straightedge. Figure 4.11 represents the lesson plan developed by Virginia to teach angle-terms. This lesson plan was given to each student to facilitate individual learning. The lesson plan was developed to include the writing of the definition of the angle terms (e.g., right angle, obtuse angle, measurement of an angle).

Angles - terms, theorems, postulates 09-1113

TERM	DESCRIPTION	NAMING	SYMBOLIC REPRESENTATION
Angle	Definition		$\angle ACB$ $\angle A$
Measurement of an angle			$\angle TND = 26^\circ 8'$
Acute angle			
Conditional statement			
Right angle			$\angle SUN$ is right $\angle SUN = 90^\circ$ $\angle U = 90^\circ$
Conditional statement			

For each definition, teacher ask the conditional statement



Obtuse angle		None of the above $\angle MON$ is obtuse $\angle O$ is obtuse
Conditional statement	$90^\circ < \angle < 180^\circ$	
Congruent Angles		
Conditional statement	Two angles that have the same measurement, these are congruent angles	
Angle Bisector		50°
Conditional statement	Students construct the conditional statement according to the definition	

Figure 4.11: Virginia's angles term lesson plan

Virginia: Lesson Plan Implementation

Virginia implemented these lesson plans according to the TEKS. TEKSG.11.C is stating that students can develop, apply, and justify triangle similarity relationships, such as right triangle ratios, trigonometric ratios, and Pythagorean Theorem using a variety of methods.

Additionally, these lesson plans were implemented with the intent to formulate and test conjectures about the properties and attributes of polygons and their component parts based on explorations and concrete models.

Virginia implemented this lesson plans explaining each example and exercise in front of the white board. She preferred to refer to the angle definitions on a math web page rather than using her own. The definitions of the angle terms were provided before the beginning of the lesson, as a handout, so that students would be familiar with the terms and could read and write the definitions. Virginia implemented these lessons using the white board and didactic material for the construction of the angles. The handouts included space so that students could write each angle definition and take the handout home to study.

Characteristics of the Borderland Pedagogy Found in Virginia's Lesson Plan Development and Implementation.

As a product of the developed and implemented lesson plans on segment and angles-terms, some characteristics of Borderland Pedagogy were identified. The characteristics found in Virginia's lessons were: Lack of flexibility, Passion, Culture, and Language. These characteristics were also found in some of the additional data sources used for data collection. The following Table 4.6 exhibits the characteristics enacted in lesson plan development demonstrated by Virginia.

Table 4.6: Characteristics' Enacted in Lessons Plan Development and Implementation: Virginia

Characteristics	Description
Lack of Flexibility	Design curriculum lacked uncreatively to incorporate the students' backgrounds and ways of knowing the world
Passion	Ability to help all students at different levels
Culture	Teacher has to understand the dynamics of how the communities in the border area work and the needs they have.
Language	Teacher needs to understand the acquisition of languages and the language used by these communities. Teacher needs to have knowledge of Spanish and English.

Lack of flexibility: Design curriculum lacked uncreatively to incorporate the students' backgrounds and ways of knowing the world

Virginia had a traditional understanding of geometry teaching. Throughout the lesson plan development and implementation, Virginia emphasized that Geometry can be taught in a particular order, teaching some concepts and topics, such as the process of solving equations to find the segments of the line, as well as by memorizing the description of acute angle, right angle, obtuse angle and congruence angle with the conditional statement for each one. This was observed in the method that she used to explain when two triangles are congruent as exhibited below:

“If and only if $m \angle JAK \approx m \angle LPM$, it means that $\angle JAK = 40^\circ$ $\angle LPM = 40^\circ$.” (Observation, September, 13, 2013, Appendix, C).

However, this seemed to contradict what she expressed in the interview about another geometry class she taught. Virginia said:

“I try to make the activity manually and touchable so that the students not only do the procedure, one after another, after another, if they do something with their hands and remember what they are doing, using different colors.” (Interview October 29, 2013, Appendix, A).

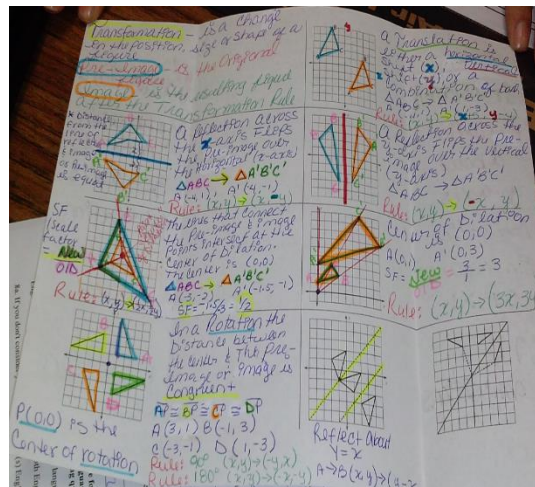


Figure 4.12: Geometry activity implemented by Virginia

The above figure represented one of the activities that Virginia incorporated into lesson plan development and implementation for another geometry class that was not observed in this study.

In the class observation, Virginia also made reference to finding the segment of the line:

“... it is important to give an algebra review.” (Observations conducted September 10, 2013, Appendix, C).

Virginia solved two similar problems to know if the students remember how to find the value of x solving the equation. Virginia conducted the algebra review by jogging student memory:

“Do some of you remember how to find the x values of this equation $4(3x+4)=2(5x+3)$. Students pointed out that they need to eliminate the parenthesis ($12x+16= 10x+6$) and

after that add the x 's and the constants. ($12x-10x=6-16$). , Finally, find the x 's value ($2x=-10$, $x=-5$)” (Observation September 13, 2013, Appendix, C).





Virginia knew that her students understood how to solve equation when they responded to her questions about solving this algebra equation. She followed by asking the students:

“Please solve the following equation and tell me what is the x value ($8x+2+4x+3=20x+8$)” (Observation September 13, 2013, Appendix C). The students solved the equation individually and then Virginia solved it on the white board.

Virginia’s strategies in planning for the lesson plan about angle terms, theorems and postulates did not change throughout the observation. She gave some hand-outs to each student in order for them to write the definition, description, naming, and symbolic representation of an angle, acute angle, right angle, and others. Through this handout (Figure 4.12) the students worked individually. Virginia reviewed students’ definitions by having students read the definitions in groups and write it on the hand-outs. Virginia’s specific instructions to the students were

“Please read the angle definition and the symbolic representation and write it on the hand-outs” (Observation September 11, 2013, Appendix, C).

09-11-13

Angles - terms, theorems, postulates			
TERM	DESCRIPTION	NAMING	SYMBOLIC REPRESENTATION
Angle	Definition		$\angle P$ $\angle A$ $\angle B$
Measurement of an angle			$\angle TMD = 26^\circ$
Acute angle			
Conditional statement			
Right angle			$\angle SUN$ is right $\angle SUN = 90^\circ$ $\angle U = 90^\circ$
Conditional statement			

For each definition, teachers ask the conditional statement



Obtuse angle		Name of the angle $\angle MON$ $\angle O$ $\angle A$ is obtuse $\angle A > 90^\circ$
Conditional statement	$90^\circ < \angle < 180^\circ$	
Congruent Angles		
Conditional statement	Two angles that have the same measurement, then are congruent angles	
Angle Bisector		50°
Conditional statement	Students construct the conditional statement according to the definition	

Figure 4.13: Hand-out used by Virginia during lesson plan implementation

Table 4.7 represents the actions most frequently enacted under the characteristics of lack of flexibility demonstrated by Virginia and obtained from the three data sources.

Table 4.7: Actions Most Enacted Under the Characteristic of Lack of Flexibility: Virginia

Characteristic	Description	Data Sources	Frequency
Lack of Flexibility	Design curriculum lacked uncreatively to incorporate the students' backgrounds and ways of knowing the world	Lesson Plan	4
		Observations	7
		Interview	2

Passion: Ability to help all students at different levels

Although using the traditional way to teach geometry, Virginia showed an ability to facilitate student learning as evidenced in their questions about finding the x value, or about the obtuse angle constructions using a straight edge, compass and protractor and her responses to identified areas of confusion. As Virginia walked around the classroom observing the students' work, she observed that a student had made some errors in the construction; she again explained the obtuse angle definition by stating

... we need to remember what is the definition of obtuse angle...according to this definition what are the characteristics' of it? An obtuse angle is one which is more than 90° but less than 180° . In other words, it is between a right angle and a straight angle. Draw this angle with this characteristic (Observation September 13, 2013, Appendix, C). Virginia was able to answer all student questions about the solution of the segment, as

well as the representation of an obtuse angle through the definition and the geometric representation. Table 4.8 represents the actions most frequently enacted under the characteristic of Passion demonstrated by Virginia.

Table 4.8: Actions Most Frequently Enacted Under the Characteristic of Passion: Virginia

CharacteristicDescription		Data Sources	Frequency
Passion	Ability to help all students at different levels	Observations	4
		Interview	1

Culture: A teacher has to understand the dynamics of how the communities in the border area work and the needs they have

Virginia was the only participant who had posters about algebra and geometry concepts in both English and Spanish throughout her classroom. Below is a journal entry after the observation of Virginia:

For example, the figure below represents some of the posters found in Virginia's classroom. Virginia used this tool as a visual resource to help the students to learn some mathematics concepts with visuals objects, since 90% of her students are Hispanic and have difficulty with some math definitions. Virginia used appropriate tools in order for the language to not be a barrier to student learning. These images not only contain mathematical terms in both languages, but they are represented by creative images that help students learn in a fun way. For instance, the sign of the parabola was represented by a sad or happy face, which it refers to when the parabola has a positive or negative first coefficient (Observation September 10, 2013, Appendix, C).

Virginia expressed in the interview

“I know the necessity that have my students have since one time I passed by the same situation. I came to El Paso, TX from Mexico to study in high school at the same age of my students” (Interview October 29, 2013, Appendix, A).

Virginia considered bilingual material necessary to teach algebra and geometry since most of her students are Hispanic and they are not proficient with English. For example, Figure 4.14 is a visual image of selected mathematics concepts in both languages posted in Virginia’s classroom. This teaching approach and strategy helps facilitate English Language Learners’ understanding of academic math vocabulary definitions in both languages .

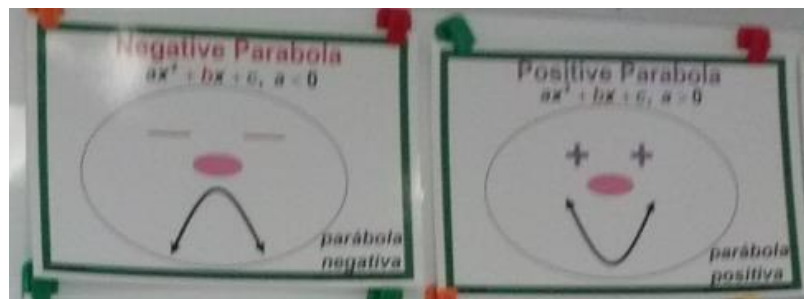


Figure 4.14: Parabola concept represented by a sad and happy face.

Table 4.9 represents the actions most frequently enacted under the characteristic of Culture demonstrated by Virginia.

Table 4.9 Actions Most Frequently Enacted Under the Characteristic of Culture: Virginia

Characteristic	Description	Data Sources	Frequency
Culture	Teacher has to understand the dynamics of how the communities in the border area work and the needs they have.	Observations	5

Language: Teacher needs to understand the acquisition of languages and the language used by these communities and Teachers need to have knowledge of Spanish and English

Virginia expressed the importance of having knowledge of Spanish and English to teach geometry lessons in a border region. She supported her assertion of importance based on the demographics of her students or the fact that 90% of the students came from Juarez and were taking English classes. She said:

“I have posters of the mathematics concepts in both languages with the purpose that students can learn and understand the mathematics concepts in both languages. I try to tell the things in English with the purpose that students will learn English, but if I observe that some of them didn't understand, I translate it to Spanish. But most of the time I explain the instructions and the geometry problems in English.” (Interview October 29, 2013, Appendix, A).

Virginia identified two students who have difficulties with the English language. First, she attributed students' difficulties to a lack of understanding of the acute angle definition in English. However, she needed to make sure that her students would understand this definition. For this reason in her explanations of acute angle, Virginia asked one student:

“...translate the acute angle definition to Spanish, por favor”. (Observation, September, 11, 2013, Appendix, C).

However, Virginia seemed to think students would understand the acute angle not only by the definition, but also by the construction representation of an acute angle. She said:

“We will learn how to construct an acute angle through a ruler, compass and protractor” (Observation September 11, 2013, Appendix, C)

Virginia understood the importance of teachers having knowledge of Spanish and English in the Borderland area. She stated:

“I understand the situation of my students because I was in the same situation, I came to study the high school at the same age as them, and I understand the difficulty of the language they are just learning. I tell them, please not to despair. I can help them if they have problems with some definitions in English”. (Interview October 29, 2013, Appendix, A).

Virginia showed an interest in helping students understand geometry concepts in both languages, given that some of the students had difficulty with some terms. She expressed the importance of working with Hispanic students’ needs since she went through the same situation.

Table 4.10 represents the actions most frequently enacted under the the characteristic of Language demonstrated by Virginia.

Table 4.10: Actions Most Frequently Enacted Under the Characteristic of Language: Virginia

CharacteristicDescription		Data Sources	Frequency
Language	Teacher needs to understand the acquisition of languages and the language used by these communities.	Observations	8
		Interview	3
		Observations	10
		Interview	5
	Teacher needs to have knowledge of Spanish and English.		

Summarizing the data from Virginia’s case study, demographic information was provided, as well as language and communication profiles. There was a description of the classroom, including the importance of having posters of mathematics concepts in both languages. As noted in the observations, Virginia taught geometry topics using traditional methods. However, in the interview there was contradictory evidence to the observation data

based on her usage of foldables in another class – a non-traditional method. Finally, a report of the findings for the Virginia case study was included in this section. The characteristics of Borderland Pedagogy found in her teaching were presented and included in frequency tables of the pedagogical actions demonstrated by Virginia.

To summarize, in the U.S. case study there were some similarities and differences in the characteristics of Borderland Pedagogy incorporated by the participants in their lesson plan development practices. For example, according to characteristic of flexibility, Esteban designed and developed lesson plans in order to stimulate the students' interest and learning. However, Virginia was influenced by the method where she considered follow up instruction important; in other words, Virginia didn't consider it important for students to discover the concepts taught in the geometry lesson plan she functioned in a traditional teacher-directed modality.

In regard to characteristics of passion, both mathematics teachers demonstrated the content knowledge to help students according to their identified learning needs. For example, Esteban showed the ability to explain the behavior of the graph through Cassandra's Problem. As well, Virginia showed the ability to explain the obtuse angle definition through the construction of it, using a protractor and ruler.

Finally, regarding the characteristic of cultural sensitivity and pluralistic language orientation, both mathematics teachers considered it important to know about the diversity of cultures, since most of the students who attended their classes were Hispanic students. They considered it important to know about Latino/a culture. As well, they considered it important to speak both languages in the classroom since 90% of the students who attended the class were not proficient in using the English language.

4.4.2 Mexican Mathematics Teachers: Angelica's and Yolanda's Case Study

Angelica: Demographic Information

Angelica, one of the Mexican participant was between the ages of 40-50 at the time of the study. She identified herself as Hispanic/ or Latina. She studied at the University in Ciudad Juarez. She studied engineering in electronics and digital systems. She had fourteen years of experience as an electronics teacher and one year as an algebra mathematics teacher at the high school level.

Angelica: Language and Communication Profile

Angelica considered Spanish her native language and English her second language. She did not speak any additional languages. She mentioned that the language used with her students in the classes and outside the class is Spanish since the students in the class are monolingual. As a point of clarification, Spanish was the language used by the students, teachers, and principals in her school. When she meets with other teachers or principals, discussion of the lesson plans is in Spanish.

Angelica: Classroom Description

Angelica's classroom (Figure 4.14 visual image) was a large classroom; each student had an individual work table. There was no technology utilized or mathematics posters explaining concepts in the classroom; nor were there any pictures on the walls. This classroom had more than 50 chairs since in Ciudad Juarez the number of students per class is greater than in the U.S. Also, in front of the classroom, there was a green chalk board, where Angelica could explain the examples and exercises to the students.



Figure 4.15: Angelica's classroom

Angelica: Lesson Plan Development

Angelica taught the different cases of factorization: notables' products, product of a binomial with a common term, square binomial, and conjugate binomial. Angelica developed these lesson plans following the examples and exercises from the algebra book. Angelica used the same algebra book provided by Secretaria de Educacion Pública (SEP), but she used different exercises and examples. These lessons plans were developed with the purpose that students would develop mathematical reasoning and solve problems applied to real life (Secretaria de Educacion Pública or SEP, 2013). In addition, the lesson plans were developed with the objective that students could build mathematical models by applying arithmetic and algebraic comprehension. These lesson plans were developed according to the algebra book established by the Secretaria de Educacion Pública (SEP, 2013). Embedded in the lesson plans were some activities assigned as homework for the students. These activities were found in the algebra book

used by Angelica. The assignments consisted of solving some exercises about the product of a binomial with a common term (Figure 4.15).

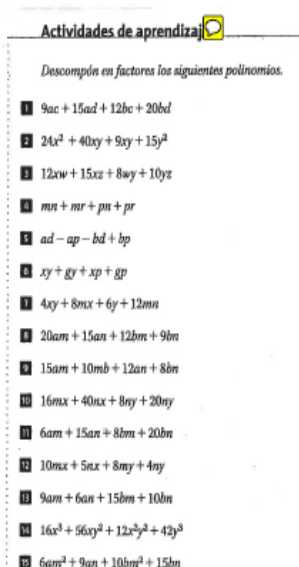


Figure 4.16: Angelica's assignment: product of a binomial with a common term

Although Angelica developed the lesson plan following the algebra book, Angelica implemented these topics through presentations given by the groups. She formed 8 groups, and each group consisted of 4 students. Each group worked out problems together and after that, they presented 1 or 2 cases of factorization. Each group provided the definition, one example, and one exercise. These factorization presentations took seven days to implement. Angelica gave a review of the case of factorization every Wednesday to facilitate student understanding of these topics. Figure 4.16 represents how Angelica formed the groups to implement the learning of the cases of factorization.

Monday	Group 8 case 2 and Group 2 case 3
Tuesday	Group 3 case 4 and Group 4 case 5, 1 st part and group 5 2 nd part
Wednesday	questions, and exercises about the cases presented for the groups
Thursday	Group 6 case 6
Monday	Group 7 and 1 case 7
Tuesday	questions, and exercises about the cases presented for the groups
Wednesday	Group 9 &10 case 8

Figure 4.17: Formation of student groups to implement factorization.

Characteristics of the Borderland Pedagogy Demonstrated by Angelica's in the Lesson Plan Development and Implementation.

In the lesson plan developed and implemented by Angelica, she demonstrated some of the characteristics of Borderland Pedagogy. The lack of characteristics demonstrated by Angelica were: Lack of Flexibility, Lack of Passion, Lack of Cultural Sensitivity and Lack of Pluralistic Language Orientation. These characteristics' were found in the three data sources. The following Table 4.11 shows the characteristics enacted in lesson plan development and implementation demonstrated by Angelica.

Table 4.11: Characteristics Enacted in Lesson Plan Development and Implementation: Angelica

Characteristics	Description
Lack of Flexibility	Design curriculum lacked uncreatively to incorporate the students' backgrounds and ways of knowing the world
Lack of Passion	Lack of ability to help all students at different levels
Lack of Cultural Sensitivity	Teacher doesn't have to understand the dynamics of how the communities in the border area work and the needs they have. Teacher doesn't need to know and learn about different cultures
Lack of Pluralistic Language Orientation	Teacher doesn't need to understand the acquisition of languages and the language used by these communities. Teacher doesn't need to have knowledge of Spanish and English

Lack of flexibility: Design curriculum lacked uncreatively to incorporate the students' backgrounds and ways of knowing the world

Angelica showed a traditional understanding about the teaching of conjugate binomials, notables products, and the product of a binomial with a common term. She considered that the traditional way was the most convenient since it was her first year teaching Algebra; previously she had 14 years of experience teaching Electronics. She expressed a fear of creating different activities in front of the group to teach this subject since she did not have experience of teaching algebra.

"It is my first year that I teach Algebra, so, I don't have an idea how this subject will be taught" (Interview October 30, 2013, Appendix A).

According to this statement, she was afraid to develop and implement these lessons in a different way. She mentioned that because of her lack of teaching experience with the subject, occasionally she met with and collaborated with another teacher, on development and implementation of lesson plans.

Angelica considered the lesson plan implementation through student presentations to the group to be appropriate. The students were presenting 1 or 2 cases of factorization. Forming groups, Angelica thought that the students "...could explain the topics well," (Source); however it did not happen with the first two groups that presented the notables products topic and conjugate binomials, as delineated in the observation field notes:

For example, when the first group finished explaining the notables product exercise, some of the students did not understand the concept and the problem solution. The students began to ask questions of the group, but they did not know how to respond. For this reason, Angelica tried to explain the same examples in front of the group using the same strategies that the group used. Despite the explanation given by Angelica, some of the students did not understand yet. For this reason Angelica stated "On Wednesday I will give a review of this topic" (Observation conducted 24, 2013, Appendix B).

As observed, Angelica showed a lack of knowledge to teach algebra in front of the group. She thought that through the presentations, the explanation of these topics would be easier for her. Table 4.12 represents the actions most frequently enacted under the characteristic of lack of flexibility demonstrated by Angelica.

Table 4.12: Actions Most Frequently Enacted Under the Characteristic of Lack of flexibility:

Angelica			
CharacteristicDescription		Data Sources	Frequency
Lack of Flexibility	Design curriculum lacked uncreatively to incorporate the students' backgrounds and ways of knowing the world	Lesson Plan	12
		Observations	8
		Interview	4

Lack of Passion: Lack of ability to help all students at different levels

On the presentations of notables product and conjugate binomial, Angelica did not show the ability to clarify the students' doubts in reference to these topics.

During her explanation the students did not understand Angelica. However Angelica didn't change her method or strategy to explain the problem to the students, she preferred to continue with the class. (Observation conducted October 24, 2013, Appendix B).

During the three days of observation, Angelica used the same methods and strategies to teach cases of factorization. The solution that she gave to the students' questions was to give a review of the topics taught by the students, in order to respond to the students' questions, however it did not work. She thought that it was the best way to teach algebra. Table 4.13 presents the actions most frequently enacted under the characteristic of lack of passion enacted by in Angelica's data.

Table 4.13: Actions Most Frequently Enacted Under the Characteristic of Lack of Passion:

Angelica			
CharacteristicDescription		Data Sources	Frequency
Lack of Passion	Lack of ability to help all students at different levels	Observations	3

Lack of Cultural Sensitivity: Teacher doesn't have to understand the dynamics of how the communities in the border area work and the needs they have; teacher doesn't need to know and learn about different cultures

Angelica, gave an opinion (without elaboration) about the need to understand the necessities of communities in the border area. She stated:

“In my personal opinion, since I am working in Ciudad Juarez, I didn’t consider it necessary to understand these necessities” (Interview October 30, 2013, Appendix A).

She showed a disinterest in this topic. She preferred to not continue with the interview.

Table 4.14 represents the actions most frequently enacted under the characteristic of lack of cultural sensitivity demonstrated by Angelica.

Table 4.14: Actions Most Frequently Enacted Under the Characteristic of Lack of Cultural

Sensitivity: Angelica

Characteristic	Description	Data Sources	Frequency
Lack of Cultural Sensitivity	Teacher doesn’t have to understand the dynamics of how the communities in the border area work and the needs they have.	Interview	1
	Teacher doesn’t need to know and learn about different cultures	Interview	1

Lack of Pluralistic Language orientation: Teacher doesn’t need to understand the acquisition of languages and the language used by these communities. Teacher doesn’t need to have knowledge of Spanish and English

Angelica expressed that the necessity to speak English and Spanish in the high school was not required, that is, it was not necessary to know the two languages to teach algebra.

“It is not a private high school where you need knowledge of the two languages, For example, in a private school, sometimes the mathematics classes are taught in English, but it is not our case” (Interview October 30, 2013, Appendix A).

Angelica's lessons were explained and implemented in Spanish; given it was a monolingual class. Table 4.15 represents the actions most frequently enacted under the characteristic of Lack of Pluralistic Language Orientation demonstrated by Angelica.

Table 4.15: Actions Most Frequently Enacted Under the Characteristic of Lack of Pluralistic Language Orientation: Angelica

Characteristic	Description	Data Sources	Frequency
Lack of Pluralistic Language Orientation	Teacher doesn't need to understand the acquisition of languages and the language used by these communities.	Interview	1
		Interview	1
	Teachers don't need to have knowledge of Spanish and English.		

Angelica's case study showed that the designs of the lesson plan were not creative with the explanation that it was the first year that she taught algebra. She considered it convenient to form groups to explain the case of factorization; however this strategy did not work. Also, this participant did not consider it necessary to speak both languages with the students, given that the dominant language in Mexico is Spanish.

Yolanda: Demographic Information

Yolanda is between the ages of 30-40 years old. She identified herself as Hispanic/or Latina. She studied at the University at Ciudad Juarez, Chihuahua. She specialized in Industrial Engineering. She continued her studies at the University at Ciudad Juarez gaining a Master's Degree in Industrial Engineering Sciences. Yolanda has experience working as a mathematics teacher at Ciudad Juarez, Chihuahua. She has seven years' experience teaching Algebra, Geometry, Trigonometry, Calculus, and Statistics and Probability.

Yolanda: Language and Communication Profile

Yolanda considered her native language to be Spanish and identified herself as monolingual because she does not consider English as her second language. She communicates using the Spanish language inside and outside of the classroom. When Yolanda meets with other mathematics professors to develop algebra lesson plans, she converses with them in Spanish.

Yolanda: Classroom Description

The characteristics of Yolanda's classroom are very similar to Angelica's classroom due to the fact that they taught in the same high school in Ciudad. Juarez. It was a larger classroom where the walls were painted in white color. There were more than 50 chairs to accommodate a large the number of students. Each student had a chair where they could work individually. There were no posters about any mathematics concepts in the Spanish language. Also, it was an accessible classroom, like Angelica's, because Yolanda could walk around the classroom to check the students' assignments. Similarly, it had the same inconveniences as Angelica's classroom; when the students worked in groups, there was not enough space for Yolanda to walk without difficulty to check the students' work. This classroom did not have technology, like graphing calculators, with which the students could work individually, nor did it have an electronic white board or a personalized computer where Yolanda could grade the students' assignments.

Yolanda: Lesson Plan Development

Yolanda developed the lesson plans about cases of factorization such as notables products, product of a binomial with a common term, square binomial, and conjugate binomial. These lessons were developed for students who were taking Algebra at high school level. Also,

these subjects are taught in the second year of high school at Ciudad Juarez. Yolanda used different lesson plans during the three days that I observed her.

Yolanda developed these lesson plans according to the objectives dictated by the Secretaria de Educacion Pública (SEP). These lesson plans were developed with the purpose that the students could develop a mathematical model and solve problems applied to real life in and out of a mathematical context.

The purpose of these lesson plans was to provide students with background knowledge facilitating building mathematical models and interpreting applications of arithmetic procedures, algebraic, and analyses compression for real situations. These lessons plans were developed according to the algebra book established by the Secretaria de Educacion Pública (2013).

The lesson plans were developed with activities assigned as homework for the students. These homework assignments consisted of a list of problems about notables' products, product of a binomial with a common term, square binomial; conjugate binomial requiring student solution. Figures 4.18, 4.19 and 4.20 represent the lesson plans developed by Yolanda about notables' products, product of a binomial with a common term, conjugate binomial during the three days of classroom observation, including the examples solved by Yolanda. These examples were taught and solved in front of the group of students following traditional teaching procedures. Figures 4.21 and 4.22 represent the homework assignments given by Yolanda to students. These exercises covered the topics conjugate binomial and binomial with a common term. The homework problems were taken from the algebra book established by the Secretaría de Educación Pública.

Productos Notables

Copiar lo de productos notables de la pág 65.
Dar un copia de los pág 81 al final los formularios.

Cuadrado de un Binomio

$$(a+b)^2 = a^2 + 2ab + b^2 \quad (1)$$

$$(a-b)^2 = a^2 - 2ab + b^2 \quad (2)$$

De donde son la fórmula?

$$(a+b)^2 = (a+b)(a+b) \quad (a-b)^2 = (a-b)(a-b)$$

$$\begin{array}{r} a+b \\ a+b \\ \hline a^2 + b^2 \\ a^2 + ab \\ \hline a^2 + 2ab + b^2 \\ (1) \end{array}$$

$$\begin{array}{r} a-b \\ a-b \\ \hline a^2 - b^2 \\ -ab + b^2 \\ \hline a^2 - 2ab + b^2 \\ (2) \end{array}$$

Figure 4.18: Yolanda's lesson plan: notables' products

Binomios Conjugados

$$(a+b)(a-b) = a^2 - b^2 \quad (3)$$

$$\begin{array}{r} a+b \\ a-b \\ \hline -ab - b^2 \\ +a^2 + ab \\ \hline a^2 - b^2 \\ (3) \end{array}$$

Ejemplos

$$\begin{aligned} \star 1 \quad (a+b)(a-b) &= a^2 - b^2 \\ (3a+5b)(3a-5b) &= (3a)^2 - (5b)^2 \\ &= 9a^2 - 25b^2 \end{aligned}$$

$$\begin{aligned} \star 2 \quad (a+b)(a-b) &= a^2 - b^2 \\ (2x^3+3y^2)(2x^3-3y^2) &= (2x^3)^2 - (3y^2)^2 \\ &= 4x^6 - 9y^4 \end{aligned}$$

Figure 4.19: Yolanda's lesson plan: conjugate binomial

Producto de un Binomio con un termino comun

$$(a+b)(a+c) = a^2 + a(b+c) + bc \quad \textcircled{5}$$

Ejemplos

$$\star 1 \quad \begin{array}{cc} (x+y) & (x+z) \\ \begin{array}{c} a & b \\ a & c \end{array} & \end{array} = \begin{array}{c} a^2 + a(b+c) + bc \\ \boxed{x^2 + x(y+z) + yz} \end{array}$$

$$\star 2 \quad \begin{array}{cc} (x+3) & (x+2) \\ \begin{array}{c} a & b \\ a & c \end{array} & \end{array} = \begin{array}{c} a^2 + a(b+c) + bc \\ \boxed{x^2 + 5x + 6} \end{array}$$

$$\star 3 \quad \begin{array}{cc} (x+7) & (x-2) \\ \begin{array}{c} a & b \\ a & -c \end{array} & \end{array} = \begin{array}{c} a^2 + a(b+c) + bc \\ \boxed{x^2 + 5x - 14} \end{array}$$

Figure 4.20: Yolanda's lesson plan: product of a binomial with a common term

Actividades de aprendizaje

1 Encuentra los productos de las siguientes multiplicaciones.

a) $(3a + 3b)(3a - 3b)$

b) $(8x + 3y)(8x - 3y)$

c) $(3x + 2)(3x - 2)$

d) $(2m - 3)(2m + 3)$

e) $(4x - 2)(4x + 2)$

f) $(2x - 5y)(2x + 5y)$

g) $(3x^2 + 2y^3)(3x^2 - 2y^3)$

Figure 4.21: Yolanda's homework assignment: conjugate binomial

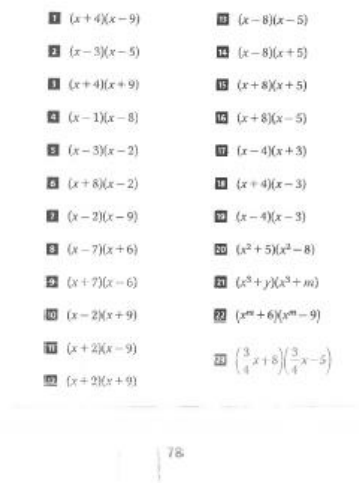


Figure 4.22: Yolanda’s homework assignment: product of a binomial with a common term

Yolanda: Lesson Plan Implementation

Yolanda implemented these lesson plans using only a green chalk board where she explained the examples and exercises assigned to the class. She did not embed technology or hands-on activities to teach these subjects. For Yolanda, implementation of the lesson plans in a traditional way was important to her.

“Algebra class is a subject that will be taught without technology, nor hands-on activities” (Interview October 30, 2013, Appendix, A).

Characteristics of the Borderland Pedagogy Demonstrated in Yolanda’s Lesson plan development and implementation.

In the lesson plan developed and implemented by Yolanda, she used some of the Characteristics’ of Borderland Pedagogy. The characteristics demonstrated by Yolanda were: Lack of Flexibility and Passion, Absence of Cultural and Linguistic Sensitivity. These characteristics were identified in the lesson plan collection, observations, and interview. Table

4.16 depicts the characteristics enacted in lesson plan development and implementation demonstrated by Yolanda.

Table 4.16: Characteristics Enacted in Lesson Plan Development and Implementation: Yolanda.

Characteristics	Description
Lack of Flexibility	Design curriculum lacked uncreatively to incorporate the students' backgrounds and ways of knowing the world
Lack of Passion	Lack of ability to help all students at different levels
Lack of Cultural Sensitivity	Teacher doesn't have to understand the dynamics of how the communities in the border area work and the needs they have. Teacher doesn't need to know and learn about different cultures
Lack of Pluralistic Language Orientation	Teacher doesn't needs to understand the acquisition of languages and the language used by these communities Teacher doesn't need to have knowledge of Spanish and English

Lack of Flexibility: Design curriculum lacked uncreatively to incorporate the students' backgrounds and ways of knowing the world

Yolanda's attitude showed disinterest in the use of technology or hands-on activities to develop algebra lesson plans, primarily due to the high school not having enough resources to incorporate into algebra instruction. She pointed out

“... the technology is not a tool where I can teach algebra or implement in my lesson plans” (Interview October 30, 2013, Appendix, A).

For Yolanda, technology is not a significant or necessary tool to teach algebra. Yolanda limited herself to developing lesson plans according to the content of the algebra textbook. She did not design lesson plans using different tools or materials to teach the lessons. All the examples and exercises were obtained from the algebra textbook, as evidenced in the following statement:

“the notables’ product, conjugate binomial lessons were developed through of the definitions and examples mentioned by the book” (Interview October 30, 2013, Appendix, A).

Yolanda’s educational experiences were vivid for her and strongly influenced both her view of the way to teach algebra, as well as her orientation toward teaching. Yolanda liked mathematics, and attributed her like for mathematics to one particular teacher that she had in middle school. From this experience, she remembered how her mathematics teacher taught and she expressed that it was the best method to teach algebra since she learned that way. Yolanda said that practice was the best manner in which she learned mathematics. These experiences helped to her to decide to be a mathematics teacher. She believed that the most important method of teaching mathematics was through practice with a long list of problems.

”... algebra can be taught through practice. It's a matter of practice, practice, practice. This is the way I learned when I was student algebra” (Interview October 30, 2013, Appendix, A). Yolanda was positive about her experience in the lesson plan development for algebra.

“I enjoy to every day developing the algebra lesson plan” (Interview October 30, 2013, Appendix A). However, her experience on algebra lesson plan development was procedural and rule based. (Lesson Plan collected on October 24, 2013, Appendix L).

Yolanda asserted algebra should be taught following mathematical procedures, as well using manipulative symbols, signs, variables. Accurately applying rules to obtain correct answers was important. She saw the formulas as procedures (Figure 4.23).

$$\begin{aligned}
 & \star 3 \quad (a+b+c)(a+b-c) \\
 & \quad \quad \quad \begin{matrix} (a+b) & (a-b) \\ ((a+b)+c) & ((a+b)-c) \end{matrix} = \begin{matrix} a^2 - b^2 \\ (a+b)^2 - (c)^2 \end{matrix} \\
 & \quad \quad \quad \frac{a^2 + 2ab + b^2 - c^2}{a^2 + 2ab + b^2 - c^2} \\
 & \star 4 \quad (2x+3y+z)(2x+3y-z) \\
 & \quad \quad \quad \begin{matrix} (a+b) & (a-b) \\ ((2x+3y)+z) & ((2x+3y)-z) \end{matrix} = \begin{matrix} a^2 - b^2 \\ (2x+3y)^2 - z^2 \end{matrix} \\
 & \quad \quad \quad \frac{a^2 + 2ab + b^2}{(2x)^2 + 2(2x)(3y) + (3y)^2} \\
 & \quad \quad \quad \frac{4x^2 + 12xy + 9y^2 - z^2}{4x^2 + 12xy + 9y^2 - z^2} \\
 & \star 5 \quad [(x+y)+(m+n)][(x+y)-(m+n)] \\
 & \quad \quad \quad \begin{matrix} (a+b) & (a-b) \\ (x+y) & (x-y) \end{matrix} = \begin{matrix} a^2 - b^2 \\ x^2 - y^2 \end{matrix} \\
 & \quad \quad \quad \frac{(a+b)^2}{(x+y)^2} = \frac{a^2 + 2ab + b^2}{x^2 + 2xy + y^2} \\
 & \quad \quad \quad \frac{(a-b)^2}{(x-y)^2} = \frac{a^2 - 2ab + b^2}{x^2 - 2xy + y^2} \\
 & \quad \quad \quad \frac{a^2 + 2mn + n^2}{m^2 + 2mn + n^2} \\
 & \quad \quad \quad \frac{(x^2 - 2xy + y^2) - (m^2 + 2mn + n^2)}{(x^2 - 2xy + y^2) - (m^2 + 2mn + n^2)}
 \end{aligned}$$

Figure 4.23: Exercises developed by Yolanda

Yolanda's strategy in planning the lessons about algebra was in consideration of: (1) traditional teaching methods, (2) personal tutoring implemented by her or by another students and (3) tutoring with another mathematics teacher. Yolanda expressed:

"traditional refers when I am in front the group explaining the exercises, and the students listening me and they copy the problem in their notebooks" (Interview October 30, 2013, Appendix, A). For example, when covering the topic Conjugates Binomial, Yolanda solved two similar examples. The strategies that she used to teach these examples were to explain these examples in front of the group of students. After that, she provided two similar exercises for the students to work individually. One of the examples explained by Yolanda was the following:

"What is a binomio?" No one responded, then she stated "It has two equal terms".

"For example, $(a+b)(a+c) = a^2 + a(b+c) + bc$. "a" is the equal term"

And she introduced another example: $(x+y)(x+z)$, "what is the common term?"

And one student responded: "x"

Yolanda asked, “so how is the answer?”

And one student responded “ $x^2+xy+xz+yz$ ”

(Observation, October 28, 2013, Appendix, F).

Yolanda utilized a second way to prepare her algebra class through personal tutoring given by other students or by her. She said:

“If the student doesn’t understand with the traditional form, I explain personally or I leave that another of their peers explain the exercise” (Interview October 30, 2013, Appendix, A).

Finally, Yolanda expressed that:

“The last option is that students came to tutoring on Saturdays” (Interview October 30, 2013, Appendix, A). Every Saturday there was tutoring for the algebra class, and Yolanda mentioned to the students that if they did not understand the lesson, please attend tutoring on Saturday. These tutoring sessions were conducted by another mathematics teacher. Table 4.17 represents the actions most frequently enacted under the characteristic of lack flexibility demonstrated by Yolanda.

Table 4.17: Actions Most Frequently Enacted Under Characteristic Lack of Flexibility:

Yolanda			
Characteristic	Description	Data Sources	Frequency
Lack of Flexibility	Design curriculum lacked		
	uncreatively to incorporate the	Lesson Plan	22
	students’	Observations	15
	backgrounds and ways of	Interview	3
	knowing		
	the world		
	.		

Lack of Passion: Lack of ability to help all students at different levels

On the first and second day of observations, Yolanda showed a lack of ability to assist students with the factorization problems. For example, on the second day, Yolanda began her algebra class teaching a new topic; the topic was “the square of the binomial”. She explained two examples step by step using the green chalk board but without adequate content knowledge.

For the example Yolanda mentioned “we can’t write a formula with negative signs (-), because we have a lot of formulas. In other words, we will have different combinations. Examples, $a+b+c$, $a-b+c$, $a+b-c$, $-a+b-c$, $a-b-c$, etc.” (Observation, October, 24, 2013, Appendix B). After this explanation, some of the students asked the same questions and she did not know how to explain the same example using a different method or strategy. Table 4.21 represents the actions most frequently identified as the characteristics No Passion demonstrated by Yolanda.

During the explanation of the square of the binomial, Yolanda did not change her strategy in explaining why the case of factorization could not contain a negative sign. She continued presenting the content in the same manner with the class (Table 4.18).

Table 4.18: Actions Most Frequently Enacted Under the Characteristic of Lack of Passion:

Yolanda			
Characteristic	Description	Data Sources	Frequency
Lack Passion	Lack of ability to help all students at different levels	Observations	5

Lack of Cultural Sensitivity: Teacher doesn't have to understand the dynamics of how the communities in the border area work and the needs they have; teacher doesn't need to know and learn about different cultures

Yolanda showed disinterest in the questions about the necessity to understand the communities in the border area. She was of the understanding that it was not necessary to understand the necessities of a border area since she is working in Ciudad Juarez as a mathematics teacher. In Ciudad Juarez there are not students who came from El Paso, Texas to study at the high school where Yolanda teaches.

“It doesn't happen on this side of the border area. For example, in El Paso, Texas there is most common that student decide continue their studies at El Paso, Texas than in Ciudad Juarez. In my opinion, the teachers who need to understand this situation are those Mexican teachers who work at the border area as teachers. They need to know the necessities of the community as well they need to know and understand the diversity of cultures. I think that in El Paso, Texas there is a diversity of cultures and you need to understand and respect it. In my personal opinion, this teachers need to have a special training in order to learn about the diversity of culture” (Interview October 30, 2013, Appendix, A). Table 4.19 represents the Characteristics found in Yolanda.

Table 4.19: Actions Most Frequently Enacted Under Characteristic of Lack of Cultural

Sensitivity: Yolanda

Characteristic	Description	Data Sources	Frequency
Lack of Cultural Sensitivity	Teacher doesn't have to understand the dynamics of how the communities in the border area work and the needs they have.	Interview	1
	Teacher doesn't need to know and learn about different cultures	Interview	1

Based upon the ideas and opinion that Yolanda expressed, it was clear she did not consider it necessary to understand border community necessities due to the fact that she was working as mathematics teacher in Ciudad Juarez and in her opinion, diversity of cultures is not a factor there.

Lack of Pluralistic Language Orientation: Teacher doesn't needs to understand the acquisition of languages and the language used by these communities & teacher doesn't need to have knowledge of Spanish and English

Yolanda did not express interest in speaking English in the classroom because Yolanda's class is monolingual. The primary language spoken between students and teachers is Spanish (Survey conducted October 30, 2013, Appendix, G). During the three days of observation, Yolanda explained the examples and exercises in Spanish. She did not encourage speaking another language with her the students. For example, she stated

“The language talked with my students is Spanish, I don't have the necessity to talk another language different to Spanish since we are in Mexico” (Interview October 30, 2013, Appendix, A).

Table 4.20 represents the actions most frequently enacted under the characteristic of the lack of pluralistic language orientation demonstrated by Yolanda.

Table 4.20: Actions Most Frequently Enacted Under the Characteristic of Lack of Pluralistic

Language Orientation: Yolanda

Characteristic	Description	Data Sources	Frequency
Lack of Pluralistic Language Orientation	Teacher doesn't needs to understand the acquisition of languages and the language used by these communities.	Interview	2
	Teacher doesn't need to have knowledge of Spanish and English.	Interview	1

Included in Yolanda's case study were descriptions of how she implemented and developed her lesson plan to teach the different case of factorization. She did not consider it necessary to implement technology since, in her opinion, algebra class should be taught through the usage of procedures. Also, Yolanda, in some exercises, did not show adequate knowledge to change strategies to explain the notables' products.

In the second case study represented by Mexican mathematics teachers, there were some similarities between the characteristics of Borderland Pedagogy, i.e. the absence of some characteristics incorporated by them in their lesson plan development practices. For example, Yolanda used three different methods to teach her algebra class: a traditional approach, advising students individually, or recommending tutoring with a different mathematics teacher. Her teaching methods resulted in a lack of opportunity for the students to discover the meaning of the algebra concepts. As well, Angelica used the Algebra book as a primary resource to develop and implement her lesson plan, including use of the examples from the standardized textbook.

Both mathematics teachers demonstrated lack of passion and did not show enough desire to facilitate learning beyond direct instruction and to help students to be able to solve problems about case of factorization. The lack of passion appears to be linked to the fact that both mathematics teachers didn't demonstrate enough content and/or pedagogical knowledge to explain the different cases of factorization. Finally, referring to lack of cultural sensitivity and lack of pluralistic language orientation, both Mexican teachers showed disinterest in the questions about the necessity to understand the communities in the border area. They mentioned that it was not necessary to understand the needs of a border area. In Ciudad Juarez there are not students who came from El Paso, Texas to study at the high school where they teach. As well, both mathematics teachers expressed in the interview that they did not need to speak or have knowledge of English language since the main language spoken in their classrooms was Spanish.

4.4.3 Transitioning Mathematic Teacher: Carlos' Case Study

Carlos: Demographic Information

Carlos was between the age ranges of 30-40 years old at the time of the study and identified himself as Hispanic/Latino. He attended the University at Ciudad Juarez, Chihuahua, studying Electronic Engineering. Continuing his studies, Carlos obtained a Master's in Education from the University of Texas at El Paso (UTEP). He worked as a high school mathematics teacher for one year in Ciudad Juarez. (It is not known what class he taught in Ciudad Juarez, Chihuahua). After teaching for one year in Ciudad Juarez, he decided to come to El Paso, Texas and work as a mathematics teacher. At the time of the study, he had ten years of experience as a mathematics teacher in the U.S.

Carlos: Language and Communication Profile

Carlos considers Spanish his native language and English his second language. He mentioned that he uses both Spanish/ English in and outside of the classroom. Finally, he stated that when he meets with other teachers, principals, and staff about lesson plan development he seldom uses his first language, and also uses the English language to develop lesson plans.

Carlos: Classroom Description

Carlos's classroom, located in a portable building, was very different from Esteban's, Virginia's, Yolanda's, and Angelica's classrooms. Although it is a portable classroom, the classroom is equipped with the tools necessary to implement the mathematics class. Being a portable building is the major difference in comparison to Esteban's, Virginia's, Yolanda's, and Angelica's classrooms. Carlos' classroom is large, allowing Carlos to walk around the classroom to check the assignment of each student. Each student has an individual chair to work in and the classroom was equipped with technology to develop the lessons. For example, there was a large whiteboard where Carlos can do exercises and check the answer with his students. In front of the classroom, Carlos had a personal computer connected to the internet to check e-mail, or to grade students' assignments. Also, each student had access to an individual calculator where they could perform operations and graph any function. Next, in the classroom, you can observe posters of mathematics concepts in the English language and the TEKS, as well as protractors, compasses, rulers, and other mathematics instruments to develop the lesson. Also, in Carlos' classroom there were student computers on the sides. On these computers some of the students could take an online math class; in other words, there were some students who took the class face-to-face and other students who took the class on the computer. Carlos' role was as a mathematics teacher who taught the class face-to-face and assisted the students who were taking the class on the

computer. At the end of the classroom, there were some books available about Pre-calculus and Algebra for the students to use in the Pre-calculus class as needed. Figures 4.24 and 4.25 are visual images of Carlos' classroom.

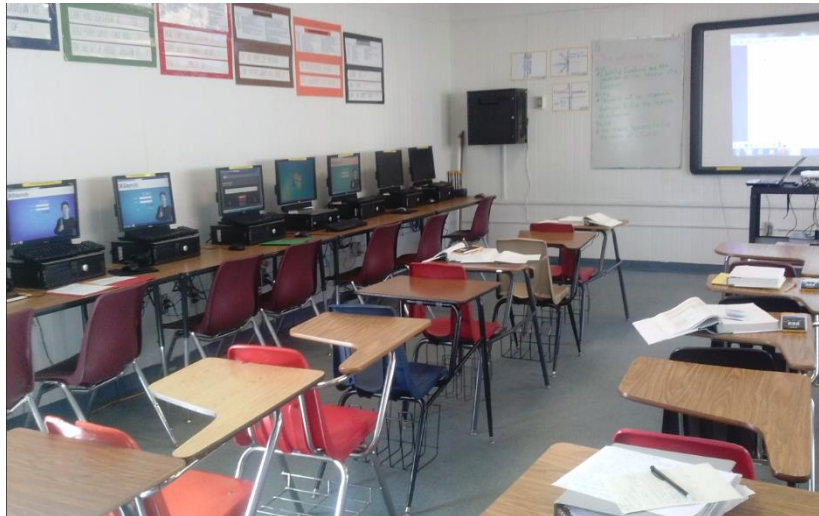


Figure 4.24: Carlos' classroom: side view



Figure 4.25: Carlos' classroom description arrangement: front wall

In Figure 4.24 there are shown some computers available for the students who were taking the online classes in mathematics. Each of the students had access to one computer. Figure 4.25 shows the white board where Carlos implemented his Pre-calculus class. On the white board, Carlos solved the inverse and composite function problems. Also, aside from the

white board, there were some teaching materials (e.g., protractor, ruler) that Carlos used to teach pre-calculus.

Carlos: Lesson Plan Development

Carlos developed a lesson plan about composition/ inverse functions. These lesson plans were developed for students who were taking Pre-calculus at the high school level. Carlos used different lesson plans during the three periods of observation.

The purpose of the inverse function lesson plan required students to identify functions and the existence of the inverse of a function using the vertical and horizontal line tests. Also, the students used algebraic techniques to find the inverse of a function. Students used graphing and tabular techniques to find the inverse of a function. The purpose of the composition function lesson plan was to facilitate students' use age of algebraic techniques to find the composition of two functions, as well as learning to use tabular and graphical techniques to find the composition of two functions. In general, the emphasis of the inverse and composition functions lesson plan was to allow students to prove that two functions are the inverse of each other by composition and with the use of graphs and the tables of values.

Figures 4.26, 4.27, and 4.28 present the lessons that Carlos developed to teach the inverse function topic. Figure 4.26 presents the list of inverse function problems assigned to the students as homework; this lesson was taken from the pre-calculus book. Figures 4.27 and 4.28 present the lesson plans that Carlos developed about inverse functions through the graph representation. The first exercise was solved in the classroom with Carlos' help. Finally, Figure 4.29 presents the composition function lesson plan. This lesson plan was designed by Carlos with the purpose of facilitating student understanding of the topic through a story problem.

Find the inverse of the following functions (don't work with the rational functions). Is the inverse a function? Sketch the graphs reflected across the line $y=x$

$f(x) = 5x^3 - 5$
 $f(x) = 4x^2 - 3$
 $f(x) = \frac{9x-3}{7x+6}$
 $f(x) = 6x - 4$
 $f(x) = 7x - 9$
 $f(x) = 7x + 4$
 $f(x) = 3x^5 - 9$
 $f(x) = 6x + 7$
 $f(x) = \frac{4x+2}{4x+3}$
 $f(x) = 5x^2 + 4$
 $f(x) = \frac{4x-1}{2x+2}$
 $f(x) = \sqrt[3]{8x-3}$
 $f(x) = \sqrt[3]{-6x-4}$
 $f(x) = \frac{8x-7}{3x-6}$
 $f(x) = \sqrt[3]{-3x-5}$

Figure 4.26: Inverse function lesson plan-1

Solving Equations (pp. 1 of 4)

Use your knowledge of transformations of parent functions to sketch the graph of each function below. Describe the transformations used.

1) Parent: $y = x^3$ (dotted)
Function: $f(x) = -(x+2)^3 + 4$

Transformations:

-
-

Solution:

Solving Equations (pp. 2 of 4)

3) Parent: $y = \frac{1}{x}$ (dotted)
Function: $f(x) = \frac{4}{x-5} + 2$

Transformations:

-
-

Solution:

2) Parent: $y = \sqrt{x}$ (dotted)
Function: $f(x) = 2\sqrt{x-3} - 5$

Transformations:

-
-

Solution:

4) Parent: $y = x^2$ (dotted)
Function: $f(x) = \frac{1}{2}(x+3)^2 - 4$

Transformations:

-
-

Solution:

Figure 4.27: Inverse function lesson plan-2

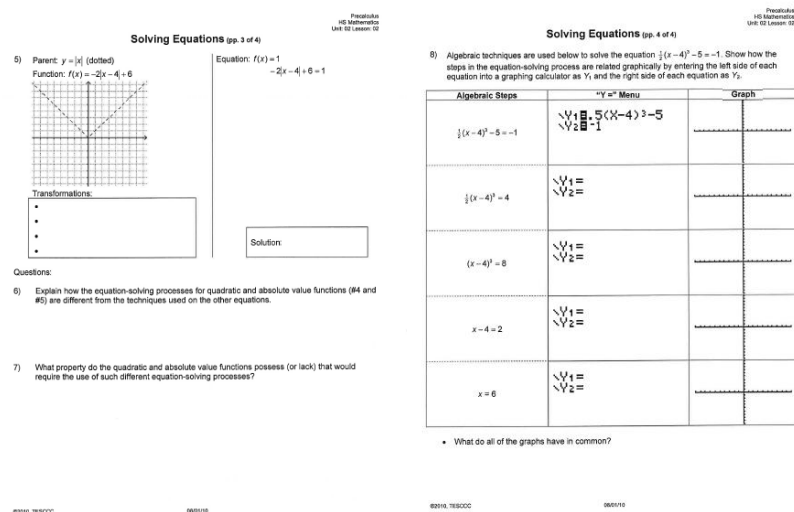


Figure 4.28: Inverse function lesson plan-3

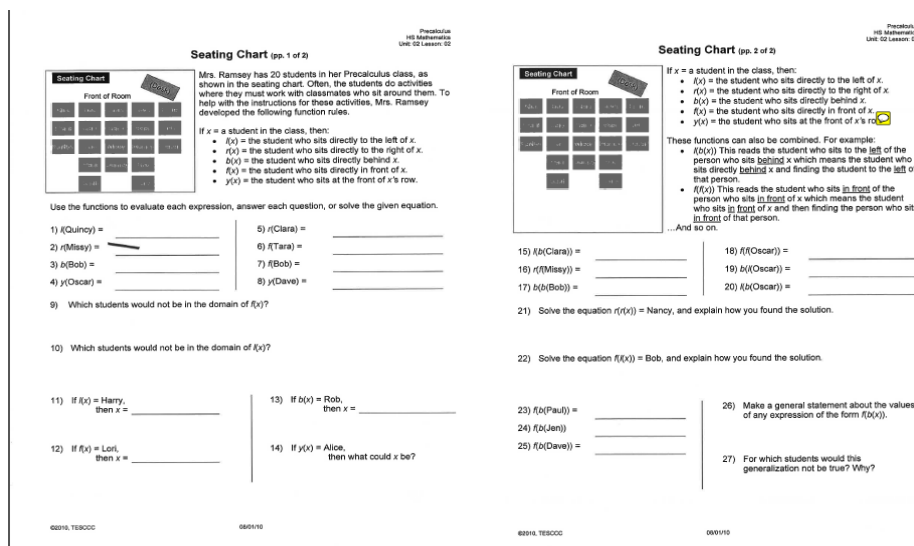


Figure 4.29: Composition of functions lesson plan

The lesson plans were developed with an activity as homework for the students. These assignments consisted of students finding the composition and inverse of the functions. These assignments were taken from the lesson plans development since they did not finish these activities in the classroom.

Carlos: Lesson Plan Implementation

Carlos implemented the lesson plan according to the TEKS (2012): “The student will apply mathematical processes to understand that functions have distinct key attributes and understand the relationship between a function and its inverse. The student is expected to: graph the functions $f(x)=\sqrt{x}$, $f(x)=1/x$, $f(x)=x^3$, $f(x)=\sqrt[n]{x}$, $f(x)=bx$, $f(x)=|x|$, and $f(x)=\log_b(x)$ where b is 2, 10, and e , and, when applicable, analyze the key attributes such as domain, range, intercepts, symmetries, asymptotic behavior.;

- (A) Graph and write the inverse of a function using notation such as $f^{-1}(x)$;
- (B) Describe and analyze the relationship between a function and its inverse (quadratic and square root, logarithmic and exponential), including the restriction(s) on domain; and
- (C) Use the composition of two functions, including the necessary restrictions on the domain, to determine if the functions are inverses of each other.”

Carlos implemented these lesson plans using technology such as the white board, and TI Inspire Calculator. Also, he used graphic organizers, and power point presentations. Carlos implemented different strategies to teach these lesson plans. For example, he used the strategy of “we do”, “you do” and “closure” The part of “we do” refers to the idea that students will practice the concepts to be learned with direct feedback from the teacher including clarification of possible misconceptions. “You do” means that students will work individually on practice problems to include multiple representations, graphing by hand, elaborating tables of values and working on application problems selected by the teacher. Finally, “closure” refers to writing a reflection about the lessons implemented during the week.

The journal reflection contained questions such as the following:

What I know about _____ so far is _____

What I'm still not sure about is _____

What I'd like to know more about is _____ (Lesson plan collected October 30, 2013, Appendix, J)

Additional examples are found in the data demonstrating how Carlos implemented the lesson plans to teach inverse/composition function to the pre-calculus class.

Characteristics of the Borderland Pedagogy Found by Carlo's Lesson Plan Development and Implementation

In the lesson plan developed and implemented by Carlos, he made use of some of the characteristics of Borderland Pedagogy. The characteristics identified for Carlos were Flexibility, Passion, Culture, and Language. These characteristics were found in the lesson plan collection, observations, and interview. Table 4.21 illustrates the characteristics enacted in lesson plan development and implementation demonstrated by Carlos.

Table 4.21 Characteristics Enacted in Lesson Plan Development and Implementation: Carlos

Characteristics	Description
Flexibility	Design curriculum creatively to incorporate the students' backgrounds and ways of knowing the world.
Passion	Ability to help all students at different levels
Culture	Teacher needs to know and learn about different cultures Teacher in the border region needs to have contact with people of different socioeconomic classes, with people from distinct cultures and languages, and preferably to have an experience personally living in another country to better understand cultural values different from his or her own .
Language	Teacher needs to understand the acquisition of languages and the language used by these communities. Teacher needs to have knowledge of Spanish and English.

Flexibility: Design curriculum creatively to incorporate the students' backgrounds and ways of knowing the world

Carlos' lesson plan development was grounded in and strongly influenced both by his experience as a mathematics teacher in Mexico, as well as his experience as an Electronic Engineer. He intimated that he was always attracted to the teaching of mathematics, not because the content; rather, he wanted to apply his mathematics knowledge and his experience as an engineer to lesson plan development. For example, he stated

“My experience as an electronic engineer for seven years helped me to change the way I teach mathematics. In other words, I learned that some electronic concepts can be taught through some topics in mathematics like the behavior of functions; I got this experience when I worked as an Engineer in a fabric in Juarez”. (Interview, October 22, 2013, Appendix, A). Throughout this explanation, Carlos remembered his experience as an Electronic Engineer, and he explained how this experience helped him to apply some mathematics concepts to real life. However, the researcher noted that he demonstrated some characteristics of a traditional teacher. For instance, one component of his lesson plan was to solve a long list of inverse function (Figure 4.26).

In college, Carlos got a Bachelor's Degree in Mathematics and Physics. Although he always experienced success in mathematics, he remembered little content from his mathematics classes and mentioned that he had the opportunity to change his methods and strategies to teach mathematics in front of the group at the high school level. Throughout his experience, he felt it necessary to design lesson plans using different methods and strategies. For example, he expressed

“When I was in high school in Juarez, all mathematics lessons were developed in a traditional way. If you studied in Juarez, you remember that the lesson plans are developed with the objective that the students solve a long list of problems and finally they can apply the mathematics concepts to real life. It never happens, because time doesn’t permit” (Interview October 22, 2013, Appendix A). Through this explanation, Carlos expressed the opportunity that he had to change the teaching methods in order for students to learn mathematics concepts and apply them to real life. For this reason, he developed the inverse/ composition function through a story. He mentioned:

“When I came to El Paso, Texas to work as a mathematics teacher, I could apply all my knowledge since the TEKS are developed in order that students can learn mathematics differently from how they learn in Juarez” (Interview October 22, 2013, Appendix A).

Finally, through the lesson plan of inverse function, Carlos began his lesson by explaining the function definition. He considered this explanation as necessary, since in mathematics it is fundamental to know what a function is and how it can be represented before advancing to learn inverse/composition function. The following excerpts exemplify this notion:

Carlos: “What is a function?”

Carlos: “Any idea?”

Carlos: “You learned about quadratic function, logarithm function.”

Carlos: “But what is a function?” (Observation, 24, 2013, Appendix, D). Since nobody responded to the questions, Carlos developed a lesson where the students could learn the function definition through the three representations (Table, Graph, and Mapping diagram). Figure 4.30 presents the lesson developed by Carlos to teach function definition.

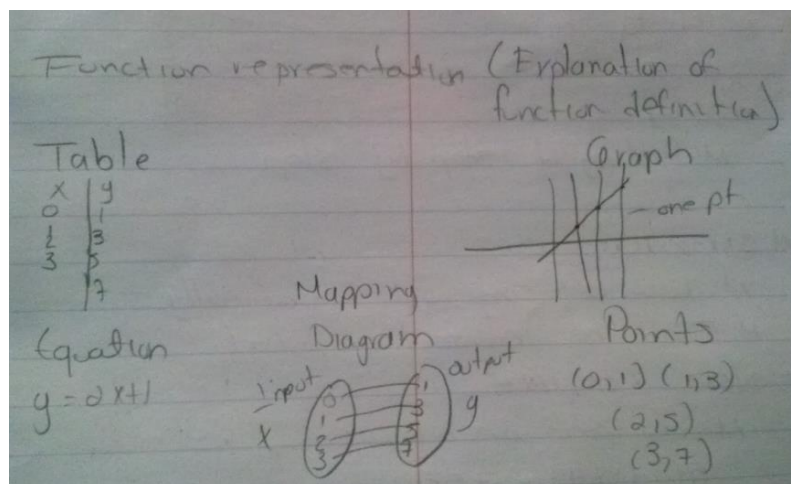


Figure 4.30: Lesson plan about function definition.

Table 4.22 represents the actions most frequently enacted under the characteristic of Flexibility demonstrated by Carlos.

Table 4.22: Actions Most Frequently Enacted Under the Characteristic of Flexibility: Carlos

Characteristic	Description	Data Sources	Frequency
Flexibility	Design curriculum creatively to	Lesson Plan	23
	incorporate the students'	Observations	12
	backgrounds and ways of knowing the world.	Interview	6

Passion: Ability to help all students at different levels

During the three days that Carlos was observed, he showed the ability to respond to students who had some questions about inverse function questions and questions about the use of technology (calculator TI Inspire). For example, on the second day of observation, it was interesting to see the explanation that Carlos gave to one student about why these two mathematics expressions are the same " $3\sqrt{8} \neq 3\sqrt{x}$ " (Observation October 31, 2013, Appendix, D). Carlos explained with patience why these two expressions are not the same. Table 4.23

represents the actions most frequently enacted under the characteristic of Passion demonstrated by Carlos.

Table 4.23: Actions Most Frequently Enacted Under the Characteristic of Passion: Carlos

Characteristic	Description	Data Sources	Frequency
Passion	Ability to help all students at different levels.	Lesson Plan	0
		Observations	9
		Interview	2

Culture: Teacher needs to know and learn about different cultures and Teachers in the border region need to have contact with people of different socioeconomic classes, with people from distinct cultures and languages, and preferably to have an experience personally living in another country to better understand cultural values different from his or her own

Throughout the interview, Carlos expressed an interest in learning about different cultures since he was working as a mathematics teacher in the border area. For example, he mentioned:

“When I came to work as a mathematics teacher, I received special training at Region 19. This training helped me to understand the importance to respect, learn and understand the diversity of cultures of the students. It was surprising for me to know what kind of cultures you can find here in El Paso, Texas. It is surprising to observe that in El Paso, Texas some classrooms are formed by a diversity of cultures, something that is not seen in Ciudad Juarez.” (Interview October 22, 2013, Appendix, A).

He expressed that his experience as a transitioning teacher and of living in another country (Ciudad Juarez) had helped him to dominate both languages. He pointed out:

“Some teachers ask for my help since they can’t help Hispanic students since they are not using both languages. So, my experience as a mathematics teacher in Ciudad Juarez

and in El Paso, TX, has helped me to understand them by knowing their culture and language”. (Interview October 22, 2013, Appendix, A).

He mentioned that through his experience as a mathematics teacher in Ciudad Juarez and El Paso, TX, he could understand the needs of Hispanic students’ diversity of cultures that are seen in some high schools. Table 4.24 represents the actions most frequently enacted under the characteristic of Culture demonstrated by Carlos.

Table 4.24 Actions Most Frequently Enacted Under the Characteristic of Culture: Carlos

Characteristic	Description	Data Sources	Frequency
Culture	Teacher needs to know and learn about different cultures	Interview	3
	Teacher in the border region needs to have contact with people of different socioeconomic classes, with people from distinct cultures and languages, and preferably to have an experience personally living in another country to better understand cultural values different from his or her own	Interview	4

Language: Teacher needs to understand the acquisition of languages and the language used by these communities and Teacher does need to have knowledge of Spanish and English

Carlos expressed that it is necessary to know both languages since he is working as a mathematics teacher in the borderland area. He stated:

“It is important to speak both languages since we are in the border area. Some of my students are not proficient with the language” (Interview October 22, 2013, Appendix, A). He confirmed the importance of speaking both languages in the classroom. He

understood its importance because he had the opportunity of speaking both languages with the students. Unfortunately, it does not happen with all the mathematics teachers who work in the same high school as Carlos. He stated:

“Some teachers who are not proficient in Spanish, and they are working at the border are asking me to help them with some students who didn’t talk English” (Interview October 22, 2013, Appendix, A).

Since Carlos considered it important to talk about language with his students, Carlos could also explain lessons to students in both languages without difficulty. For instance, he said: “Dejame lo hago para que lo veas, si? Student: Si, thank you” (Observation October 29, 2013, Appendix D). Table 4.25 represents the actions most frequently enacted under the characteristic of Language demonstrated by Carlos.

Table 4.25: Actions Most Frequently Enacted Under the Characteristic of Language: Carlos

Characteristic	Description	Data Sources	Frequency
Language	Teacher does needs to understand the acquisition of languages and the language used by these communities.	Observations	4
		Interview	2
	Teacher needs to have knowledge of Spanish and English.	Observations	5

As the transitioning teacher case study participant, Carlos demonstrated that he was flexible while designing his lesson plans. His experience as an Electronic engineer helped him to work successfully with the mathematics problems, and to apply them to real life. In reference to the characteristics of cultural sensitivity and pluralistic language orientation, Carlos mentioned that the border area is formed by a diversity of cultures, especially by the Latino/a culture requiring a teacher to be aware of the students’ needs. Supporting this assertion, he mentioned

the importance of speaking both English and Spanish with his students, since he is working as a teacher in the border area and some of his students are not proficient with the English Language. Carlos demonstrated all of the characteristics of Borderland Pedagogy as identified in observations, interviews and lesson plan development. As a transitioning teacher from Mexico to the U.S., Carlos became aware of the need for and developed his methods and skills through additional educational experiences and professional development.

4.5 SUMMARY

Based on the description of the importance of presenting findings as case studies, this chapter provided a thorough description of the findings. This chapter presented the five case studies, as well as the characteristics of Borderland Pedagogy identified for each participant. Each case study provided a description of the classroom, the lesson plan development and its implementation. For each participant, different positive and negative characteristics of Borderland Pedagogy were identified. These positive and negative characteristics were presented in frequency charts including the data sources for the characteristics found.

Chapter 5: Discussion, Implications, and Conclusion

5.1 INTRODUCTION

In this chapter, the three case studies of this mixed methods research are presented. This mixed methods study utilized case study as the methodology in the qualitative portion with frequencies as the main findings for the quantitative component. The findings were analyzed and discussed based on each research question. Each research question contained the Characteristics of Borderland Pedagogy that emerged as demonstrated by each participant, as well as the graphic representation of the frequencies. The following research questions guided the study:

1. What are the lesson plan developments and implementation practices incorporating Borderland Pedagogy used by U.S. high school mathematics teachers in El Paso, Texas?
2. What are the lesson plan developments and implementation practices incorporating Borderland Pedagogy used by Mexican high school mathematics teachers in Ciudad Juarez, Chihuahua?
3. How does a transition from Mexico (Ciudad Juarez, Chihuahua) to the United States (El Paso, TX.) impact high school mathematics teacher's incorporation of Borderland Pedagogy into their lesson plan development and implementation practices?

In this chapter, I build upon the preceding chapters' presentation of key findings, explicating the meanings behind those findings through discussion of each research question. The data gathered in this study that answered the three research questions were: three observations of each participant, one semi-structured interview with each participant and lesson plan collection from each participant.

The closing section of this chapter presents a discussion of the implications for research, theory, and practice as well as the potential for the findings of this study to contribute to research

and practice. The assumptions and limitations of the study and a conclusion are also presented in this chapter.

5.2 DISCUSSION OF FINDINGS

As mentioned before, the discussion of the findings are presented in relation to each research question. Each research question was responded to according to the analysis of the findings linked with the literature review and theoretical framework. The first research question was answered by the first and second case studies (Esteban and Virginia), as these were the U.S. mathematics teachers. The second research question was answered based on the findings of the third and fourth case studies (Yolanda and Angelica) as these were the Mexican mathematics teachers. Finally, the third research question was answered based on the findings of the fifth case study (Carlos), as this participant was the transitioning mathematics teacher. These research questions were discussed and analyzed according to the positive or negative characteristics for Borderland Pedagogy and lesson plan design as theoretical framework.

5.3 WHAT ARE THE LESSON PLAN DEVELOPMENT AND IMPLEMENTATION PRACTICES INCORPORATING BORDERLAND PEDAGOGY USED BY U.S. HIGH SCHOOL MATHEMATICS TEACHERS IN EL PASO, TEXAS?

The first research question was answered using the analysis of the first and second case studies. The first case study is attributed to Esteban and the second is attributed to Virginia. These two participants had worked as mathematics teachers in El Paso, Texas for several years. For example, Esteban has ten years of experience as a mathematics teacher at the high school level and Virginia has thirteen years of experience as a mathematics teacher at the high school level. During the semester the teachers were observed, they were teaching Geometry classes to 9th grade high school students.

Through the analysis of the three data sources, it was found that when Esteban and Virginia developed and implemented their lesson plan practices to teach Geometry, they incorporated some Characteristics of Borderland Pedagogy. Esteban and Virginia's case studies coincided with positive characteristics identified as Passion, Culture, and Language. The descriptions of these characteristics are presented in Table 5.1. However, they did not demonstrate the characteristic identified as Flexibility. Virginia demonstrated a negative characteristic of flexibility when she developed the lesson plan development as presented in Table 5.1. Table 5.1, additionally, presents the actions most frequently demonstrated by these participants, in relation to Characteristics of Borderland Pedagogy. These frequencies emerged through the different data sources for each participant lesson plan collection, three observations and one semi-structured interview. The discussions of the analysis of these characteristics are presented in the same order that is represented in Table 5.1. This table represents the Characteristics of Borderland Pedagogy Predominantly Demonstrated by Esteban and Virginia.

Table 5.1: Frequency Table of the Characteristics of the Borderland Pedagogy
Predominantly Demonstrated by Esteban and Virginia.

Characteristic	Description	Data Sources	Participant	Frequency
Flexibility	Respects students, without caring about their race, religion, or socioeconomic status.	Observation	Esteban	5
		Lesson Plan	Esteban	14
		Observations	Esteban	11
		Interview	Esteban	7
Lack Flexibility	Design curriculum lacked uncreatively to incorporate the students' backgrounds and ways of knowing the world	Lesson Plan	Virginia	4
		Observations	Virginia	7
		Interview	Virginia	2
		Observations	Esteban	14
Passion	Ability to help all students at different levels.	Observations	Virginia	4
		Interview	Virginia	1
Culture	Teacher in the border region needs to have contact with people of different socioeconomic classes, people from distinct cultures and languages, and preferably to have personally experienced living in another country to better understand cultural values different from his or her own.	Observations	Esteban	2
	Teacher has to understand the dynamics of how the communities in the border area work and the needs they have.	Observations	Virginia	5
Language	Teacher needs to understand the acquisition of languages and the language used by these communities.	Observations	Virginia	8
		Interview	Virginia	3
	Teacher needs to have knowledge of Spanish and English.	Observations	Esteban	4
		Interview	Esteban	2
		Interview	Virginia	10
		Interview	Virginia	5

Esteban's and Virginia's Case Studies

Flexibility: Design curriculum creatively to incorporating the students' backgrounds and ways of knowing the world

In the survey completed by Esteban, he mentioned that his ten years of experience as a mathematics teacher had helped him to design and develop lesson plans in order to stimulate the students' interest and learning. This experience has helped him to know different methods and strategies that he can use to creatively develop lesson plans, as reflected in the following quotation:

"I develop the lesson plan in order for students to be able to reflect on the subject being taught. For example, I let them create their own problems. This is a method I like to implement a lot, I like to use this a lot for students to create their own situation to see what they are thinking mathematically" (Interview, October 15, 2013, Appendix, A).

Research conducted by Shulman (2008), Lit et al. (2009), and Cline and Necochea (2006) has asserted that teaching needs to be connected to the students. With this teaching approach, students will connect their understanding of differences that may arise from culture, family experiences, and approaches to learning, as well the community necessities. Cline and Necochea (2006) maintained "... an effective borderland teacher should design curriculum creatively the lesson plan to incorporate the students' backgrounds and ways of knowing the world" (p.272).

Through his experience, Esteban designed and developed the lesson plan using the creativity of the students. For example, through the creativity of the lesson plan, Esteban gave the freedom to the students to design their own problem in a group (e.g. the Casandra trip problem). The Casandra problem was created with the objective that students can understand the rates of change and area underneath the curve.

Scholars (Stigler & Hiebert, 2004; Li et al, 2009; Shulman, 1986) have emphasized that a lesson plan should be created according to the students' expectations, and expectations of the knowledge, attitude and skills required for the lesson. In other words, the expectations should be linked with the lesson objectives and/or learning outcomes. The Casandra math problem was designed by Esteban and the students in order to facilitate student understanding of the concept of function. This math problem provided the students an opportunity to understand when a function was increasing, decreasing, and when it equals zero. Correspondingly, the students were able to understand how to obtain the range and the domain of the function.

As a result of a group discussion of the Casandra problem, Esteban was able to know what the students were thinking mathematically and to determine their mathematical skill levels. Formative evaluation was facilitated through the design of the mathematics problem in the form of posed critical questions students reflected upon and answered. Buckley (2010) maintains that standards-based curriculum in the United States is designed in order for teachers to engage students. Utilizing a standards-based curriculum can facilitate the teacher's assessment of instruction or content as relevant to enabling students to participate in activities and answer questions. Similarly, scholars (Alwehaibi, 2012; Barnett & Francis, 2012; Choy & Oo, 2012), have stressed that lesson planning needs to be designed to develop student critical thinking skills. The objective of lesson planning should be to provide opportunities for students to develop a mental ability that allows them to interpret complex ideas, evaluating function arguments as evidence. Additionally, Lopez (2012) pointed out that the mission of the mathematics teacher was not so much to teach students a multitude of knowledge in highly specialized fields, but above all, to promote learning to learn, to ensure that the student acquires an intellectual

autonomy. This can be achieved by incorporating the development of higher order thinking skills such as critical thinking.

Through analysis of observations, it was found that Esteban's lesson plan development embedded the use of technology in the form of calculator use, TN-Spire, and a white board. Through the use of technology, the students showed motivation to learn the function concept, particularly when using the calculator. It was observed that the students learned the calculator function according to their advancement in the mathematics topic. Through technology, the students did not just learn the function definition, but they also learned the representation graphically, as well as the table representation of the equation. As noted by Esteban,

“The use of technology has led to substantial changes in the way students learn mathematics. The use of technology can be used to provide conditions for students to identify, review and communicate different mathematical ideas (Interview, October 15, 2013, Appendix, A).

Some Scholars (Seamon, 1999; Spooner et al 2007; Cline & Necochea, 2006) ascertained that an effective lesson plan design should be designed or developed without losing sight of the purpose and objective of the lesson plan. As Cline and Necochea (2006) mentioned that creatively developing the lesson plan means that the teacher did not try to follow a program step by step, but rather that the teacher created the lesson plan according to students' interests and needs. The interpretation of this analysis was that students have the opportunity to broaden their learning experience by using new technologies such as constructivist learning tools. These tools offer options through which the traditional classroom can become a new learning space. These new learning spaces provide innovative activities with collaborative and creative ways that allow students to consolidate what they learn while having fun. Within this new learning environment,

students themselves are able to build new knowledge with the teacher as a guide and mentor. As Diaz et al. (1999) suggested, some teaching strategies created by teachers can help students to develop learning by bringing activities into the classroom in which teachers involve the students' own knowledge. New technologies may bring opportunities for active participation of students in the experimental design and reflection, with access to the same tools that many professionals use today. In Mexico, teaching roles promote integration of activities that link the students' prior knowledge with learning objectives, as well as developing a proper school climate. For example, collaborative work among students through activities and work dynamics encourages active participation in class (Saiz, 2002, SEP, 2010; 2012).

In the interview analysis, some strategies and methods in Esteban's lesson plan development practices functioned while other strategies did not. In the interview, Esteban said,

"I was not afraid if some of the lessons did not work...it could happen, but it is not a barrier to continue and change the lesson" (Interview October 15, 2013, Appendix, A).

The strategies that worked were used according to the different ways that students learned. For example, if some of the students needed to know more about the function concept, Esteban designed the strategies according to those students' needs. To design and develop the lesson plan, Esteban conducted a pre-analysis of what the students already knew about the topic. According to this pre-analysis, he then developed the lesson plan. The pre-analysis consisted of having an open discussion with the group to ascertain what the students knew about the topic (function). Studies conducted by Stigler and Stevenson (2004), and Li et al (2009) found that developing open discussions with the students is essential to achieving effective lesson plan design and classroom instruction. Esteban's case study confirmed different methods and

strategies were used to creatively design the lesson plan as evidenced in the description of the Characteristics of Flexibility.

Lack of Flexibility: Design curriculum lacked uncreatively to incorporate the students' backgrounds and ways of knowing the world

Virginia's lesson plan development and implementation practices were very different from Esteban's. In the analysis of Virginia's case study, the characteristic of flexibility did not emerge. During the analysis, it was found that Virginia was influenced by traditional methods of teaching Geometry. Virginia designed and developed the segments and angles lesson plans employing the memorization of terms and procedures such as following instructions to solve a math problem. Research conducted by Mubark (2011) surmised that high school level mathematics teachers need to develop their lesson plans and practices in consideration of promoting students thinking and reasoning. Students reason in a logical manner whereby they begin to formulate and test conjectures, make sense of things, justify judgments, and make conclusions. Research conducted by Maat and Zakaria (2010) established that teachers need to discover and use concrete materials for teaching practices to improve mathematic achievement. Furthermore, scholars (Abdelfattah et al., 2012; Li & Li, 2009) suggested that, to incorporate teaching practices effectively, teachers need to know and understand the mathematics they are teaching in order to apply his or her understanding in a variety of formats. Shimizu (2008) mentioned that sometimes the mathematics teacher can be influenced by the autonomy of the textbook. Other researchers (Li & Li, 2009) have claimed that textbooks are one of the major curriculum resources not only to facilitate lesson planning, but also in implementing classroom instruction. This research found that the relationship between textbooks and teachers depends on how teachers interpret and react to students in the context of the classroom setting. The study

conducted in 2009 by Li and Li found that individual teachers used different criteria when choosing upon what to base their lesson plans. For instance, some teachers may rely heavily on textbooks while others do not, depending on their different beliefs, knowledge, and experiences.

An interesting approach found in the analysis of Virginia's case study was the inclusion in the lesson plan of student-created hands-on materials such as "foldables". The "foldables," created manually by the students, were used to explain translation, rotation, and reflection, where each of these concepts was represented by different colors. The objective of the "foldables" was to avoid students having to do procedures or operations. It was designed with the sole purpose of facilitating students' memory of each math concept definition through the different representation of the color.

This was found to be contradictory to the lesson plan that Virginia developed in another Geometry class. Virginia mentioned that:

"...through this procedure, students can get significant learning because if students learn only through listening to the mathematics teacher, only 20 % of the students can learn and understand the lesson, but the other 80% does not" (Interview October 29, 2013, Appendix, A).

As per the TEKS (2010), Geometry lesson plans will be taught through dynamic mathematics problems. These mathematics problems will be designed with the objective that students can apply their knowledge in other areas as well.

According to Sun, Kulm, and Capraro (2009) and Remillard (2005), curriculum or lesson planning was regarded as an experience where teachers encounter an educational setting, including national/state standards, textbook, lesson plans, and classroom instruction. Some researchers, such as Abdelfattah et al. (2012), have claimed that textbooks are one of the major

curriculum resources not only to facilitate lesson planning, but also in implementing classroom instruction. Mathematics researchers Stigler and Stevenson (2004) have affirmed that teachers need to be creative; they need to provide students with the skills necessary to face new situations successfully. Creativity is one of the tools in this task. For teachers to succeed in the task of helping students to develop their creativity, they must also be creative, have a number of personality traits, and be clear on the importance of their educational mission.

During the interview analysis, it was found that Virginia had a traditional understanding about the teaching of Geometry. She preferred for students to solve problems through procedures whereby students memorize the concepts about angles and the procedures to find the segment of two points.

Flexibility: Respect students without caring about their race, religions or socioeconomic status

During the three days that Esteban's teaching practices were observed, he never demonstrated disrespect toward his students. Esteban's support for the students was gentle and respectful. During observations it was noted

“Esteban never insults the students, nor shouts at them when they do not understand the concept of Piece-Wise function. On the contrary, whenever students needed help, Esteban approached them to assist them with their questions. Also, he never discriminated students who asked questions in Spanish” (Observations conducted Sep 11-Sep 13, 2013, Appendix, B).

Cline and Necochea (2006) affirmed, “an effective teacher should have respect for students, without caring about their race, religion, or socioeconomic status, since in the border region, the majority of students are Hispanic” (p. 272). Similarly, numerous scholars (Romo &

Chavez, 2006; Fiume, 2005), averred that teachers who teach in the border regions should respect the students regardless of their culture or language.

Passion: Ability to help all students at different levels

As was determined in the initial analysis of these case studies, Esteban and Virginia revealed knowledge and ability to help students by embedding technology to address some mathematics questions. In evidence it was identified that Esteban had more actions in accordance with the Characteristic of Passion. He demonstrated fourteen actions found in the classroom observation and Virginia demonstrated only four actions in classroom observation and two in the interview.

Within Esteban's case study, some students had questions about how to insert functions into the TI-Nspire, and in response these questions, Esteban explained in detail "Please go to the function TI and give a click insert function" (Observation conducted September 11, 2013, Appendix, B). During whole-class instruction, Esteban showed knowledge about the lesson plan that he developed on Piece-Wise function analytic Geometry. Through the development of this lesson plan some of the students asked questions about the function behavior. For example, they asked, "why does a function increase in specific interval $[2,3]$ " (Observation conducted September 11, 2013, Appendix, B). In order to address the student's question, Esteban answered "Please calculate the slope between this point and tell me the sign that slope will have" (Observation conducted September 11, 2013, Appendix, B). According to this explanation, Esteban considered it necessary that students understand the sign of the slope through knowing the formula and through the graph analysis. The objective was to help the students understand and learn why this interval of the slope was positive or negative. Esteban's approach was in accordance with the TEKS (2012) and SEP (2012) or students will use mathematical

relationships between the different representation of the problem to generate solutions and make connections and predictions. The students will analyze mathematical relationships to connect and communicate mathematical ideas and will display, explain, or justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

Similar to Esteban, Virginia also demonstrated having the ability to respond to some student mathematical questions. These questions emerged according to the topic taught that day (Observations conducted September 13, Appendix, E) while Virginia conducted a lesson on the angles-terms definitions. On that particular day, the students learned the obtuse angle definition, but some of the students did not understand the definition. In response, Virginia considered it necessary to explain the definition through the construction of the obtuse angle. She used didactic material such as a protractor, a straight edge, and a compass. As Fiume et al (2005) and Hooks (1994) point out, some techniques used by the teachers encourage student initiative and promote their understanding through their explanations, and these explanations facilitate the students' understanding. Also, through these explanations, when conducted by the borderland teacher, the students explore the answer and they can discuss it in a group. As Cline and Necochea (2006) state that an effective teacher in the borderland area should be passionate about the work that they develop in front of the group they will do whatever it takes to help transitional students succeed.

Culture: Teachers in the border region need to have contact with people of different socioeconomic classes, with people from distinct cultures and languages, and preferably to have an experience personally living in another country to better understand cultural values different from his or her own

Another characteristic of Borderland Pedagogy demonstrated by Esteban was “culture”. Esteban expressed that since most of the students who were registered in his Geometry class are Hispanic, he needed to know about the diversity of cultures. Through the interview analysis, Esteban affirmed “...having a master’s degree from UTEP had helped me to know about the diversity of cultures that exists in El Paso, Texas. (Interview October 15, 2013, Appendix, A). As supported by Cline and Necochea (2006), an effective teacher is someone who is cultural sensitive to the students. Being cultural sensitive is when the teacher has a background different from their students.

Culture: A teacher has to understand the dynamics of how the communities in the border area work and the needs students have

In the analysis of the data sources, it was evident that Virginia displayed Culture as described within the Characteristics of Borderland Pedagogy. Virginia knows the necessities of the community in the border area and was the only participant who had posters on Algebra and Geometry concepts in Spanish and English. Virginia used this tool as a visual resource to help the students learn mathematics concepts with visuals objects, based in the knowledge that 90% of her students were Hispanic and have difficulty with some math definitions. Virginia used appropriate tools to minimize language as a barrier to student learning. These posters not only contained mathematical terms in both languages, but included creative images that helped students learn in a fun way. For instance, “the sign of the parabola was represented by a sad or

happy face, which refers to when the parabola is positive or negative” (Observation September 10, 2013, Appendix, C). The importance of using tools, such as posters, to learning is supported by Christensen (1994), Peterson (1994), and Major and Brock (2003): educators must be prepared and able to create significant learning and elaborating environments that are culturally compatible with the students’ races, culture, language, and background.

Virginia expressed in the interview that she knows the necessities of the student needs since she herself experienced their same situation. She came to El Paso, Texas and enrolled in high school “... at the same age of her students” (Interview October 29, 2013, Appendix, A). As Cline and Necochea (2006) affirm,

“...an effective teacher for the border region needs to have contact with people of different socioeconomic classes, with people from distinct cultures and languages, and preferably to have an experience personally living in another country to better understand cultural values different from his or her own” (p. 278)

Virginia demonstrated an ability to empathize with her students based on her personal experiences.

Virginia considered it important to incorporate activities or visual objects that would help students with their learning. She considered it important to put into her practice the influences of their experiences in order to facilitate the understanding of the diversity of cultures, especially the Latino/a culture.

Language: Teacher does need to understand the acquisition of languages and the language used by these communities and Teachers do need to have knowledge of Spanish and English

In consideration of the characteristic of “language”, Esteban and Virginia demonstrated attributes that coincided with the characteristic’s description. Based on the classroom

observations, Esteban gave his students the freedom to speak both languages: Spanish and English. Cline and Necochea (2006) stated that "... a teacher in the border region should speak two languages to help the students with the classroom instructions" (p. 279). For example, if one of Esteban's students did not understand (e.g. what is the slope?), Esteban translated the question into Spanish for student to understand the mathematical term. One particular mathematical term that students had difficulty understanding was the word intersects; it is surmised that the difficulty in understanding is based in translation to Spanish that is, in Spanish it means intersection. To translate these mathematical terms was not difficult for Esteban because he expressed in his interview that his first language is Spanish. Therefore, he did not have difficulty understanding both languages. Also, it was observed that Esteban gave the instructions in English, but frequently he used some Spanish words (e.g., *que pasa* or *con la pendiente*). As Romo and Chavez (2006) mentioned "... most teachers are likely to ask more complex questions, provide more praise, use a wider variety of strategies, provide more opportunities to learn, and positively evaluate students whose culture and first language are most like their own" (p. 151).

Virginia mentioned that 90% of her current students lived in Ciudad Juarez; these students crossed the bridge every day to attend school. She also noted that most of her students are learning English as a second language and to address this need, she communicates with them in both languages, communicating first in English, so that students can familiarize themselves with that language. When some of the students did not understand the explanation or any concept, Virginia then translated to Spanish, with the objective that students learn and understand the concept she was explaining. Cline and Necochea (2006) stated, "... teachers who prepare themselves continuously will obtain those teaching methods necessary to understand or

resolve the different situations of each student in the border region” (p. 275). Virginia was able to address each learning situation based on student needs in order to facilitate learning.

In the interview analysis, it was found that Virginia considered it important to present visuals or posters to the students. It is surmised that through her thirteen years of experience as a mathematics teacher, Virginia had learned that the students should learn through the use of visuals objectives, like posters. Thus, she incorporated figures and pictures of mathematics concepts in both languages into the classroom. Researchers (e.g. Cline & Necochea, 2006; Romo & Chavez, 2006; Fiume, 2005) have determined that borderland teachers need to understand the needs of each student; this implies it is necessary to know the community needs as well as language skills. Virginia’s strategies were designed with the purpose that students who are not proficient in the English language can learn the math vocabulary. In the interview conducted with Virginia, she mentioned that “90% of the students who are taking Geometry class came from Juarez. They cross the bridge every day. These students are learning English as a second language” (Interview October 29, 2013, Appendix, A). From Virginia’s perspective, it was important to address language needs, as well as mathematical learning.

5.4 WHAT ARE THE LESSON PLAN DEVELOPMENT AND IMPLEMENTATION PRACTICES INCORPORATING THE BORDERLAND PEDAGOGY USED BY MEXICAN HIGH SCHOOL MATHEMATICS TEACHERS IN CIUDAD JUAREZ, CHIHUAHUA?

The second research question was answered through the analysis of the third and fourth case studies (Yolanda and Angelica). These two participants had worked as mathematics teachers at Ciudad Juarez teaching Algebra for several years. In Ciudad Juarez, Algebra is taught in the third semester of high school. Angelica was the participant who had little experience as a mathematics teacher; she had been a mathematics teacher for only 1 year at the

high school level, but had been an Electronic engineer for 14 years. Yolanda had 7 years of experience as a mathematics teacher at the high school level. During the semester that they were observed, both Yolanda and Angelica taught Algebra at the high school level in the same classroom, but at different times.

In the analysis of the lesson plan collected, classroom observations and one interview, it was found that Yolanda and Angelica weren't influenced by some characteristics of Borderland Pedagogy when they designed and implemented their lesson plans. The characteristics found in the case studies of Yolanda and Angelica were Lack of Flexibility, Lack of Passion, and Lack of Pluralistic Language Orientation. These two participants used different strategies and methods to teach algebra; however they were influenced in how they taught this subject following the examples and exercises of the algebra book. The descriptions of these characteristics are presented in Table 5.2. which depict the actions that most frequently emerged from these participants, based on the negative Characteristics of Borderland Pedagogy. These frequencies were identified through different data sources for each characteristic: lesson plan collection, observation, and one semi-structured interview per participant. The analyses of these characteristics are discussed in the same order that is presented in Table 5.2.

Table 5.2: Frequency Table of the Characteristics of the Borderland Pedagogy
Predominantly Demonstrated by Yolanda and Angelica

Characteristic	Description	Data Sources	Participant	Frequency
Lack of flexibility	Design curriculum lacked uncreatively to incorporate the students' backgrounds and ways of knowing the world	Lesson Plan	Yolanda	22
		Observations	Yolanda	15
		Interview	Yolanda	3
		Lesson Plan	Angelica	12
		Observations	Angelica	8
		Interview	Angelica	4
Lack of Passion	Lack of ability to help all students at different levels.	Observations	Yolanda	5
		Observations	Angelica	3
Lack of Pluralistic Language Orientation	Teacher doesn't need to understand the acquisition of languages and the language used by these communities.	Lesson Plan	Yolanda	0
		Observations	Yolanda	0
		Interview	Yolanda	1
		Lesson Plan	Angelica	0
		Observations	Angelica	0
		Interview	Angelica	1
	Teacher doesn't need to have knowledge of Spanish and English.	Lesson Plan	Yolanda	0
		Observations	Yolanda	0
		Interview	Yolanda	1
		Lesson Plan	Angelica	0
		Observations	Angelica	0
		Interview	Angelica	1

Yolanda's and Angelica's Case Studies

Flexibility: Design curriculum lacked creativity to incorporate the students' backgrounds and ways of knowing the world

In the analysis of the case study of Yolanda, she did not demonstrate flexibility when she designed and implemented her lesson plan. According to Cline and Necochea (2006) flexibility was defined as "... design curriculum creatively to incorporate the students' backgrounds and ways of knowing the world" (p. 272). In the interview analysis, it was found that Yolanda used three different methods to teach algebra class. The first method was based on the traditional way. This method consisted of solving and explaining some exercises on the green board in front the group. All the instructions were verbatim from the algebra book. Yolanda stated "The examples that I explained in front the group are taken from the algebra book...I used this method to teach the different case of factorization" (Interview October 30, 2013, Appendix, A). It is presumed Yolanda used this method since she was influenced by a mathematics teacher who taught her algebra using this method. According to Sun, Kulm, and Capraro (2009) and Remillard (2005), curriculum or lesson planning is regarded as an experience that teachers encounter in an educational setting. Sometimes mathematics teachers are influenced by the national/state standards, textbooks, or other lesson plans, that help them to develop their lesson plan. These strategies or teaching methods are followed from past teaching paradigms, or in other words, a teacher will teach an algebra or Geometry class utilizing the same methods used with them as students. Some mathematics teachers use these strategies because they do not have the time and enough resources to teach in a different way. As Yolanda mentioned, the algebra book is designed for the students to follow the instructions and do a long list of exercises. Scholars (Abdelfattah et al., 2012.; Li & Li, 2009) have pointed out that to incorporate teaching practices

effectively, teachers need to routinely reflect and collaborate on instructional practices, student progress, and know and understand the mathematics they will be teaching at a deep knowledge level to be able to explain and apply his/her understanding in a variety of formats. These researchers have claimed that textbooks are one of the major curriculum resources not only to facilitate lesson planning, but also in implementing classroom instruction. However, the use of these resources does not guarantee student learning. Research conducted by McCutcheon (1980) and Remillard (2005), affirmed that there was a strong relationship between textbooks and lesson planning when it came to the development and implementation of lesson plans developed by the mathematics teachers. When teachers design their lessons, teachers read and use variety textbooks, substantially or not. This research found that the relationship between textbooks and teachers depends on how teachers interpret and react to students in the context of the classroom setting. The studies also found that individual teachers use different criteria when choosing what to base their lesson plans on.

The second method, implemented by Yolanda, peer collaboration, provided that students could help each other when they had doubts. Remillard (1999) mentioned that sometimes the students can assist each other in helping to clarify their doubts, supporting Yolanda's approach. Research conducted by McCutcheon (1998) discovered that some practices, strategies, and methods that U.S. and Mexico high school teachers employ when they develop lesson plans do facilitate students having success in their learning. Some strategies, practices and methods were: working in group, discovering the meaning of some mathematics concepts.

The last observed method implemented by Yolanda consisted of having students tutored by a different mathematics teacher. Yolanda mentioned "... if some students have problems with the topic taught in my classroom, they need to come to tutoring on a Saturday...because this

tutoring is implemented by another mathematics teacher” (Interview October 30, 2013, Appendix, A). As Cline and Necochea (2006) mentioned “... an effective educator in the borderlands creatively look for ways to help students access the core curriculum and learn the skills necessary for success” (p. 274). Yolanda clearly identified another method of helping students through tutoring arrangements.

Yolanda expressed disinterest in the use of technology, based on her statement, “Algebra is a subject that will be taught through procedures and the solution of a long list of problems. She mentioned that I can’t use technology” (Interview October 30, 2013, Appendix A). For example, she mentioned that mathematics teachers cannot be creative to teach algebra, stating that “... algebra will be taught through of the practice and repetitions, because in algebra the teacher can’t be creative” (Interview October 30, 2013, Appendix A). However, she affirmed that in Geometry class, the mathematics teacher has the most opportunity to be creative. In the interview she stated that to teach Geometry, she uses a creative method to teach the “bisectriz, mediatris, baricentro of the triangle (bisectriz, mediatris and baricentro)” (Interview October 30, 2013, Appendix A). To facilitate a Geometry lesson, the students can use the outside school premises, such as the yard, to find the distance between three points. As several authors point out (e.g., Paine & Ma, 1993; Stigler & Hiebert; 1999; Stigler & Stevenson, 2004), if an effective mathematics teacher designs and develops well-thought-out and high quality lesson plans as a process of curriculum planning, the mathematics teacher should create a solid base of classroom instructions, such as how Yolanda’s lesson plan was developed.

In the analysis of Angelica’s case study, it was found that she used similar strategies compared to Yolanda. In the analysis of the interview and lesson plan development, Angelica used the algebra book to guide teaching the different cases of factorization. Cases of

factorization were taught through reading, writing, and student presentations in front of the class. She formed groups of four students and each group presented one case of factorization. As identified in the observation protocol analysis, Angelica tried to demonstrate that through the listening and writing examples presented by their peers the students could learn the mathematics topic. In the interview, Angelica stated, "... the students can learn if they present the mathematics problem using their own words" (Interview October 30, 2013, Appendix, A). Contradictory to Angelica's approach, Lewis (2002) and Johnson asserted, that sharing of one's teaching through of the discussion and observation of the lesson, the lessons will be worry some and ineffective to the students. For this reason, it is important that lesson plan will be developed in order that students can construct and discovery the mean of some mathematics concepts.

In the lesson plan analysis, Angelica did not design creatively the lesson plans according to Cline and Necochea (2006) description. This finding was confirmed during the interview when she mentioned that the "... lesson plan did not need to be developed in detail" (Interview October 30, 2013, Appendix, A). From the analysis of interview date, it was extrapolated that Angelica was not creative in lesson planning as a result of it being the first year that she had taught algebra. Angelica mentioned that she "... did not feel confident to create activities where the students can apply their knowledge in other subjects" (Interview October 30, 2013, Appendix, A). Angelica did not consider herself a person creative enough to develop and implement the lesson. Scholars, Johnson (2000) and Kulm (2009), reported that mathematics teachers need to create and develop lesson plans in order to develop an effective assignment. Effective assignment of mathematics learning must be performance-based, use multiple strategies, and employ more open-ended assignment tasks than have been used in the past. Rice

(1999) states that effective assignment is essential to support mathematics instruction that produces improved student performance.

Finally, in lesson plan development, Yolanda and Angelica did not have access to sources such as use of technology, manipulatives, or other learning objects to embed in their lesson plan design. They only had a computer lab, but this computer lab was used just for the students who are taking informative class. O'Donnell and Taylor (2006) pointed out if teachers can design a high quality lesson plan, they can build a solid base for classroom implementation using some tools like technology. Quality instruction is, therefore, more likely to occur. Furthermore, Li et al. (2009) stated that "... a better understanding of teachers' lesson planning would require further specifications of different system contexts and their nature" (p. 718).

Lack of Passion: Lack of passion understanding for each individual student

In the analysis of these two case studies, Yolanda and Angelica did not exhibit having the ability to explain some students' questions about cases of factorization. For example, Yolanda did not know how to explain that "the square of the binomial" couldn't have a negative sign. She stated "... we can't have a negative sign because we will have different combinations" (Observation 24, 2013, Appendix, F). As well, Angelica did not change the strategy to explain the topic notables' products, since she preferred to continue with the class. As Cline and Necochea (2006) and Lit et al (2009) mentioned that the teacher need to be prepared to help to students in different levels, Angelica and Yolanda did not demonstrate the level of knowledge to address student needs at different levels.

Lack of Pluralistic Language Orientation: Teacher doesn't needs to understand the acquisition of languages and the language used by these communities and teacher doesn't need to have knowledge of Spanish and English

Yolanda and Angelica expressed that they did not need to speak or have knowledge of the English language since the main language spoken in their classrooms was Spanish. They did not consider it necessary to speak with students or other faculty in English, since they were not in a private school. In Mexico private schools, the mathematics, science, and chemistry classes are taught in English; since Yolanda and Angelica were working in a public school, it is not in the curriculum or required for them to speak any other language but Spanish. This is in contradiction to the assertion of Cline and Necochea (2006) and Fiume (2005) who maintain that it is necessary for teachers who work in the border area to encourage students speak multiple languages.

5.5 HOW DOES A TRANSITION FROM MEXICO (CIUDAD JUAREZ, CHIHUAHUA) TO THE U.S. (EL PASO, TX.) IMPACT HIGH SCHOOL MATHEMATICS TEACHER'S LESSON PLAN DEVELOPMENT AND IMPLEMENTATION PRACTICES INCORPORATING THE BORDERLAND PEDAGOGY?

This research question was answered according to the three data sources for the fifth case study. The fifth case, Carlos, was identified as a transitioning teacher. Studies conducted by Duchesne and Stitou (2010) describe a transitioning teacher as a person who migrates to another country and is influenced by ethnicity, and culture in education. This transition can be influenced by their educational background and experiences as mathematics teacher.

The characteristics that emerged during the study and that answered this research question were: Flexibility, Passion, Culture, and Language. Within Table 5.3 are described each of the Characteristics of Borderland Pedagogy, paired with the actions that most frequently emerged in presentation of respective the Characteristics of Borderland Pedagogy. These frequencies emerged from the three data sources: lesson plan collection, observation, and

interviews. Analysis of these characteristics is discussed sequentially in the order that data is presented in this table 5.3.

Table 5.3: Frequency Table of the Characteristics of the Borderland Pedagogy

Predominantly Demonstrated by Carlos

Characteristic	Description	Data Sources	Frequency
Flexibility	Design curriculum creatively to incorporate the students backgrounds and ways of knowing the world.	Lesson Plan	23
		Observations	12
		Interview	6
Passion	Ability to help all students at different levels.	Lesson Plan	0
		Observations	9
		Interview	2
Culture	Teacher needs to know and learn about different cultures	Interview	3
	Teacher in the border region needs to have contact with people of different socioeconomic classes, with people from distinct cultures and languages, and preferably to have an experience personally living in another country to better understand cultural values different from his or her own	Interview	4
Language	Teacher does need to understand the acquisition of languages and the language used by these communities.	Interview	4
		Observation	2
	Teacher does need to have knowledge of Spanish and English.	Observations	5

Flexibility: Design curriculum creatively to incorporate the students' backgrounds and ways of knowing the world

Carlos, the transitioning teacher, mentioned that he worked in Ciudad Juarez as mathematics teacher in high school and as electronics engineer for seven years. Through the analysis of this case study, based on his previous experience, Carlos developed his practices and implementation of the lesson plan as applied to real life situations. Carlos worked in a factory manufacturing electronic products, and as such, his college background was in physics and mathematics. These experiences as a mathematics teacher in Ciudad Juarez and as an engineering worker experience helped him to develop lesson plans with the perspective of making connections with all the applications that are associated with mathematics in the real life. It was not difficult for him and he mentioned that the curriculum (TEKS) is focused and linked to applications of the mathematics concepts. As Elbaz-Luwisch (2007) affirmed:

It is through their storing of their experiences of work, relationships, history and power that the immigrant teachers manage to create meaningful accounts of themselves as teachers in their new settings. Their stories are meaningful because they are stories of making a difference. For these teachers, making a difference is important: they care about helping individual pupils to make something of their lives, and to this end they invest their energies in initiating projects, confronting authorities and trying to bring about changes in their schools. (p. 410).

On the second day of observation, Carlos developed the composition function lesson plan with the purpose of having students understand the meaning of this mathematics concept through the use algebraic, graphics and tabular techniques. Before teaching the inverse and composition lessons, Carlos considered it important to begin with an introduction about the

function meaning. In mathematics, it is important to understand this term in order to understand the inverse and composition meaning. He explained the function definition through three representations (“Table”, “Mapping Diagram,” and “graphic”) (Observation October 24, 2013, Appendix D). Supporting this approach, Choy and Oo (2012) and Korkmaz (2012) have contended that if lesson plans can be designed with the objective that the students develop critical thinking, the students develop a mental ability that allows them to interpret complex ideas, and evaluate function arguments in his evidence. Critical thinking can help students to distinguish between something that is reasonable and something that is not. Similarly, studies conducted by Li et al. (2009), affirmed that “... if teachers can design a well thought-out and high-quality lesson plans, as a process of curriculum planning at the micro-level, they build a solid base for classroom implementation. Quality instruction is, therefore, more likely to occur” (p.717).

In analyzing the findings obtained in the interview conducted with Carlos, his experience as a mathematics teacher in Ciudad Juarez somewhat influenced his teaching of Pre-calculus in El Paso, Texas. Evidence of this influence is presented in one of the student assignments requiring them to solve a long list of composition function. As Carlos mentioned, “... in Mexico, the mathematics, teachers implement and design the lesson plan through procedures and memorization. Although the curriculum designed by the Secretaria de Educación Publica (SEP) state the opposed” (interview October 24, 2013, Appendix A). SEP (2012) policy documents stated that the signature of mathematics should be implemented with the purpose that

...es que el alumno desarrolle el razonamiento matemático, haga uso del lenguaje algebraico a partir de la resolución de problemas de la vida cotidiana, dentro y fuera

del contexto matemático, representados en modelos donde se aplican conocimientos y conceptos algebraicos. (p. 1)

Translated this means "... that students develop mathematical reasoning, make use of algebraic language from the resolution of problems of everyday life, inside and outside the mathematical context, represented in models where algebraic skills and concepts are applied" (SEP, p. 1).

In Carlos' experiences as a mathematics teacher in Ciudad Juarez, he mentioned that the lesson plans were designed in a traditional way, where the teacher talks in front of the group, that is, the lesson plans in Ciudad Juarez are designed through procedures where the students learn listening to the teacher. Based on this experience, Carlos considered it necessary to change his strategy or methods to design mathematics lesson plans. Authors (Romo & Chavez, 2006; Cline & Necochea, 2006; Fiume et al., 2005) noticed that many immigrant teachers come to another culture in which education includes different teaching styles like memorization. These authors mentioned that memorization was the principal learning strategy and knowledge assessment was achieved through standardized testing. The scholars felt that mathematics teachers can make a difference by changing both their practices and their educational beliefs to meet the expectations of the educational system. As Cline and Necochea (2006) stated "The educational systems in the United States and Mexico were so distinct and disconnected from one another that they may as well be in two separate worlds" (p. 269). For example, Carlos was assimilating into the U.S approach bringing his experience as an engineer.

Analyzing data sources obtained from the classroom observation, it was found that Carlos designed a lesson plan creatively according to the students' interests. For example, he designed a lesson plan with applications of conic sections in sound, in aerospace, in the emission of sound

signals, and electromagnetic signals. In the interview Carlos mentioned that he designed a lesson plan applying conic sections in relation to American football, since students were familiar with and watched the sport. Carlos explained that,

“Football reporters have a plate and point to where the players are. Those plates are parabolic dishes and point to where players are. Then what happens is that when the player is screaming, not 100 meters from the parabolic dish, the point and the sound travel in waves bounce off the parabola and is reflected in what is called the focus of the parabola. The focus is located on a microphone that amplifies the sound and that's how all the bad words on TV can be heard” (Interview October 22, 2013, Appendix A).

Cline and Necochea (2006) affirmed that a borderland teacher who is open minded, can develop or “...design curriculum creatively with the purpose to incorporate the students’ interest” (p. 273), an open-mindedness displayed by Carlos in the example given above.

As evidenced in the interview analysis, Carlos mentioned that since the language of mathematics is universal, he did not have difficulty if the students didn’t understand some mathematics symbols. For example, when he came to work as a mathematics teacher in El Paso, Texas, he obtained certification training as a mathematics teacher in Region 19 and through the process, he learned about the educational system in Texas. Carlos mentioned that there is a difference between mathematics teachers who live in the border region to a mathematics teacher who lives in another place that is not a Texas border area.

Also, through this interview analysis, it was found that his experience as a transitioning mathematics teacher helped Carlos identify the difficulty that students who live in El Paso, Texas have as opposed to the students who came to the United States to study. As a Cline and Necochea (2006) affirmed:

“An effective borderland teacher has to understand the dynamics of how the communities in the border area work and the needs they have. The teacher needs to understand the acquisition of languages and the language used by these communities. This understanding will allow him or her to give the students what they need to be successful”. (p. 277).

Summarily, Carlos was identified as the transitioning participant who worked as a mathematics teacher in Ciudad Juarez. His previous experience as a mathematics teacher in Ciudad Juarez and engineering worker helped him to design mathematics problems based on real life and engineering.

From three data sources, Carlos demonstrated that he designed creatively lesson plans in order to incorporate different backgrounds of students. He designed the lesson plan to encourage students to apply their acquired knowledge in real-life applications. Carlos designed the composition and inverse lesson plans based on his experience as an Electronic Engineer and mathematics teacher in Ciudad Juarez, Chihuahua. This evidence supports identification of the characteristic of Borderland Pedagogy, as influenced by transition from a differing career and from Mexico to the U.S.

Passion: Ability to help all students at different levels

Carlos was one of the participants who was shown to have the ability and knowledge to respond appropriately to students with mathematics questions. For example, Carlos provided reflection questions helping students to understand the procedures necessary to solve inverse/composition problems, in addition to acquiring the tools and mathematical knowledge through communication and interpretation of procedures to solve problems. He was able to communicate with the students in English and Spanish. Cline and Necochea (2006) stated

...passion is the energy that sustains many borderland teachers, ongoing professional development is a critical component for effective instruction. Teachers need professional growth activities that will provide them with the instructional practices necessary for the diverse students in their classrooms. (p. 275)

Through these observations, Carlos demonstrated knowledge to teach function definition through table, graph and mapping diagram. Carlos showed the ability to facilitate student learning of composition function, as well as respond to student questions about graphing in the math program. (The students who were taking the class on the computer were using a special math program.)

Culture: Teachers in the border region need to have contact with people of different socioeconomic classes, with people from distinct cultures and languages, and preferably to have an experience personally living in another country to better understand cultural values different from his or her own

Similar to Esteban, Carlos mentioned that the border area is formed by a diversity of cultures, especially by the Latino/a culture. He mentioned “I have contact with Hispanic students, and I identified with them, since I came from Juarez too” (Interview October 22, 2013, Appendix A). Cline and Necochea (2006) stated “ Borderland teachers are unique” (p. 277), since they learn about the different cultures and customs. The authors mentioned that teachers learned of the diversity of cultures to credit each student with what he or she brings to the classroom. Researchers, Cline and Necochea (2006), Fiume et al (2005), and Romo and Chavez (2006), pointed out that thinking in the diversity of cultures gives to teachers a new perspective from which to value diversity and able to work with it. This allows teachers to exploit the

enormous potential that knowledge of diversity gives to social beings in general, and as agents of the educational system in particular.

Language: Teacher does needs to understand the acquisition of languages and the language used by these communities and teachers do need to have knowledge of Spanish and English

In the analysis of the interview data, in transition to work as a mathematics teacher in El Paso, Texas, Carlos adapted to the culture and the language. Prior experience and factors weren't barriers to design and teaching of mathematics in the high school level. Not only wasn't it a barrier, it was an asset since the TEKS are designed with the purpose of applying mathematics concepts to real life. Numerous researchers (Necochea & Cline, 2006; Fiume, 2005; Romo & Chavez, 2006; Elbaz-Luwisch, 2007) determined that the teacher was a main factor in the educational system at any level of school. These researchers mentioned that teachers who migrate from one country to another are faced with some challenges such as that culture and language that must be faced when teaching in a new school system, but as Elbaz-Luwisch (2007) stated "... teachers who have made a transition from one cultural setting to another are likely to have developed an awareness of teaching and schooling in the new culture that other teachers may not have" (p. 387).

It was found that culture was not a barrier for Carlos, since he lived in the border area and he knew a little about the diversity of cultures. Supporting this finding, Elbaz-Luwisch (2007) purport that place concept is a main factor to understand the postmodern time that we are living.

Carlos mentioned the importance of speaking both languages (English and Spanish) with his students due to the fact that he is working as a mathematics teacher in the border area and some of his students are not proficient with the English Language. Carlos said "... it is important talk both languages since we are in the border area.Sometimes the students ask me some

questions in Spanish and I need to answer them in Spanish so they can understand the lesson” (Interview, October 22, 2013, Appendix A). Carlos believed that language wasn’t a barrier since his first language is Spanish and English is his second language. (Survey October 22, 2013, Appendix C). Carlos stated he can communicate in both languages with the students, meeting the characteristic of effectiveness defined by Cline and Necochea (2006): “The characteristic of an effective teacher for the border region is to have knowledge of Spanish and English, because to be bilingual means you can communicate in two languages” (p. 277).

Through his experience as a transitioning mathematics teacher, Carlos can identify the difficulties experienced by students who were born and raised in the United States as compared to the difficulties experienced by students who migrated to the United States to study. Cline and Necochea (2006) and Fiume (2005) contend a borderland teacher needs to understand the different dynamics and requirements of how the community in the border area works. Carlos mentioned that students who all their life had studied in El Paso, Texas, have more difficulty in solving the procedures of some problems compared to other students.

In conclusion, Carlos was identified as the transitioning teacher participant since he worked as a mathematics teacher in Ciudad Juarez and continued working as a mathematics teacher in El Paso, TX. In the analysis of all data sources of this case study, major points of analysis centered on Carlos’ consideration that it was important to make changes in strategies and methods to successfully teach mathematics at the high school level in a border region. His experience as an engineer helped him to apply and design mathematics problems to the real life, helping students make connections and facilitate learning.

5.6 SUMMARY

In this chapter the main findings of each case study were discussed. This chapter provided an explanation of how each core category emerged and the categories and subcategories related to core categories. The findings of each case study, with each data collection source of this mixed methods study were presented with supporting evidence.

5.7 IMPLICATIONS

Following is a discussion of the implications of the findings for theory, research, and practice from data analysis. Based on the analysis of findings from this study, it is recommended that Secretaria de Educación Pública (SEP) and Local Education Agencies (LEAs) examine different Borderland Pedagogy characteristics in order to better support teachers in their practices in developing mathematics lesson plans. This Borderland Pedagogy comparative case study and lesson plan design review arrived at different outcomes; however, all are connected to previous theory and studies conducted by other researchers (Cline & Necochea, 2006; Romo & Chavez, 2006; Shulman, 2008; Elbaz-Luwisch, 2007; Li et al., 2009). The findings help researchers, SEP and educational agencies to have a better understanding of how practices and development of the lesson plans are experienced from the perspective of the mathematics teacher. Findings were supported by a comparative analysis of the different methods and strategies that the participants used to design and develop their lesson plan using multiple data sources: collection of the lesson plans, interviews and classroom observations.

Finally, the findings revealed that in the setting where the Mexican teacher participants were observed, they didn't develop and implement the characteristics of Borderland Pedagogy in their lesson plan. The two specific characteristics of Borderland Pedagogy not incorporated by Mexican participants were flexibility and passion. Incorporation of the characteristic of

flexibility in the public school setting was inhibited by the lack of resources to implement and develop lesson plans using different tools and strategies, like technology, or hands on activities. As well, they didn't have content professional development or special pedagogical training to improve their lesson plan practices. Additionally, the mathematics teachers showed lack of academic language registers and content knowledge at the level of reasoning as exemplified by the explanation of the case of factorization.

Another specific characteristic not incorporated by Mexican mathematics teacher participants was passion. Results of the study showed that Mexican participants lack content knowledge during the explanation of the lesson plan. Moreover, Mexican and transitioning teachers expressed their concern that the lesson plans developed by the SEP were designed traditionally for classroom practice, yet according to the SEP (2010; 2012), lesson planning is designed with the purpose that students apply such concepts discovering the mathematics concepts to real life indicating a contradiction between practice and policy based in these two statements. It is a commonly held educational belief among scholars that lesson plans should be designed with applications where the students can develop critical thinking to be prepared for the future, yet this is not evident in the practices of Mexican teacher-participants.

This research brings several contributions to the field of mathematics education research specific to Mexico and internationally. In addition, this study has several implications for policy and/or practice and for further research.

5.7.1 Implications for Practice and Policy

Supported by the data collected in this study, the U.S. and transitioning mathematics teachers had recommended ways to improve practices and development of the Geometry and Algebra lesson plans created by SEP. The recommendations were the following: design the

lesson plan through problems applied to the real life, use hands-on/ manipulatives to elaborate these activities, use technology to motivate students, as well as connecting new concepts with previous knowledge. Although these topics have familiar resonance from research, what is different is the findings in this study came from the teachers' voices. Teachers believe that implementing these recommendations could enhance student learning to perform in a better environment that encouraged mathematical reasoning and critical thinking.

Additionally, the study found that although the transitioning teacher worked as mathematics teachers in Ciudad Juarez, he considered it important to incorporate the Characteristics of Borderland Pedagogy in his lesson plan development and implementation practices as opposed to the different perspective of the Mexican teacher participants. For instance, Carlos mentioned that as a transitioning teacher, it was important to take special training in order to learn about the diversity of cultures that is present in El Paso, TX. Special training provided Carlos an opportunity to expand his content knowledge and pedagogical resources that he could implement in his lesson plan practices. These resources helped him to develop and implement lesson plans that encouraged student discovery and construction of the mathematics concepts (e.g., composition and inverse function definition). From his transitional experiences, Carlos learned about the educational environment in a border and considered it important to speak both languages English and Spanish in the classroom. Moreover, specialized training helped him to be better prepared to perform his job as mathematics teacher in the U.S.. Conversely, Yolanda and Angelica didn't consider it important to incorporate the characteristics of Borderland Pedagogy since they mentioned that Algebra lesson plan should be taught through the practice without use of didactic material by following examples standardized by SEP. The Mexican participants had not had the same professional development opportunities that were

available to Carlos as a transitioning teacher. It is recommended and reiterated, based on the findings of this study, that SEP take into consideration revisions to policy that would encourage the inclusion of Borderland Pedagogical characteristics in lesson plan development practices.

5.8 RECOMMENDATIONS FOR POLICY AND/OR PRACTICE

There are some recommendations for policy/practice that arose from this study. First, mathematics teachers should receive special training to be aware of designing their lesson plans to incorporate students' needs as pertinent to borderland factors. Second, implementation and development of professional development focusing on the importance of the design of the lesson plans to teach mathematics in classrooms in the border region are needed in Mexico and the U.S. This structure/approach could be achieved through the implementation of professional development that includes didactic activities that allow mathematics teachers to identify their abilities to design lesson plans incorporating Borderland Pedagogy characteristics. Furthermore, mathematics teachers should be aware of the students' diverse linguistic and cultural needs that can arise in their classroom on a daily basis. Finally, the mathematics teachers should receive a special training to learn and know about the diversity of cultures as well, the Hispanic students need.

5.9 LIMITATIONS

This study provided a snapshot of the comparative analysis of lesson plan development practices utilized by Mexican, Transfronterizo, and U.S. mathematics teachers. The results of this study do not indicate what factors cause the mathematics teachers to teach their lesson plan in a traditional method. Each of the case studies presented unique strategies and methods to design and implement lesson plans. The results of this study do not indicate other contributing factors that explain why the two Mexican teachers use different strategies and methods to implement

lesson plans, although they are teaching the same subject, using the same lesson plan format in the same public school environment. If this study were to be replicated in other parts of Mexico, results might be different. There were several extraneous limitations identified in the study such as language, the instrument, and sample, which are further described.

Finally, there were some limitations of the theoretical framework in terms of characteristics of Borderland Pedagogy. For example, one of the characteristics of Borderland Pedagogy that was not operationally defined was flexibility. This characteristic was not operationalized for measuring extraneous factors, such as access to resources. In the setting where I observed the Mexican participants, this high school didn't have resources (e.g., technology, TI-Nspire calculators) available that the mathematics teachers could use to develop and implement approaches in their algebra class in a creative way. It is not clear if the Mexican teachers would have demonstrated a presence of the characteristics of flexibility if they had had access to resources. Based on the defined characteristics, Mexican participants demonstrated a lack of flexibility.

Another characteristic of Borderland Pedagogy that was not clearly operationalized was passion. Generally, this term is defined as "any powerful or compelling emotion or feeling, as love or hate". However, Cline and Necochea (2006) in characterizing the term passion posed the interpretation that the characteristic of passion exists when the "teacher must have the ability to help all students at different levels" (p. 275). For example, both Mexican mathematics teachers didn't show the ability or the content knowledge to interpret and explain the case of factorization using different strategies or methods. Whereas, the U.S. and transitioning participants did demonstrate the ability and content knowledge according to the prescribed definition in the theoretical framework. Moreover, the terms passion and ability are mutually independent: one

might have a passion but do not possess ability and vice versa. For future studies, it will be important to operationalize characteristics that reflect generally held perceptions of the definition and still allow for identification and measurement of Borderland Pedagogy characteristics.

The theoretical framework was applicable and operationalized from the perspective of one side of the border (U.S.), since the transitioning teacher incorporated more frequently these characteristics in his lesson plan development and implementation practices, following his transition to the U.S. side. This is exemplified by the observation that Carlos considered it important to develop his lesson plans incorporating reading (word) problems, as well, he considered important to provide the instructions or the dialogue with the students in both languages recognizing that most of his students were English Language Learners (ELLs). Thus, the theoretical framework is better suited to study transitioning teachers on the U.S. side of the border. The framework would require adaptations to allow for deeper comparative analysis of Mexican mathematics teachers.

5.9.1 Language

Language was the principal limitation in this study. The native language of all the participants was Spanish; all participants wanted to respond to interview inquiries in Spanish, since they felt more confident speaking their native language. Thus, the first language issue found was one of translation, that is, the study was written in English. To analyze the interviews, each transcript was translated into English making the skills of the translator an important tool and variable in the study.

One of the advantages of the translator was that the translator and the researcher were the same person, so there was the contextual advantage of having access to the ideas and experiences of the participants. Furthermore, this positioning as translator and researcher facilitated the ease

of interpretation of the findings. However, a limitation to the study was the possibility of losing some information or meaning through the translation process. Another limitation related to language, was the lesson plan collection process and translation. The lesson plans of the two Mexican participants were written in Spanish; therefore, these instruments were translated to Spanish and a group of persons who were dominant in the English language verified the translations. The limitation being that there is the possibility that important information of the translation might have been lost, which in turn might have affected the analysis and interpretation of the mathematics teachers' lesson plans. However, verification of translations minimized this potential effect.

5.9.2 Sample

Another limitation was the selection of the sample. The original intent was to select two Mexican mathematics teachers who taught a Geometry class in El Paso and taught previously in Mexico at the high school level. Case study selection was complicated due to few participants with these characteristics teaching in the El Paso Independent School District. A single participant with these characteristics was identified. Another sampling limitation was the ability to find a high school administration in Mexico who was willing to work with the researchers in attaining data. The principal of the first high school gave authorization to conduct the study in one of the high schools in Juarez, but after four months the permission was rescinded due to complications associated with the approval process. After searching, administration of another high school in Mexico with the same characteristics did agree to allow the research to be conducted with two of its teachers. However, the process of selection could be considered a limitation.

5.10 CONCLUSIONS

The research topic emerged from a discussion during a meeting with Dr. Tchoshanov, Dissertation Chair. This topic emerged from the identified necessity to know and learn more about the practices and implementation of the lesson plans of Mexican and United States high school mathematics teachers. It was important to ascertain what were the practices and implementation of mathematics teachers to comparatively identify the practices of the transfronterizo mathematics teacher. Based on this identified research gap, two Mexican participants and two U.S. participants were selected. Lesson plan development and practices have exerted a great impact in the educational mathematics field, but had not been fully researched from this perspective.

Investigations conducted by mathematics educators indicate that lesson plan development is a topic challenged by educators (Li, et al, 2009), since some of them design their lesson plan according to what they learned or they dare to change their methods or strategies to teach mathematics. Also, research by Beyer, Davis, Forbes, and Stevens (2012) pointed out that "... lesson plans play a central role in guiding teachers' practice" (p. 798). This explanation was discussed by the Mexican teachers. Thus, further research was needed to address why Mexican teachers, possessing an engineering background, designed lesson plans in a traditional way in comparison to transitioning teacher.

This study was conducted in Ciudad Juarez, Chihuahua, located on the border with the United States and El Paso, Texas. Significant strategies and methods necessary to design lesson plans in order to facilitate students application of the mathematics' concepts were identified. Not all observed methods taught by the mathematics teachers proved to be successful into the classroom.

Considering the relevance of studying and analyzing the lesson plan development and practices, this study was an attempt to measure high school mathematical teachers' lesson plan development and relationship to teaching strategies and methods that were used to teach in Geometry and algebra classes in Mexico and the United States. The broader educational purpose of this study was to determine how a transition from Mexico (Ciudad Juarez, Chihuahua) to the United States (El Paso, TX) impacted high school teachers' lesson plan development practices to teach Geometry at the high school level. In order to achieve this purpose, the following research questions were addressed in this study:

1. What are the lesson plan development and implementation practices incorporating Borderland Pedagogy used by U.S. high school mathematics teachers in El Paso, Texas?
2. What are the lesson plan development and implementation practices incorporating Borderland Pedagogy used by Mexican high school mathematics teachers in Ciudad Juarez, Chihuahua?
3. How does a transition from Mexico (Ciudad Juarez, Chihuahua) to the United States (El Paso, TX) impact high school mathematics teacher's incorporation of Borderland Pedagogy into their lesson plan development and implementation practices?

This study analyzed five case studies in order to define lesson plan development and practices of mathematics teachers. Analysis provided insight into how the transition from Juarez City, Mexico to the United States (El Paso, TX) impacts high school teachers' lesson plan development practices and implementation to teach mathematics at a high school level. Cline's and Necoche's model (2006) provided the theoretical foundations for Borderland Pedagogy. Within this theoretical framework, it was necessary to investigate and analyze each of the Characteristics of the Borderland Pedagogy, in order to include lesson plan design as another

theoretical framework. The study is immersed in the characteristics of the Borderland Pedagogy established by Cline and Necochea (2006). Analyzing previous research with regard to Borderland Pedagogy, each description of each characteristic was identified and described with the purpose to link it to lesson plan design.

For the analysis of the “Characteristics of Borderland Pedagogy”, Cline and Necochea (2006) proposed a framework in which different characteristics were identified as attributes that should be demonstrated by a borderland teacher. Therefore, the nature of the characteristics depended on the ways in which a teacher designed lesson plans, and the language that they used in the classroom, as well as if they were culturally sensitive with the students. Finally, identifying if the teachers showed passion for Borderland Pedagogy was an important characteristic to evaluate. According to these characteristics of Borderland Pedagogy, lesson plan design was selected as an aligned theoretical framework. The additional framework was supported by identification of research or studies that mentioned the importance of the lesson plan design, as well as what methods or practices that were the most appropriate to implement lessons in the classroom.

Based on the theoretical frameworks and the origins of the research questions, a qualitative method emerged in order to provide credibility to conduct the study. The qualitative study was composed of three phases: classroom observation, semi- structured interview, and lesson plan collection. As this study was developed, a small component of a quantitative method was integrated. This quantitative method was developed in one format: frequency tables. The frequency table was developed for each characteristic of Borderland Pedagogy with the intent of identifying how many times a participant demonstrated characteristics of flexibility, passion, cultural sensitivity and language.

In the qualitative and quantitative phases, the three research questions were addressed sequentially to adequately respond to each one. The analysis reported an overall finding that each participant was unique and each of them designed and used different strategies and methods to implement lesson plans to teach Geometry and Algebra. The following table represent the key considerations of the findings.

Table 5.4 Key Considerations

	Ciudad Juarez, Chihuahua (Mexico)	El Paso, TX (USA)
Standards	National Standards (SEP)	State Standards (TEA)
Curriculum	Heavily Standardized by SEP <ul style="list-style-type: none"> • Prescribed Curriculum • Prescribed Textbook 	Recommended by TEA <ul style="list-style-type: none"> • Curriculum developed by LEAs • Textbooks adopted by LEAs
Lesson Planning	Heavily Standardized by SEP <ul style="list-style-type: none"> • Prescribed Lesson Plan format • Prescribed Scope and Sequence, Assessment 	Recommended by LEAs <ul style="list-style-type: none"> • Campus recommended Lesson Plan format • LEA recommended Scope and Sequence, Assessment
Language	Monolingual Instruction <ul style="list-style-type: none"> • Spanish 	Monolingual/Bilingual Instruction <ul style="list-style-type: none"> • English Language Learners (ELLs)
Culture	Predominantly Monocultural <ul style="list-style-type: none"> • Mexican Culture 	Predominantly Multicultural <ul style="list-style-type: none"> • Diversity of Cultures

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Appendix A Interview Questions

1. When you develop a lesson plan, how do you select teaching methods that are necessary to address culturally and linguistically diverse students?
2. When you develop a lesson plan, do you design activities to address diverse needs of students? Please provide an example of these activities?
3. How creatively do you develop a lesson plan in order to incorporate the students' diverse backgrounds?
4. What are strategies do you use to understand the cultural diversity, language and community into the classroom?
5. Do you consider necessary that the teachers who teach at the border region take a special training on culturally and linguistically responsive lesson plan development? Why or Why not?
6. When you design and implement a lesson plan into the classroom, do you encourage students to speak multiple languages (e.g., English / Spanish)? If yes, how?
7. How much detail do you feel is necessary in writing your own lesson plans to teach High School Mathematics? Would this change if a supervisor/ coach wanted copies of your lesson plans?
8. What aspects of lesson planning are the most daunting? How will you go about getting assistance in planning your lessons?
9. Would you prefer lesson plans to be handed to you and you closely follow it. Why or why not?
10. Would you prefer to write your own lesson plans based on available resources? Why or Why not?

Thank you for your time and participation!

Entrevista

- 1 . Cuando usted desarrolla su lesson plan , ¿cómo usted selecciona los métodos necesarios para hacer frente a diversas culturas e idiomas para la enseñanza de los estudiantes ?
- 2 . Cuando usted desarrolla su lesson plan, las actividades que diseña, las diseña para hacer frente a las diversas necesidades de los estudiantes? Proporcione un ejemplo de estas actividades?
- 3 . ¿Cómo se puede desarrollar el lesson plan, por ejemplo que tan creativamente se puede desarrollar un lesson plan de acuerdo para poder incorporar los diferentes backgrounds de los estudiantes ?
- 4 . ¿Cuáles son las estrategias que utiliza usted para comprender la diversidad cultural , el idioma y a la comunidad dentro del aula ?
- 5 . ¿Considera usted necesario que los profesores que enseñan en la frontera tomen una formación especial sobre la cultura y el desarrollo linguistico? ¿Por qué o por qué no?
- 6 . Al diseñar e implementar su lesson plan en el aula, se puede animar a los estudiantes a hablar varios idiomas (por ejemplo, Inglés / Español) ? En caso afirmativo, ¿cómo?
- 7 . ¿Qué tan detallado considera usted desarrollar sus propios lesson plan para enseñar matemáticas a nivel preparatoria ? ¿Esto cambiará si un supervisor / entrenador quería copias de sus planes de estudio ?
- 8 . ¿Qué aspectos de la planificación de las clases son los más difíciles ? ¿Cómo va a ir sobre cómo obtener asistencia en la planificación de sus clases ?
- 9 . ¿Prefiere planes de lecciones para ser entregado a usted y usted lo sigue de cerca . ¿Por qué o por qué no?
9. Would you prefer lesson plans to be handed to you and you closely follow it. Why or why not?
- 10 . ¿Prefiere escribir sus propios planes de estudio basados en los recursos disponibles? ¿Por qué o por qué no?

Thank you for your time and participation!

Appendix B Esteban's Case Study: Fieldnotes Expanded Observations

September-11-2013

Observing Esteban

On September-11, I visited for second time the Bel Air High School to observe to Mr. Salcido. First of all, I went with the secretary to get my visitor badge to have the permission to observe to Mr. Salcido. After that, I headed to Mr. Salcido classroom. When I arrived to the classroom, I found to Mr. Salcido into the classroom and we talked a little about the progress of my study. After this conversation, I sat down at the end of one of the lines to take my observations. The class began to 2:25, and the first instruction that Mr. Salcido gave to students was “please take the calculation and make log in”. So, the students took the calculator and they made log in. When the teacher give the instructions, the teacher mention “ No esperen aprender a utilizar la calculadora en un día”; “ it is a new tool”. **OC: The characteristic of an effective teacher for the border region is to have knowledge of Spanish and English, because to be bilingual means you can communicate in two languages Cline and Necochea, 2005, p. 279. This is a clear example, where the math teacher makes use of you native and second language to communicate with the students).**

Into the instructions Mr. Salcido explained how to use the calculator, especially how insert function into the calculator. Also, Mr. Salcido explained what screen the students will have in their calculator. After that, Mr. Salcido, asked to the students what is the domain of the function. Also, he asked to find the domain, range and find the equations of the function.

After the students do log in, Mr. Salcido explained the instructions and he said a story to explain the function behavior. He asked at each student with the purpose that all students can participate and can understand the function behavior. Through a conversation between teacher

and students, they can solve the problem. It shows that Mr. Salcido design creatively his activity with the purpose that students participate and can construct by themselves the function behavior. **(OC:... designs curriculum around themes (big ideas) that prompt exploration of problems, such as what is history? Sleeter, C. 2005, p. 46). OC: For these teachers, language issues allow them to approach the curriculum creatively to ensure that all students have an equal opportunity to succeed within the school system, Cline and Necochea, 2005, p. 279).**

Through the class, I observed that Mr. Salcido showed passion and tried that each student learned the lesson/ or activity. For example, teacher (Mr. Salcido) walked around the classroom to check and confirm if the students are doing well the assignment or if they need help. Also, I noticed that Mr. Salcido, talked in English and Spanish with the students. For example, if the student asks in Spanish, Mr. Salcido responded his/her in Spanish. However, the students talk at English when they are talking between them. **(OC: Effective teachers in the borderlands are passionate about the work they do, and do whatever it takes to help transnational students succeed. They have a positive attitude about working with border communities and seek instructional practices that are effective for students, Cline and Necochea, 2005, p. 273).**

Teacher motivates at the students to participate on that lesson. Also, Mr. Salcido, demonstrate enough knowledge to help to the students in different levels; for example, in special with technology or math questions. As well, Mr. Salcido made questions at all the group about the graph and if one student toward a comment, Mr. Salcido questioned the students' responds asking "why" of his/her responds.

When the students finished inserting the graph into the calculator, Mr. Salcido explained how to save the function into the calculator and the teacher mentioned "Know you know how to

do that”. When all students finish to save the function, Mr. Salcido verify if the students have some questions about how to insert and save that document into the calculator. After that, Mr. Salcido put an assignment and the assignment consisted to develop a story about the function behavior. He said “ We will create a story based on the graph”, this story is about a “Casandra trip”

September-12-2013

As the same of the two days before, I went with the secretary to get my visitor badge. I headed to Mr. Salcido classroom. Nobody was in the classroom. I sat down to prepare my observations, after 5 minutes approximately, Mr. Salcido arrived to the classroom. He explained that he is anxious to know the student's story since some students have a good imagination. Like the students were arriving to the classroom, they were taking their calculator to make log in and began the class. On the white board, Mr. Salcido knows what student is ready to begin the class and what student are not ready. For example, if some students don't make log in Mr. Salcido push them to make log in. The assignment of this class was about talk about the Casandra story.

Mr. Salcido picked up the assignment to each student, and he was surprised when he saw one student's story. The student story developed his story containing two pages.

Mr. Salcido began the story saying "Casandra decided to hike for 3 miles in 2 hrs." to begin the class. Mr. Salcido asked to each student about what is the behavior of each piece of the function. As the class was progressing, all the class developed the Casandra Story. The Casandra story was: *The paths gets a little skeper and she walks at a rate of 2 miles in 2 hrs on 1 mph. It gets a little cooler and she is able to speed up to miles per hour. She stopped for a total of 3 hrs. to eat some lunch. Casandra had to go to the restroom, so she starts jagging at a rate of 4 miles per hour. Casandra sees her friend and slows down to 1 mph and reaches her starting point.*

After to develop the Casandra story, Mr. Salcido asked "between what interval Casandra traveling the fastest [8,9] 8 th and 9 th hour", and "between wish interval is Casandra traveling the slowest [2,4]and [9,12], or "during the trip did she little stop meeting?

Through the class was progressing, I observed that Mr. Salcido, showed enough respect with the students' comments about the casandra story. **(OC: Mr. Salcido asked at each student**

about Casandra story, and if the student got a wrong answer, Mr. Salcido didn't feel bad at the student, on the contrary, Mr. Salcido, push at the student make reflection about his/her answer. “ An effective teacher must be open-minded and willing to work with people (students and their families) of different cultural backgrounds, Cline and Necoshea, 2005, p. 273)).

According to the assignment that Mr. Salcido put to the students, show that the teacher elaborate creatively his lesson plan in order the students do a reflection about the behavior of each part of the function. (OC: **The most important principle of classroom activity design is that the student's actions determine what will be learned, Walker cited by Sleeter, C., 2005, p. 45).**

Through the class was progression, Mr. Salcido showed enough ability to ask to students about the Casandra story. For example, Mr. Salcido “ at what interval Casadra go fast or slow”. In other words, teacher (Mr. Salcido) conducted students about make reflections and think about the different piece of the function. Through this conversation, the students made sense about speed definition. Also, Mr. Salcido explained with dynamic example each part of the graph. As well, Mr. Salcido showed enough knowledge to conduct to students in order to think about the function behavior. (OC: **past. Open-minded and flexible teachers will design curriculum creatively to incorporate the students' backgrounds and ways of knowing the world. Cline and Necochea, 2005, pg. 272).** (OC: **Apparent from participants' comments was that having a disposition toward new ideas and ways of approaching curriculum and educational design is very important for border communities. Participants defined effective borderland educators as those who understand the students and the community and make adaptations accordingly. Because the cultural and linguistic landscape of the borderlands is dynamic,**

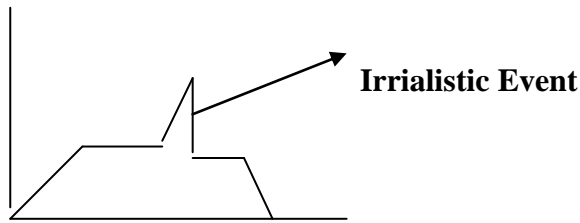
fluid, and ever-changing, teachers must address the social and academic needs of the crisol de culturas (collection of cultures) (Oppenheimer 1998; Martínez 1994; Flores and Clark 2002)).

Finally, Mr. Salcido through this conversation with the students about the Casandra story, he spoke in English and Spanish.

At 3:15, the class finished and each student turn in the calculator. For the next class, Mr. Salcido told them that they will continue to work with the Casandra story.

September-13-2013

As days before, I arrived at 2:00 p. to Bel Air high school to observe to Mr. Salcido. First of all, I addressed with the secretary to get my visitor badge. After that, I addressed to Mr. Salcido classroom to get my observation. It will be the last day to observe him, for this reason I took some pictures at the classroom. Mr. Salcido arrived few minutes later, and he greeted me, and we discussed about one of the story that one student gave him yesterday. He mentioned “ Es increíble la imaginación que tiene este alumno, me sorprendió...” . All our conversation was at Spanish, since I feel more comfortable to speak Spanish. At 2:15 the class began, and the student arrived to the classroom. Like the students were arriving, they took a calculator to make log in and to begin the class. Mr. Salcido, one day before gave an assignment were the students will create a piece-wise function and write a story that correspond to the graph. This equation must have of 6 minimum functions. The instructions were conducted for Mr. Salcido. The instructions consisted that each student must introduce their own function in the calculator. In order that Mr. Salcido gave the instructions, he explained first of all at English but he changed to finish the instructions at Spanish. Some very interesting was that nobody made questions about the instructions although that Mr. Salcido explained at English and he switch at Spanish language. **(OC: A teacher in the border should speak two languages to help the parents and the students with instructions, Cline and Necochea, 2005, p. 279).** Through of this function, the students must specify what is the interval, avg. rate of change, equation (point slope), and equation (slope-intercept). **(OC: “...teachers will design curriculum creatively to incorporate the students’ backgrounds and ways of knowing the world”, Cline and Necochea, 2005, p. 272. Mr. Salcido put this assignment in order that students can understand, what happened went the function is in this interval “ Irrialistic event”).**



The assignment consisted that students will introduce their function in the calculator. If some students didn't finish this assignment, Mr. Salcido gave them the opportunity to finish it at the classroom. Like the students were working in their assignment, Mr. Salcido walked around the classroom to check and verify the students' work. Mr. Salcido stopped with a group of 2 students and he checked the function of one of the student and Mr. Salcido asked him at Spanish about what is the range, and domain of the function, and the student responded at Spanish too. I observed that if some students have some question, Mr. Salcido helped those to get the right and Mr. Salcido was walking around the classroom and he showed respect to the students' opinion about their response, if one students have a wrong answer, Mr. Salcido respect the student work. In other words, he gave support them respecting their opinions although that some students asked him at Spanish or English. **(OC: An effective teacher should have an open mind and respect for students, without caring about their race, religion, or socioeconomic status, Cline and Necochea, 2005, p. 272).**

Appendix C Virginia's Case Study: Fieldnotes Expanded Observations

September-10-2013

Observing Virginia

On September 10, I observed for first time to Virginia. She is teaching geometry at Bel Air high school. Bel Air high school is located at 731 N Yarbrough Dr, El Paso, TX. 79915. This class was taught from 3:15-4:00 p.m. When I arrived to the classroom I talked with Virginia about the days that I will observe her class. The first thing that I observed was that most of the students prefer to talk at Spanish between them. It is interesting to know that at this class the students prefer talk in Spanish comparing to Mr. Steve's students. Also, I observed that this classroom have a lot of pictures about math concepts at both languages (Spanish and English). For example, next figure some mathematics concepts at English and Sapanish. (OC: "The research demonstrating that students learn a second language best when they build academically upon their first language", Romo and Chavez, 2006, p. 150). (OC: Effective teachers in the border region are adept at implementing flexible programs, learning schedules, and curriculum, Cline and Necochea, 2006, p. 275).

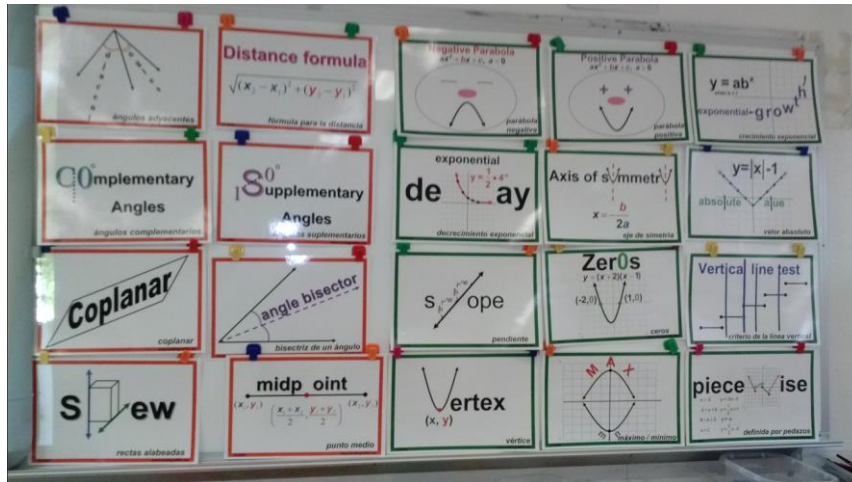


Figure 1: Math concepts at both languages

Another thing that I found was that on the wall you can see the geometry TEKS.

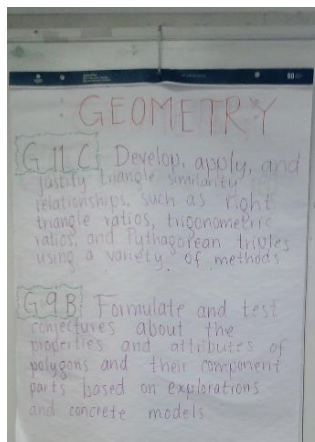


Figure 2: Geometry objectives according to TEKS.

First of all to start the class, Virginia named at each student to know if some students attended the class. She spoke at Spanish and English (e.g. Ms. Martinez, está aquí). After to name at each student, Virginia mentioned that “I will give an algebra review, ok”.

Example # 1:

$$4(3x+4)=2(5x+3)$$

$$12x+16= 10x+6$$

$$-10x-16-10x-16$$

$$2x/2=-10/2 \quad x=-5$$

Virginia explained this example detailed, asking how we can find the “x” value. After that, she explained second example:

Example #2:

$$8x+2+4x+3=20x+8.$$

At this example she explained that first you need to sum the values who have the variable “x” and after that sum the rest of the numbers. For example,

$$8x+2+4x+3=20x+8$$

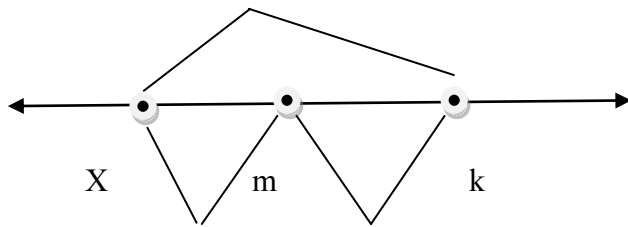
$$12x+5 = 20x+8-5$$

$$-20x-5 \quad -20x$$

$$-8x/8 = 3 \rightarrow x=-3/8$$

Then, she explained another example more complicate.

Example #3: $20x+6$



$$\begin{array}{c} \longleftrightarrow + \longleftrightarrow \longleftrightarrow = xk \\ X \quad m \quad m \quad k \end{array}$$

$$4x+3+2x+2=20x+6$$

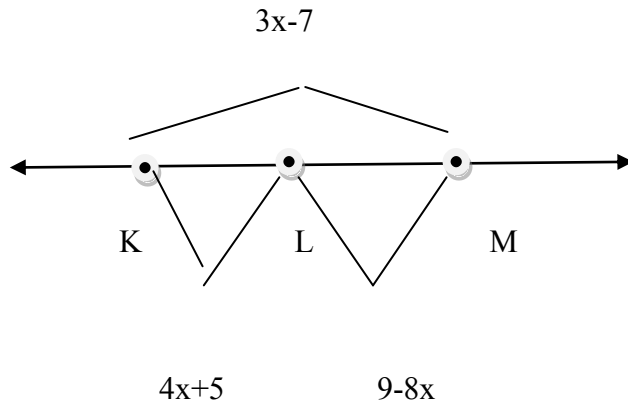
$$6x+5=20x+6$$

$$-20x-5=-20x-5$$

$$-14x=1 \longrightarrow x=-1/14$$

Virginia explained these three examples using web board. She explained step by step how to get the x value. Through her explanation, she did some questions to students to verify if they understand how to find the x value. Virginia wrote an exercise on the web board with the purpose that students can solve it individually.

The exercise is the following:



After some minutes, Virginia asked to Vanessa (student) what was the first step to solve the algebra equation?. The student did not understand the question and Virginia asked again the same questions but at Spanish. “ Cual sería el primer paso para resolver la ecuación, “ Que tendríamos que hacer primero” and Vanessa responded. (OC: An effective borderland teacher has to understand the dynamics of how the communities in the border area work and the needs they have. The teacher needs to understand the acquisition of languages and the language used by these communities. This understanding will allow him or her to give the students what they need to be successful, Cline and Necochea, 2006, p. 277).

S: “Tendríamos que juntar las ecuaciones $4x+5+9-8x=3x-7$ y sumar las x's y luego sumar las constantes.

Conversation between Virginia and Vanessa.

Virginia: Y cómo quedaría la ecuación

S: $-4x+14=3x-7$

Virginia: Aja

S: y restaríamos $-3x$ a ambos lados

$-4x+14=3x-7$

$$-3x = -3x - 14$$

$$-7x - 14 = -21$$

$$-7x = -21 / -7 = 3$$

Virginia: Muy bien, entonces el valor de KL, LM and KM es...

$$KL = 17$$

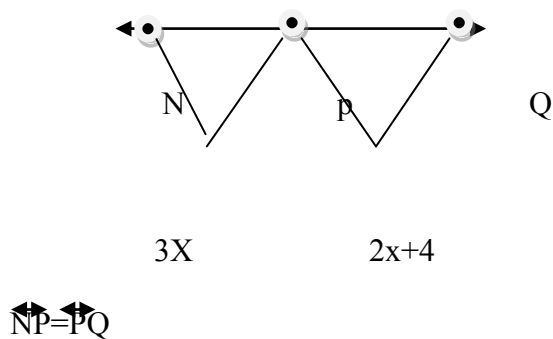
$$LM = -15$$

$$KM = 2$$

Virginia: any question about the problem.

O: nobody made questions

After that Virginia put another exercise to students. The students work individually and Virginia walk around the room to check the student work. One of the students asked to Virginia how to solve the problem like (Please miss can you please help me, porque no le entiendo). Ms. Veroncia stop with the student and she explained at Spanish.



After some minutes, Virginia asked by the “x” value, “Ms: Veronica: What is the x value?”

And one student responded S: $X=4$ and Virginia solved it on the web board to verify if it is correct. After that, Ms Veronica put homework about some similar problems. The students are dismissed from the teacher. The class ended at 4:00 p.m

September-11-2013

On September- 11, I observed for second time to Virginia at Bel Air High School. Ms Veronica showed a strong character since some students are rebels. Earlier to start the class, Virginia asked them if they finished the homework. Some of them did not finish it, for this reason she gave them some minutes to finish the homework in the classroom. She put the chronometer on the web board in order that students can know how much time they have to finish it. She observed that students did not finish with this time; therefore, she gave 5 more minutes at the students. It is interesting that with this strategy, Virginia pressures at the students to do the homework. (OC: Effective teachers in the borderlands are passionate about the work they do, and do whatever it takes to help transnational students succeed, Cline and Necochea, 2006, P. 273). After that, she picked up the homework to only the students who finished the homework. Virginia gave handouts to the students to work in the classroom.

Angles - terms, theorems, postulates 09-11-13

TERM	DESCRIPTION	MARKING	SYMBOLIC REPRESENTATION
Angle			$\angle A$ $\angle BAC$
Measurement of an angle			$\angle A = 30^\circ$ $\angle BAC = 30^\circ$
Acute angle			
Obtuse angle			
Right angle			$\angle A = 90^\circ$ $\angle BAC = 90^\circ$
Conditional statement			
Obtuse angle			$90^\circ < \angle < 180^\circ$
Conditional statement			
Congruent Angles			
Conditional statement			Two angles that have the same measurement, these are congruent angles.
Angle Bisector			
Conditional statement			Students construct the conditional statement according to the definition.

Figure 2: This figure show the handout that Virginia gave to the students.

Ms Veronica began showing the angle definition to the students. She asked them to write down the definition on their handouts that she gave at the beginning of the class. Virginia asked

at “Hispanic student” to read the angle definition at Spanish. The student translated the angle definition to the classmates. (OC: **“...pedagogy is to engage with the interface between present and past, to enact the principle that if one is to understand anything about education elsewhere one's perspective should be powerfully informed by history. So while the comparative journey in Culture and Pedagogy culminates in a detailed examination of teacher-pupil discourse for language is at once the most powerful tool of human learning and the quintessential expression of culture and identity...”** (Alexander 2001, P. 512).

(OC: **“Teachers encourage students to speak multiple languages, value every language spoken in their classroom, Cline and Necochea, 2006, p. 278).**

The next definition that students learned was acute angle. Virginia asked to another student to read the definition. The student read the definition at English; he did not translate than another student. So, it was the process how Virginia designed the class, (e.g., showing definition on the web board, students read definition, write down it on the handout and she showed how each definition is represented symbolically). She did not created some activities according to reinforce the definition. (OC: **“...lesson planning plays an important role to achieve effective classroom instruction..”, Li et al, 2009, p. 718).** (OC: **If teachers can design well thought- out and high-quality lesson plans, as a process of curriculum planning at the micro-level, they build a solid base for classroom implementation. Quality instruction is, therefore, more likely to occur, Li et al 2009, 717).** (OC: **Teachers who prepare themselves continuously will obtain those teaching methods necessary to understand or resolve them different situations of each student in the border region, Cline and Necochea, 2006, p. 275).**

When the students wrote the definition on their handouts, Virginia walked around the classroom to know if some students have some questions according with the definition. For

example, one of the students asked her about the acute angle and she made some questions at Spanish and Virginia responded her at Spanish. I observed that she did not feel uncomfortable because the student asked at Spanish. She showed respect and open-mind with the student. **(OC: The characteristic of an effective teacher for the border region is to have knowledge of Spanish and English, because to be bilingual means you can communicate in two languages, (Cline and Necochea, 2006, p. 279).**

September-13-2013

On September-13 was the last day to observe to Virginia. I arrived on time and I sat down at the end of the one line to observe the class. The class began at 3:15, some students came late and Virginia caught their attention. On this day Virginia didn't name at the students to verify if some student did not assist to the class. Virginia wrote on the white board the material that they will use to take the lesson. Like the students were arriving, they were taking the material. They used ruler, compass and protractor. They continued working with the second part of the handout.

Angles - terms, theorems, postulates			
TERM	DESCRIPTION	NAMING	SYMBOLIC REPRESENTATION
Angle	Definition		$\angle ACB$ $\angle A$
Measurement of an angle			$\angle T = 45^\circ$ $\angle R = 20^\circ$
Acute angle			
Conditional statement			
Right angle			$\angle SUN = 90^\circ$ $\angle U = 90^\circ$
Conditional statement			

Obtuse angle			
Conditional statement	$90^\circ < \angle < 180^\circ$		
Congruent Angles			
Conditional statement	Two angles that have the same measurement, they are congruent angles		
Angle Bisector			
Conditional statement	Students construct the conditional statement according to the definition		

Complementary			
Conditional statement			
Congruent Complements Theorem			
Supplementary			
Conditional statement			
Congruent Supplements Theorem			

For each definition, teachers ask the conditional statement

Figure 3: This figure show the handouts used for the teacher to teach angle, acute angle, right angle and others definitions.

The first instruction given to Virginia was that "Please draw two triangles on the notebook", and I observed that one of the students didn't understand and Virginia explained her personally at Spanish. **(OC: The characteristic of an effective teacher for the border region is to have knowledge of Spanish and English, because to be bilingual means you can communicate in two languages, Cline and Necochea, 2006, p. 279).**

When the students finish drawing the two triangles, Virginia showed a video about how to construct two congruent triangles. After that, Virginia didn't do an activity where the students using the protractor proof that the two triangles are congruent. For this activity, Virginia remind to the students that think about the conditional statement of congruent angles. For example, Ms Veronica said "When the two triangles are congruent" and she wrote on the board $m\angle JAK \approx m\angle LPM$ $\angle JAK = 40^\circ$
 $\angle LPM = 40^\circ$

After that, Virginia put an exercise where the students must to construct two congruent triangles following the conditional statement. Virginia gave at the students some minutes to do the exercise. Next, Virginia showed another video about how to construct a bisector angle and the students observe it putting attention. According to this video, Virginia asked them about "what means the purple line" and the students respond "it is the bisector angle". When the video finished, Virginia put another exercise where the students must to construct a bisector angle. Virginia walked around the classroom observing the students' work. When she observed the work of one of the students, she observed that he did some mistakes and Virginia explained how to draw angles with the protractor. She continued walking around the classroom observing the students' work.

The three days that I observed to Ms. Veroncia, I observed that she did not feel uncomfortable to work with Mexican-American students. On the contrary I observed that she out more attention at these students who have difficult with the language. **(OC: In an effective borderland school, a climate of cultural acceptance that is respectful of all members is critical. Because the borderlands are a magnet for immigrants from Mexico, Latin America, and Asia—who often are in a period of transition—effective teachers must create**

a classroom environment where all students feel accepted and respected for who they are, Cline and Necochea, 2006, p. 277).

Appendix D Carlo's Case Study: Fieldnotes Expanded Observations

October 29, 2013

Observing Carlos

On October- 29 I went to observe to Mr. Ortiz at Cesar Chavez high school. Cesar Chavez academy high school is located at Yarbrough St. El Paso, TX.. I arrived on time to high school, and I went to the central office to get my visitor badge. Mr. Ortiz teaches Pre-calculus class at small classroom with 5 students; 3 of the students worked on the computers and the rest of the students took the class face to face. I observed that the objectives of the lesson were written on white board for the students. **(OC: In teacher preparation coursework, teachers commonly learn to write lesson plans, starting with objectives. Sleeter, 2005, P. 50)**

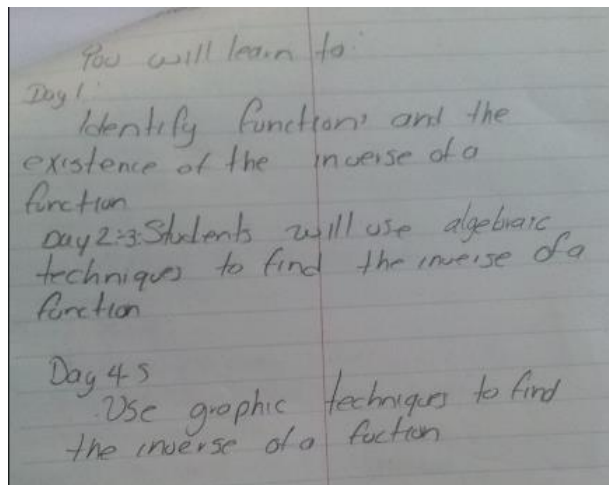


Figure 1: This figure represents the lesson objectives.

The first day of observation, Mr. Ortiz gave the algebra review to the students. Mr. Ortiz explained in detailed this example. For example, the first step that he did was: adds 5 at both sides then multiply the algebra equation by 2, after that cubic root and add 4. One of the suggestions mentioned by Mr. Ortiz was that “Mr. Ortiz: Putting it step by step”. Finally, Mr. Ortiz found the x value. “Mr. Ortiz: So, $x=6$ ”

Oct 29-13 Mr. Korin (1)
 Review to Algebra
 M. Mr. Korin explica esta eq.

$$\frac{1}{2}(x-4)^3 - 5 = -1$$

 Add 5
 Multiply by 2
 Cubic root
 add 4
 M: Putting it step by step
 M: So $x = 0$

Figure 2: This picture show the steps to solve the first example

After that, Mr. Ortiz asked at the students how they can solve this equation graphically. Through this question, Mr. Ortiz had the students reflect about how represent this algebra equation graphically. Then, Mr. Ortiz, wrote the example # 2 on the web board, he did the same process like the first example; multiply by 3, adding 4, multiply by 5 and subtracting 3.

NEXT EXAMPLE
 M: to solve an equation

$$5(3x+4) - 3 = 0$$

 multiply by 3
 Adding 4
 Multiply by 5
 Subtracting 3
 divide 3
 Subtract 4
 divide by 5
 Add 3

$$\frac{(x+3)}{5} - 4$$

 3

What is a function
 M: Any idea
 M: You learned about quadratic function,
 logarithmic function
 M: But what is a function

Figure 3: Show the steps to solve the second example

After these two example, Mr. Ortiz asked about what is a function?.

Mr. Ortiz: What is a function?

Mr. Ortiz: Any idea

Mr. Ortiz: You learned about quadratic function, logarithm function,...

Mr. Ortiz: But what is a function?

According to these conversation conducted by Mr. Ortiz, nobody responded anything. After that, Mr. Ortiz mentioned, think about how we can represent a function. Since nobody responded the question, Mr. Ortiz taught the different forms that a function can be represented.

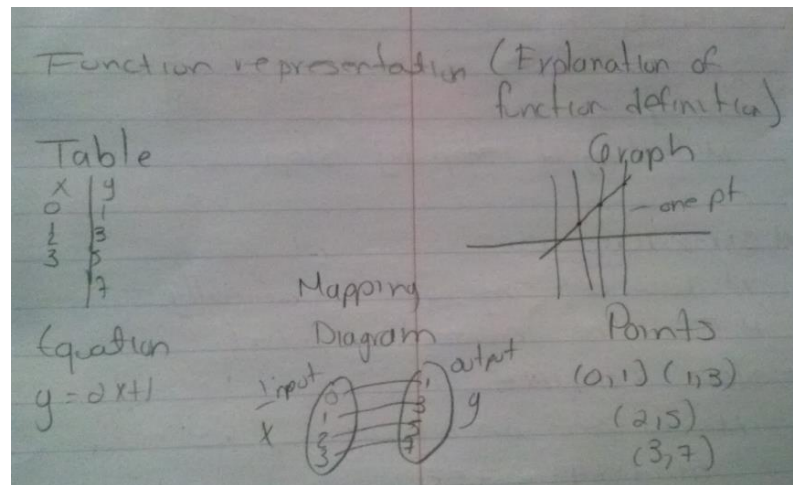
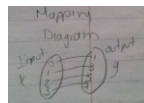


Figure 4: This picture shows the different form to represent a function

Mr. Ortiz mentioned “a function can be represented by table, graph, mapping diagram and points.



After that, Mr. Ortiz asked, it is a function? And one of the students responded “ Yes, it is a function.

Mr. Ortiz: Why it is a function?

Student: Because is one to one representation.

Mr. Ortiz: Yes, it is a function

Mr. Ortiz: Ok next example, consider some values to x and graph the function.

Mr. Ortiz: Tell me if it is a function.

Mr. Ortiz gave some minutes at the students to graph the function.

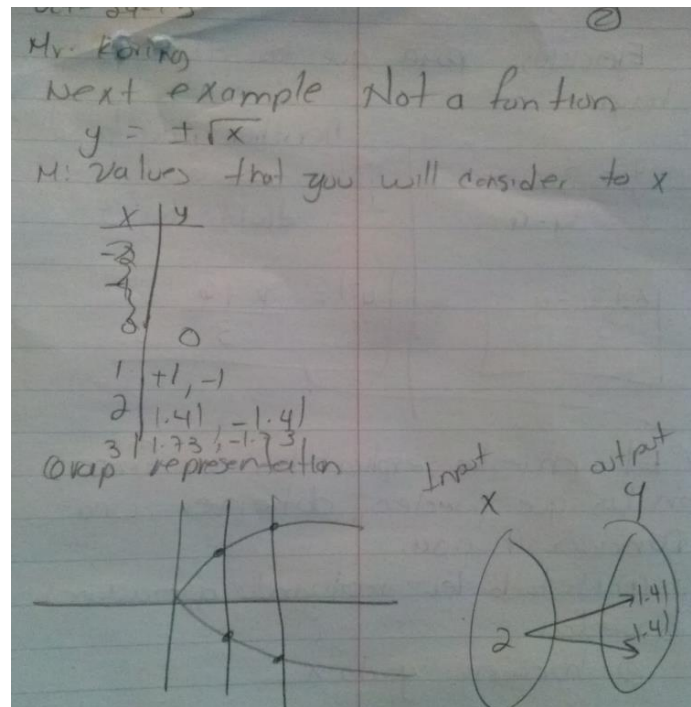
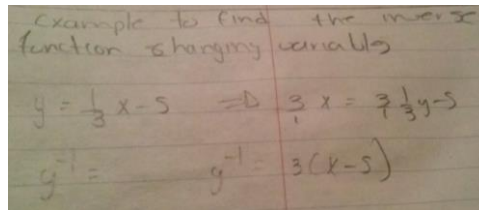


Figure 5: This picture shows another example taught by Mr. Ortiz of not a function

After some minutes, Mr. Ortiz began to graph the function, and represented it by mapping diagram. Through this example, Mr. Ortiz made some reflections about how differentiate a function and not a function. For example, he asked if it is a function, and one of the students mentioned “*Not, it is not a function because it is not a one to one representation*”. It is interesting how the math teacher began introducing a function definition and after that, he pushed at the students made reflection about the representation of not function. (OC:” ...**lesson plans; both content and process are important aspects of lesson plans. While the content aspect focuses on what to teach, it translates into a teacher’s interpretation and specification of instructional objectives and content treatment for teaching**”, Li et al, 2006, p. 719). Also, it is interesting how Mr. Ortiz designed the lesson in order that students can understand function definition and how function is represented by a table, mapping diagram and graphically.

Next, Mr. Ortiz began to teach some examples of inverse function. For example, he wrote the following example: $y = \frac{1}{3}x - 5$, and he mention we will find the inverse function with the method changing variables.



Handwritten example of finding the inverse function by changing variables:

$$y = \frac{1}{3}x - 5 \Rightarrow \frac{3}{1}x = \frac{3}{1}\left(\frac{1}{3}x - 5\right)$$

$$y^{-1} = \frac{3}{1}x - 5 \Rightarrow y^{-1} = 3(x - 5)$$

Figure 6: This picture shows the example that Mr. Ortiz did on the web board.

Finally, Mr. Ortiz gave some exercises about find the inverse fuction. The students worked on the exercises individually. Mr. ortiz walked around the classroom to check the students' work. If one of the students have some questions, Mr. Ortiz help at the students with respect and patien to explain the problem. I observed that Mr. Ortiz talk at Spanish or English with the students. It is depend how the students communicate with him. Also, I observed that Mr. Ortiz ejoy and accpet the diversity of cultures and lanaguages, since he domain and speak Spanish and English with the students.

Another interesting thing that I observed was that Mr. Ortiz showed ability to explain to students who are taking the class on the computer and the students who are taking the class face to face.

October-31-2013

On October-31 I observed for second time to Mr. Ortiz. The topic taught at this day was rational function. The students arrived to the classroom and they took their notebook, pencil and a calculator. On this day one student more attended the class. So, 3 students worked on the computer and 3 students took the class faced to face. Mr. Ortiz started the class reviewing the topic of inverse function and how you can find it. For example, Mr. Ortiz mentioned “if you switch x and y you will get the same function, so the inverse function is the same”. It is interesting how Mr. Ortiz proof it algebraically and graphically. **OC: Mr. Ortiz designed the curriculum in order that students can understand can do some algebraic operations and they can prove graphically at the calculator. As a Cline and Necochea mention, “Effective educators in the borderlands creatively look for ways to help students access the core curriculum and learn the skills necessary for success. They meet the needs of students by familiarizing themselves with and understanding the students’ families and community”, Cline and Necochea, 2006, p. 274).**

After that Mr. Ortiz wrote one example on the web board. Solve this function $y = x - 2/x + 2$, and Find Y^{-1} . Mr. Ortiz asked at the students “What we need to do first? and nobody mentioned what they need to do first to solve the irrational function. I observed that Mr. Ortiz knew that the students didn’t know how to solve the function irrational function for this reason he explained detailing the function. **OC: Before to start the class, Mr. Ortiz mentioned me that he will explain irrational functions since the students have some problems to solve this kind of function.** Since nobody says how to solve the irrational function, Mr. Ortiz began to solve it on the web board.

Algebraic what we need to do first?

Algebraic function (y+2) $x = \frac{y-2}{y+2} \rightarrow (y+2)$ $\Rightarrow xy + 2x = y - 2 \Rightarrow xy - y + 2x = -2$

$(y+2)x = y-2$ $-y -y$ $-2x -2x$ $xy - y = -2 - 2x$

$y(x-1) = -2 - 2x$ $y = \frac{-2 - 2x}{x-1}$

Graph of y function and graph of y⁻¹ function

Figure 1: This picture show the first example taught by Mr. Ortiz.

On this example Mr. Ortiz explained step by step how algebraically the students can find the inverse function. After that, Mr. Ortiz mentioned the following “ Please work with the first irrational function that do you have on your paper.

$$f(x) = -\frac{9x - 3}{7x + 6}$$

Figure 2: This picture show the first irrational function.

The students began to work on this function. Mr. Ortiz walked around to the classroom to check the student work or if they have some questions about how to solve the function. Mr. Ortiz, also walked over to check the work of the students working on the computer and, he asked at one of the student, Mr Ortiz: Pero te salio la respuesta...., como le hiciste? and the student explained to Mr. Ortiz at English language how he solved the problem. **OC: I observed that Mr. Ortiz respect the method that students use to solve any kind of problem. He respect students regardless race, idiom, religion or socioeconomic status. As well, he accept the diversity of cultures. OC: According to Cline and Necochea, an effective teacher is open mind and respect for students, without caring about their race, religion, or socioeconomic**

status. In the border region, the majority of students are Hispanic, Cline and Necochea,2006, p. 272.)

Also, I observed how Mr. Ortiz put attention at the students who are taking the class face to face and at the students who are solving the problems on the computer. For example, Mr. Ortiz explained at one of the students who are working on the computer saying “ Mr. Ortiz: Ok, la y es la dependiente y la x es la variable independiente” . Also, I observed that Mr. Ortiz explained the class at English but he communicate at both languages when he talk individually with the students. OC: Mr. Ortiz use both languages to communicate with the students. “Teachers encourage students to speak multiple languages, value every language spoken in their classroom, and frequently and openly express the advantages of speaking multiple languages in a world economy, but especially in the complex and amalgamated border region Cline and Necochea, 2004b, p. 278).

Also, I observed that Mr. Ortiz have the knowledge and ability to explain any kind of math problem at the students. For example, Mr. Ortiz explained that these two math expressions are not the same $3\sqrt{8} \neq 3\sqrt{x}$. He explained with patient about why these two expressions are not the same. Also, Mr. Ortiz showed ability to help at the students who work on the computer and who are taking the class face to face.

After some minutes, Mr. Ortiz solved this function on the we board, he explained the irrational function step by step. I observed that all students put attention about the Mr. Ortiz explanation. Nobody made question about how to solve the problem.

November-05-2013

On November 5, was the last day to observe to Mr. Ortiz. I arrived at the high school on time, and I went to the principal office to get my visitor badge. I went to Mr. Ortiz classroom; I took sit down to begin observe the class. Before to start the class, Mr. Ortiz distributed some hand out at the students and explained the instruction at English language. **OC: Language is the heart and soul of the individual and, as such, needs to be honored and respected in any educational environment, Cline and Necochea, 2006, p. 279).** He gave them a little introduction about composite function through a story. This story was about the student distribution in the classroom. **OC: Mr. Ortiz designed the lesson plan creatively in order that students can understand the composite function definition. As Li. Y. mentioned if teachers can design well-thought out and high quality lesson plans, as a process of curriculum planning at the micro level, they will build a solid base for classroom implementation. Quality instruction is, therefore, more likely to occur. Li, Y, 2007, p. 309).**

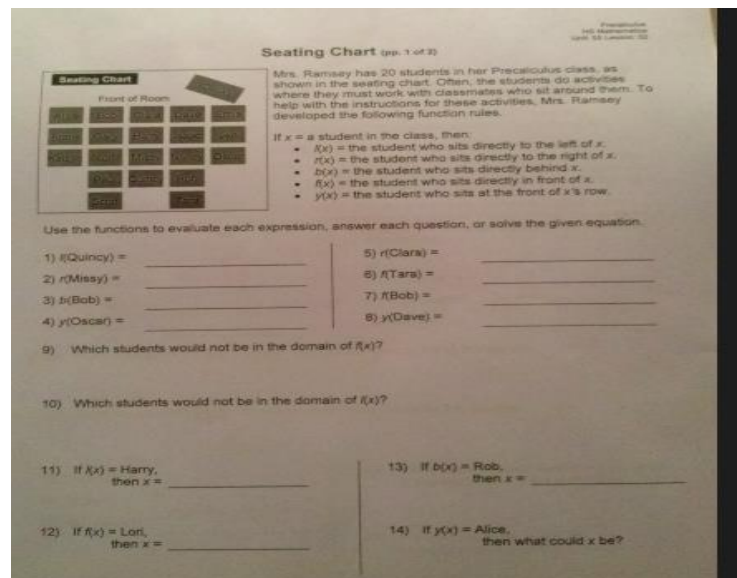


Figure 1: This picture shows Mr. Ortiz activity to explain composite function.

Mr. Ortiz gave some minutes at the students to work on the hand out. Also, Mr. Ortiz asked students to reflect on the question #9.

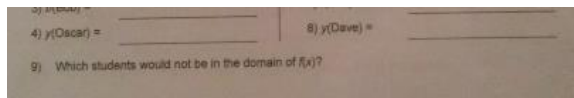


Figure 2: This picture shows question #9

After to finish the first part of the hand out, Mr. Ortiz wrote on the web board as a title “composition function” if we have $f(x)=5x+3$ and $g(x)= 4x-20$, there are two ways to find composition function. The first way is $f((g(x)))$ opposite $g((f(x)))$. Mr. Ortiz said, “it is the same if we put fog or gof. Mr. Ortiz mentioned “take care with it because it doesn’t mean a multiplication”. $f((g(x)))$ opposite $g((f(x)))$, this mean that function “g” will be inside function “f”.

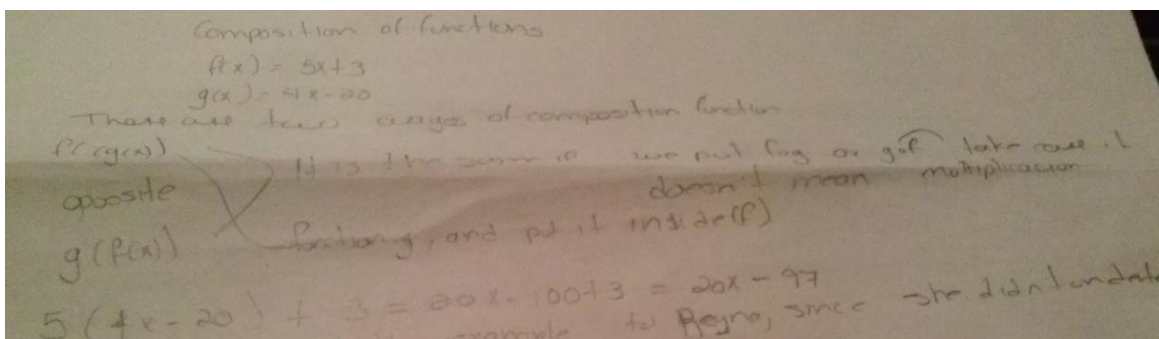


Figure 3: This picture represents the example explained by Mr. Ortiz.

After to explain it to the group, he explained the same problem to Reyna since she did not understand his explanation. He explained individually the problem at English language. **OC: As a Garza mentioned “We feel more confident and comfortable being placed into schools within the borderlands because it is not an unfamiliar environment to us. Because I live in the borderlands I am more conscious of the needs of not only Spanish speaking students but of all students that speak different languages and come from different cultures” Garza, 2005, p. 4).**

Mr. Ortiz: It is clear Reyna

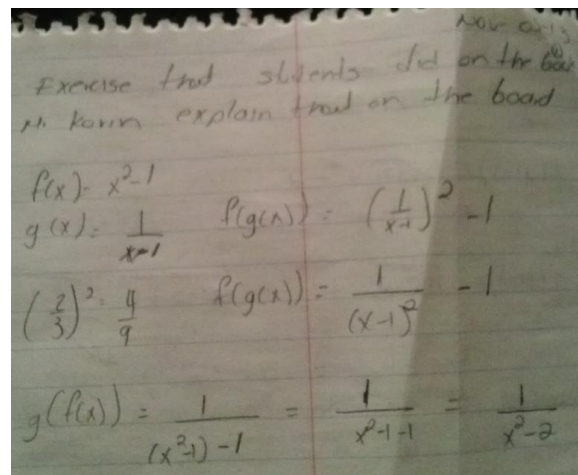
Reyna: Yes

Mr. Ortiz: Open your book on page 124; let's work on the problem 15

Reyna: All of them

Mr. Ortiz: Yes

The students began to work on some problems in the book. Mr. Ortiz gave some minutes at the students to work on the problem individually, but after some minutes he began to solve it on the web board.



Exercise that students did on the board
at home explain that on the board

$$f(x) = x^2 - 1$$
$$g(x) = \frac{1}{x-1}$$
$$f(g(x)) = \left(\frac{1}{x-1}\right)^2 - 1$$
$$\left(\frac{2}{3}\right)^2 = \frac{4}{9}$$
$$f(g(x)) = \frac{1}{(x-1)^2} - 1$$
$$g(f(x)) = \frac{1}{(x^2-1)-1} = \frac{1}{x^2-1-1} = \frac{1}{x^2-2}$$

Figure 4: This picture represents the first problem that student worked in the book.

I observed that one of the students did not understand the explanation for this reason Mr.

Ortiz explained again on the web board. He mentioned:

Mr. Ortiz: Déjame lo hago para que lo veas,

Mr. Ortiz: Si?

Student: Si, thank you

The students continue working on the book problems.

OC: "...border between Mexico and the United States represents the beginnings, endings, and blending of languages, cultures, communities, and countries, Romo and Chavez, 2006, p. 143).

Handwritten mathematical work on lined paper. The top part shows the composition of two functions:

$$f(x) = x^2 - 2$$

$$g(x) = \sqrt{x+1}$$

$$f(g(x)) = (\sqrt{x+1})^2 - 2$$

$$= x+1-2 = \boxed{x-1}$$

$$g(f(x)) = \sqrt{(x^2-2)+1} = \sqrt{x^2-2+1} = \sqrt{x^2-1}$$

Below the equations, there is a handwritten note in Spanish with an arrow pointing to the $\sqrt{x^2-1}$ expression:

One of the student ask if she can cancel el cuadrado con la raíz cuadrada
 g Mr. Ortiz le dice que no x que se está realizando una resta dentro de la raíz
 El mundo le dice que si estuviera $\sqrt{x^2}$ de ahí fuera si lo podría cancelar

Figure 5: The picture represents the next exercise did by the students

After the students solved the problem #17, Mr. Ortiz explained it on the board. One of the student asked at Mr. Ortiz “we can eliminate the square root” and Mr. Ortiz responded at the students you can’t eliminate the square root since it represents an operation into the square root.

Lunch break

At 12:55 the class continued, and the student continued working on the exercises of the book. OC: I observed that Mr. Ortiz is teachers who like to be aware of their students, for this reason he walks around the classroom to know if one of the students have some questions about the exercises. As Cline and Necochea mentioned “The sentiment that effective teachers in the border region are passionate about their work was strong among

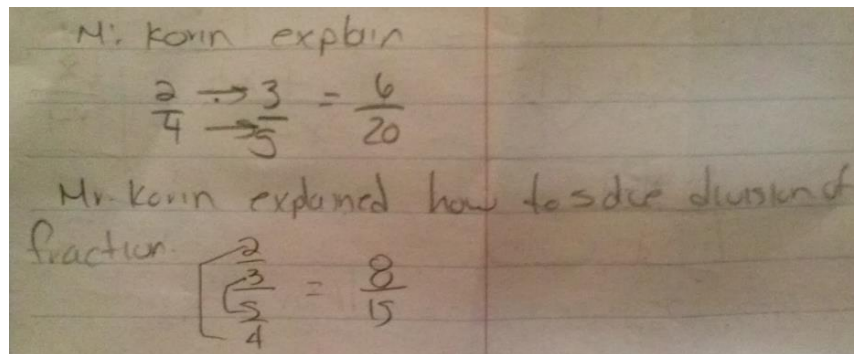
the participants. They expressed that to be effective, teachers must love what they are doing and be willing to do what it takes to help all students succeed”, 2006, p. 274).

After to solve the problem # 17, Mr. Ortiz put another exercise about solve fraction. It was as purpose that students solve the problem # 21. The problem #21 consists to solve a fraction problem. It was a purpose since the students have some difficult to solve fraction problems. (OC: It was another thing that Mr. Ortiz mentioned me before start the class. He mentioned me that students have some problems to solve (addition, multiplication, division and subtraction fractions). As Cline and Necochea affirm “ A borderland teacher needs to have passion, understanding for each individual student. This means getting to know every student through his or her family life, past experiences, language abilities, and education level. An effective teacher must have the ability to help all students at different levels, 2006, p. 275).

Mr. Kevin observation
 21 exercise
 $f(x) = \frac{1}{2x}$
 $g(x) = \frac{1}{3x}$
 $f(g(x)) = \frac{1}{2(\frac{1}{3x})} = \frac{1}{\frac{2}{3x}} = 1 \div \frac{2}{3x} = \boxed{\frac{3x}{2}}$
 $g(f(x)) = \frac{1}{3(\frac{1}{2x})} = \frac{1}{\frac{3}{2x}} = \frac{1}{1} \div \frac{3}{2x} = \boxed{\frac{2x}{3}}$
 Students work on this problem.
 Mr. Kevin explain on the board
 Mr. How we can simplify that

Figure 6: This picture represents fraction problems that students solved.

When Mr. Ortiz wrote the problem on the web board, he asked at one of the student “How we can simplify this fraction, and I was surprise since the student “men” mentioned “ I don’t know how to solve the fraction. Since the student didn’t know how to solve the fraction, Mr. Ortiz began to explain on the web board.



Mr. Kevin explain

$$\frac{2}{4} \rightarrow \frac{3}{5} = \frac{6}{20}$$

Mr. Kevin explained how to solve division of fraction.

$$\left[\frac{\frac{2}{4}}{\frac{3}{5}} = \frac{8}{15} \right]$$

Figure 7: This picture represents the fraction problem that student failed to solve.

After that, Mr. Ortiz gave another example about fraction; He said “Let’s try to do this guy”. As usual, Mr. Ortiz walked around the classroom to check the students work. He approached on one of his students to check her work and he asked her at Spanish “ Se te hizo dificil” and she responded “ yes”, and he explained personally the division fraction at English.

OC: I think that he explained at English since he responded him at English language. So, Mr. Ortiz has the facility to talk at both languages with the students. As Cline and Necochea state “ In some cases, teacher even attempt to learn words and phrases in various languages to help students feel accepted and valued as integral members of the classroom community.

When the students finished solving the fraction problems, Mr. Ortiz put another exercise and he asked them find the inverse function and find the composite function too.

Mr. Ortiz checked the students work and he gave some them some assignments about find composite and inverse function.

The image shows two pages of handwritten student work. The left page is dated November 23, 2020, and shows the calculation of the inverse of $g(x) = \frac{1}{x+1}$ as $g^{-1}(x) = \frac{x+1}{x}$. It also includes an exercise to find the inverse of $f(y) = 4x-3$, resulting in $f^{-1}(x) = \frac{x+3}{4}$. The right page shows exercises for finding inverses of $f(x) = 5x-3$ and $f(x) = x^2-3$. For the linear function, it finds $f^{-1}(x) = \frac{x+3}{5}$ and $g(f(x)) = x$. For the quadratic function, it finds $f^{-1}(x) = \sqrt{x+3}$ and $g(f(x)) = x$.

Left page work:

$$g(x) = \frac{1}{x+1}$$

$$= \frac{1}{\frac{1}{x+1}} = \frac{1}{\frac{1}{x+1}} = \frac{x+1}{1} = \frac{x+1}{x}$$

Exercise
Find the Inverse
 $f(y) = 4x-3$
 $x = 4y-3 \Rightarrow$ leave y inside
 $g(\frac{x+3}{4}) = y$
 $f(g(x)) = 4(\frac{x+3}{4})-3 = x+3-3 = x$

How to prove the inverse function
① find Inverse function
② find composite function
 $g(f(x)) = \frac{4x-3+3}{4} = \frac{4x}{4} = x$

Right page work:

Exercise
① $f(x) = 5x-3$
Find Inverse
Find $f^{-1}(x)$
 $g(f(x))$
 $y = 5x-3, x = 5y-3$
 $\frac{x+3}{5} = y$
 $f(g(x)) = 5(\frac{x+3}{5})-3 = x+3-3 = x$
 $g(f(x)) = \frac{5x-3+3}{5} = \frac{5x}{5} = x$

$f(x) = x^2-3$
 $y = x^2-3$
 $x = y^2-3$
 $\sqrt{x+3} = y$
 $f(g(x)) = (\sqrt{x+3})^2 - 3 = x+3-3 = x$
 $g(f(x)) = \sqrt{x^2-3+3} = \sqrt{x^2} = x$
at the end of the

Figure 9: This picture represents the assignment due by Mr. Ortiz.

Appendix E Angelica's Case Study: Fieldnotes Expanded Observations

October-24-2013

Observing Angelica

On October 24 at 8:10 I went to observe for second time to Ms. Angelica class. I arrived on time and I took my place to begin my observations. Ms. Angelica started the class forming the groups. The groups were formed by 4 students. Also, Ms. Angelica distributed the topics at the groups as the following.

Monday	Group 8 case 2 and Group 2 case 3
Tuesday	Group 3 case 4 and Group 4 case 5 1 st part and group 5 2 nd part
Wednesday	Doubts, and exercises about the cases presented for the groups
Thursday	Group 6 case 6
Monday	Group 7 and 1 case 7
Tuesday	Doubts, and exercises about the cases presented for the groups
Wednesday	Group 9 &10 case 8

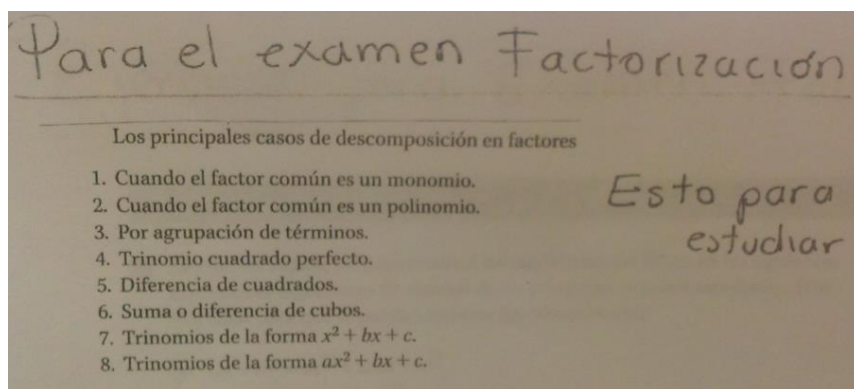


Figure 1: This picture represents the different cases of factorization presented by the groups.

After that Ms. Angelica distributed the case at the groups, the students formed the groups and they began to work on their presentation. Each student selected one exercise to explain at the whole group. **(OC: I observed that Ms. Angelica prefers that students can learn by themselves follow up the standard curriculum from Mexico. As a Sleeter mentions “...**

teachers expected to plan curriculum based on state content standard...”, “Doing this, is likely to result in boring, superficial teaching that favors memory work over understanding” Sleeter, C. 2005, p. 44).

Generally, in the whole class, the groups prepared their presentation for the next week. I observed that they discuss what exercise will present each of them.

Ms. Angelica walked around the classroom to know if the students have some questions about the exercises of the book. Ms. Angelica stopped at one of the group and one of the students asked her about one of the problem. Ms. Angelica explained at the students what mistake she did. (I did not know exactly what Ms. Angelica said since I was away from her). Ms. Angelica showed knowledge about the explanation that she gave at one of the student, but she did not show effort to explain at the students in different way to solve the problem; since Ms. Angelica explained at the students two times the same problem.

I did not want to say that Ms. Angelica is a bad teacher, but she could create some activities to explain the different cases of factorization in case those students have questions. Maybe, this explanation could be individually, if the purpose of Ms. Angelica was that the students learn by themselves the topic.

Also, Ms. Angelica explained at the groups what exercises they will explain at the group. For example, she mentioned at one group, “Ustedes van a exponer a partir de la pagina 90” “ You will present from page 90”.

I observed that when Ms. Angelica talks with their students individually, she showed respect and patient to explain if the students are doing a mistake when he/she solve a problem.

The lesson plan designed by Ms. Angelica is based on the activities, exercises of the algebra book.

October 24, 2013

On this day the groups presented the different cases of factorization. The first group presented the case named “ Cuando factor común es un polinomio”. The presentation took less than 15 minutes. Through of the presentation, Ms. Angelica showed interest about what the students were presented. The students presented this case using the diagram table that Ms. Angelica taught before. Also, the students used posters with the problem solved. The poster was created in order that group can see the steps about how to solve this problem. Ms. Angelica didn't interrupt at the group with questions. When the group finished, Ms. Angelica asked at the group if they have some questions and nobody did questions about this case. When the group finished, Ms. Angelica mentioned at the group that if it is possible to paste the poster on the wall. It is with the purpose that they can see how to solve this kind of case.

The 4 groups that presented, they pasted the poster on the wall.

Through the presentations, I observed that Ms. Angelica respects the process how the group expose the cases. If for one reason one of the student make a mistake, Ms. Angelica corrects at the student with respect, and she puts another similar example in order the group can understand better the example exposed by the group.

The lesson plan is designed following the algebra book. It means that Ms. Angelica didn't design special or different problems than of the book. All the exercises exposed by the groups were selected of the algebra book.

When the third group presented, they had some problems to solve the factorization and Ms. Angelica said “ vamos ayudarle a su compañero porque ya se atoró. Some of the students were participated in order to help at their partner. Ms. Angelica asked at the all group.
Ms. Angelica: que seguiría de ahí?,

Ms: Ya nada más sumas el 5 y el 2

Observer: Nadie respondió

Ms. Angelica: Alguna pregunta para ellos

In this presentation, some students participated, but all the explanation were conducted by Ms. Angelica.

After to the presentation of these groups, Ms. Angelica asked at the group. Quieren que haga algunos ejercicios sobre los casos que se vieron el día de hoy para que no haiga dudas.

Ms. Angelica explained three examples. Each example corresponded at each case of factorization. After that, Ms. Angelica put some exercises in order that students can solve them by themselves.

After to the explanation of the examples, the group began to work on the exercises. Ms. Angelica walked around the classroom to verify that the students were working on the problems. She stopped with some students in order to know if they solved the problem correctly.

The whole conversation between students and Ms. Angelica were at Spanish. Spanish is the primary and only language that the students and Ms. Angelica talked.

October 28, 2013

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Through the presentations, I observed that Ms. Angelica respects the process how the group expose the cases. If for one reason one of the student make a mistake, Ms. Angelica corrects at the student with respect, and she puts another similar example in order the group can understand better the example exposed by the group.

The lesson plan is designed following the algebra book. It means that Ms. Angelica didn't design special or different problems than of the book. All the exercises exposed by the groups were selected of the algebra book.

When the third group presented, they had some problems to solve the factorization and Ms. Angelica said “ vamos ayudarle a su compañero porque ya se atoró. Some of the students were participated in order to help at their partner. Ms. Angelica asked at the all group. Ms. Angelica: que seguiría de ahí?,

Ms: Ya nada más sumas el 5 y el 2

Observer: Nadie respondió

Ms. Angelica: Alguna pregunta para ellos

In this presentation, some students participated, but all the explanation were conducted by Ms. Angelica.

After to the presentation of these groups, Ms. Angelica asked at the group. Quieren que haga algunos ejercicios sobre los casos que se vieron el día de hoy para que no haiga dudas.

Ms. Angelica explained three examples. Each example corresponded at each case of factorization. After that, Ms. Angelica put some exercises in order that students can solve them by themselves.

After to the explanation of the examples, the group began to work on the exercises. Ms. Angelica walked around the classroom to verify that the students were working on the problems. She stopped with some students in order to know if they solved the problem correctly. The whole conversation between students and Ms. Angelica were at Spanish. Spanish is the primary and only language that the students and Ms. Angelica talked.

Appendix F Yolanda's Case Study: Fieldnotes Expanded Observations

Octubre -23-2013

Observing Yolanda

- Open mind
 1. At the beginning of the class, Mrs. Duarte greets at their students with respect. She began the class asking them "como estan"? , Bien?, que Bueno.
 2. Mrs. Duarte interacted with 50 students. When she talked individually with one student, she showed respect and interest in the students about how to solve the problem
 3. Mrs. Duarte designed the lesson plan according to the algebra book exercises. All the exercises and examples explained at the classroom are found at the algebra book. The topic taught was “Binomios conjugados”
 4. Mrs. Duarte solved some the algebra examples before to put the exercises.
 5. The strategies that she used to teach her algebra class were: explain 2 o 3 different examples. The explanation of these 2 o 3 examples took 20 minutes and after that, she applied some similar exercises in order that students can work individually on them.
- Passion
 1. Mrs. Duarte and the students are Mexican people. At CBTIS 128, there is not diversity for cultures.
 2. Some of the students did some questions about some exercises. Mrs. Duarte used the strategy explaining the problem solving it at the notebook.
 3. Only five students came to Mrs. Duarte asking questions about the exercises. She used the same strategy with all students.

4. If the students finish the problems before to finish the class, they will check their work with Mrs. Duarte. Mrs. Duarte revises their work, and if they have all the problems correctly, she signed them.
- Culturally
 1. Before to started the class, Mrs. Duarte talked with me and she mentioned me that she understand the different necessities of the students. For example, she mentioned me that one of the students didn't have calculator, and she did an activity to help to student buy their calculator. She mentioned me that some students have strong economic problems.
 2. Mrs. Duarte respected the different socioeconomic status to students.
 3. The students have characteristics of socioeconomic level not high, since their dresses are not for brand.
 - Language
 1. Since the class is monolingual, Mrs. Duarte explained the examples to Spanish language.
 2. The communication between students and Mrs. Duarte are at Spanish language.
 3. Since the class is monolingual, Mrs. Duarte didn't encourage to students talk another language that is not Spanish.

Octubre-24-2013

- Open mind
 1. She started her class giving welcome at the students.
 2. She gave some announcements about the “ club de matemáticas” and tutorials about mathematics. Some of the students did some questions about the schedule, and Mrs. Duarte responded the students’ questions with respect and patient. The announcements took 10 minutes.
- Passion
 1. After the announcements, Mrs. Pulido began her algebra class teaching a new topic. The topic was “ Cuadrado de un trinomio”.
 2. She explained two examples. These two examples were explained step by step of a traditional way (using the board). After to explain each example, she gave some minutes to the students to copy the problem in their notebook.
 3. After to explain the two examples, she applied 3 similar exercises. These examples and exercises can be found in the algebra book that Mrs. Duarte used to teach the class. To prepare their lectures, she solves the examples in order to can explain in front the group.
 4. At the moment that she explains the examples, she did not show enough knowledge. For example she mentioned “no podemos tener una formula con signos (-), porque tendríamos muchas fórmulas. Es decir diferentes combinaciones. Ejemplo, $a+b+c$, $a-b+c$, $a+b-c$, $-a+b-c$, $a-b-c$, etc.
 5. I observed that through this explanation about the concept of “cuadrado de un trinomio” some students had doubts.

6. Some students did questions about it, and she gave the same answer: no podemos tener una fórmula con signos (-), porque tendríamos diferentes combinaciones.

- Culturally

1. There is not diversity of culture. However the students are from different socioeconomic status.

- Language

1. According to the “club de matemáticas” announcements, Mrs. Duarte mentioned, “si algunos de ustedes no entienden bien los ejercicios que estamos viendo en clase, es importante que venga asesorías los sábados.

2. Also, she mentioned that, “si algunos de ustedes están interesados en pertenecer al club de matemáticas pueden asistir los sábados de 9-11”.

3. The instructions conducted by Mrs. Pulido were at Spanish language.

4. The communication between students-students and students-teacher were at Spanish language.

Octubre-28-2013

- Open mind

1. Mrs. Duarte started the class saluting at her students.
2. She taught new topic. The topic was “el cubo de un binomio”.
3. She designed her class, explaining three examples about “cubo de un binomio”. She explained 3 different examples using the green board. All students put attention about the Mrs. Duarte explanation. After to solve each example, she gave some minutes in order that students can copy these examples on their notebook.
4. She applied some similar exercises. Each student worked individually on these exercises.
5. While the students work on the exercises, Mrs. Duarte revised the homework.

- Passion

1. Each time that Mrs. Duarte explained individually to the students solving the problem together.

- Language

1. Mrs. Duarte started the class saying: Saquen su cuaderno. El tema de hoy es “product de un binomio con termino comun”.
2. She continued saying: Ya sabemos que es un binomio. ¿Qué es un binomio? And nobody responded, for this reason she asked “ El que tiene 2 terminos en común”. And she put one example: $(a+b)(a+c) = a^2 + a(b+c) + bc$. She put another example: $(x+y)(x+z)$, Cual es el común? And one student responed: x es el comú
Mrs. Duarte asked: Entonces como va a quedar este
Mrs. Duarte gave the answwer: $x^2 + xy + xz + yz$

Mrs. Vamos hacer otro ejemplo: $(x+3)(x+z) = x^2 + x(3+2) + (3)(2)$, $x^2 + x5 + 6$, $x^2 + 5x + 6$

3. All the instructions and explanation were conducted at Spanish language. The dominant language in Mrs. Duarte class is Spanish.

Appendix G Survey

Part I. Demographic Information. Please tell us some characteristics about yourself. Mark the choice that most accurately describes your background.

1. Gender:

Male: _____

Female: _____

2. Age:

Under 30:_____ 30-40:_____ 40-50: _____ 50-60: _____

3. Do you identify yourself as Hispanic or Latino/-a?

Yes: _____

No (Respond Q3): _____

3a. If you don't identify as Hispanic or Latino/-a, how do you identify yourself ethnically/racially?

4. Where did you attend the University?

Ciudad Juárez, México: _____ El Paso, TX.: _____ Another Place

(Respond Q4a)

4a. If you did not attend university in Juárez or El Paso, where did you attend it?

5. What is your Degree or Major/ Minor?

6. Years of Experience teaching mathematics: a) In USA_____

b) In Mexico_____

7. What grade level do you teach? _____

8. What subject do you teach? _____

Part II: Language and Communication Profile. The following questions are designed to give us a better understanding of your experience with languages. We ask that you be as accurate and as thorough as possible when answering the following questions.

8. . What language(s) do you consider your native language(s)?

English: _____ Spanish: _____ Both English and Spanish: _____

Other(respond Q8a)

8a. If you don't consider your native language(s) English/ Spanish, what language(s) do you consider your native language(s)?

9. What is your second language?

English: _____ Spanish: _____ Other (respond Q9a) Not applicable: _____

9a. If you don't consider English/ Spanish as your second language, what is your second language?

10. Do you speak any additional languages?

Yes: _____ No: _____

10a. What additional language(s) do you speak?

11. What language(s) do you use most with your students?

English: _____ Spanish: _____ Both English and Spanish: _____ Other

(respond Q11a)

11a. What other language(s) do you use most?

12. What language(s) do you use most outside of the classroom?

English: _____ Spanish: _____ Both English and Spanish: _____ Other

(respond Q12a)

12a. What other language(s) do you use most?

Part III. Please rate the statements on a scale: Never , Seldom, Frequently, Very frequently, Always

13. When you talk with other teachers, principal, and staff about lesson plan development, how frequently do you use your first language?

Never: _____ Seldom: _____ Frequently: _____ Very frequently: _____

Always: _____

14. When you talk with other teachers, principal, and staff about lesson plan development, how frequently do you use your second language?

Never: _____ Seldom: _____ Frequently: _____ Very frequently: _____

Always: _____

Parte I. Información Demográfica. Por favor díganos algunas características acerca de usted y marque mejor opción que describa su formación.

1. Sexo: a). Hombre: _____ b). Mujer: _____

2. Edad: a). Menos de 30 años: _____ b). 30-40: _____ c). 40-50: _____

d). 50-60: _____

3. ¿Se identifica como hispano o latino /-a?

Sí: _____ No (Responda Q3) : _____

3a. Si no se identifica como hispano o latino / -a, ¿cómo se identifica etnicamente? _____

4. ¿Dónde estudia o estudio la Universidad?

Ciudad Juárez, México: _____ El Paso, TX.: _____

Otro Lugar (Responda Q4a)

4a. Si usted no estudio la universidad en Juárez o El Paso, donde estudio usted la universidad?

5. ¿Cuál es especialidad?

6. Años de experiencia en la enseñanza de las matemáticas: En

a). USA _____

b) En México _____

7. ¿Qué nivel de grado enseña usted ? _____

8. ¿Qué materia enseña? _____

Parte II : El lenguaje y perfil de comunicación . Las siguientes preguntas están diseñadas para darnos una mejor comprensión de su experiencia con los idiomas. Le pedimos que elija la mejor opción.

8. ¿Qué idioma (s) considera usted como su idioma nativo?

Inglés: _____ Español: _____ Inglés y Español: _____ Otros (responder P.8a)

8a. Si su idioma nativo no es Inglés / Español, díganos cuál es su idioma nativo?

9. ¿Cuál es su segundo idioma?

Inglés: _____ Español: _____ Otros (responder Q9a) No aplicable : _____

9a. Si usted no considera Inglés / español como su segunda idioma, que idioma considera usted?

10. ¿Habla algún idioma adicional?

Sí: _____ No: _____

10a. ¿Qué idioma adicional (s) habla usted?

11. ¿Qué idioma (s) utiliza usted más con sus estudiantes?

Inglés: _____ Español: _____ Inglés y Español: _____ Otros (responder Q11a)

11a. ¿Qué otro idioma (s) utiliza usted más con sus estudiantes?

12. ¿Qué idioma (s) usa usted con más fuera del aula?

Inglés: _____ Español: _____ Inglés y Español: _____ Otros (responder Q12a)

12a. ¿Qué otro idioma (s) utiliza usted más fuera del aula?

Parte III. Por favor identifique las siguientes declaraciones en una escala: Nunca, Rara vez, frecuentemente, muy frecuentemente, siempre

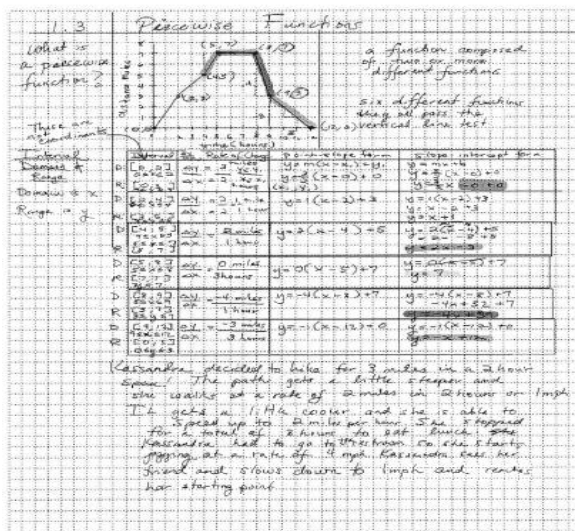
13. Cuando usted habla con otros profesores, el director y el personal sobre el desarrollo del plan de la lección, ¿con qué frecuencia utiliza su lengua materna?

Nunca: _____ Rara: _____ frecuentes: _____ Muy frecuentes: _____ Siempre: _____

14. Cuando usted habla con otros profesores, el director y el personal sobre el desarrollo del plan de la lección, ¿con qué frecuencia utiliza el segundo idioma?

Nunca: _____ Rara: _____ frecuentes: _____ Muy frecuentes: _____ Siempre: _____

Appendix H Esteban's Lesson Plan



1) Between which interval is Karina traveling the fastest?

She is moving the fastest between the $[8, 9]$ or 8th and 9th hour.

Q) Between which interval is time traveling the slowest.

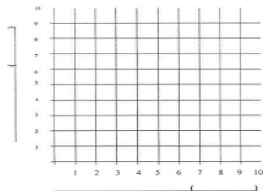
Step 2: moving the zeroes to $[2, 4]$,
 $[9, 10]$

3) During the trip, did she ever shop around?
Yes, between [5 & 7]

4.) What is the area underneath the function (Curve) for the interval $[0, 12]$?

Name _____ Date _____ Period _____

Create a piece-wise function and write a story that corresponds to the graph. (Must have minimum of 6 functions)

[illegible]

Appendix I Virginia's Lesson Plan

Segments _____ Name _____ Date _____ Period _____

SHOW ALL WORK IN THE SPACE PROVIDED.

1. Use a compass and straightedge to construct segment XY congruent to segment UV .

2. Use a compass and straight edge to construct the midpoint of segment UV .

4. Write the equation you would use to find a . Find VX and XY .

5. Write the equation you would use to find y . Solve. Find ST and SU .

6. P is the midpoint of NQ . Write the equation you would use to find x . Solve. Find NP , PQ and NQ .

For #5-6 H is between I and J . For all problems below, draw and label a diagram to fit each problem.

8. $HI = 3.9$ and $HJ = 6.2$. Find IJ .

7. $JI = 25$ and $IH = 13$. Find HJ .

8. H is the midpoint of IJ , and $HI = 0.75$. Find IJ .

9. H is the midpoint of IJ , and $IJ = 9.4$. Find IH .

10. Y is between X and Z . Find the value of x and YZ if $XY = 12$, $YZ = 4x$ and $XZ = 20$.

11. B is between A and C and B is the midpoint of AC . Find AB if $AC = 18$.

12. D is the midpoint of PQ . Find x if $PQ = 40$ and $PD = 2x + 6$.

13. L is between K and M . Find x , KL , LM and KM if $KL = 4x + 5$, $LM = 9 - 6x$, $KL = 30$ and $KM = 3x - 7$.

14. A pole-vaulter uses a 15-foot-long pole. She grips the pole so that the segment below her left hand is twice the length of the segment above her left hand. Her right hand grips the pole 1.5 feet above her left hand. How far up the pole is her right hand? Draw and label a diagram to fit the problem then solve.

15. On a subway route, station C is located at the midpoint between stations A and D . Station B is located at the midpoint between stations A and C . If the distance between stations A and D is 2.4 kilometers, what is the distance between stations B and D ? Draw and label a diagram.

Defined terms - All other terms in geometry must be definable and a definition includes a category and then a list of critical attributes.

TERM	DESCRIPTION	NAMING	SYMBOLIC REPRESENTATION
Space			
Collinear			
Conditional statement			
Coplanar			
Conditional statement			

Parallel lines			
Conditional statement			
Perpendicular			
Conditional statement			
Skew lines			
Conditional statement			

Angles - terms, theorems, postulates 09-1113

TERM	DESCRIPTION	NAMING	SYMBOLIC REPRESENTATION
Angle	Definition		$\angle A$ $\angle BAC$
Measurement of an angle			$\angle H$ $\angle DHE$
Acute angle			
Conditional statement			
Right angle			$\angle S$ $\angle USV$
Conditional statement			

For each definition, teacher ask for conditional statement

Obtuse angle		None at this time. Right is obtuse. Right is obtuse.
Conditional statement		$90^\circ < \angle < 180^\circ$
Congruent Angles		
Conditional statement	Two angles have the same measurement, then are congruent angles	
Angle Bisector		
Conditional statement	Students construct the conditional statement according to the definition	

Vertical angles				Complementary			
Conditional statement				Conditional statement			
Vertical angle Theorem	9/11/13 Hado equi.			Congruent Complements Theorem			
Linear Pair				Supplementary			
Conditional statement				Conditional statement			
Linear Pair Theorem				Congruent Supplements Theorem			

Foundations Independent Practice Name _____ Date _____ period _____

Use proper notation and figure #1 to answer each of the following questions.

1. Name two different sets of coplanar points.

Set 1:

Set 2:

2. Name two different sets of non-coplanar points.

Set 1:

Set 2:

3. Name a pair of parallel lines.

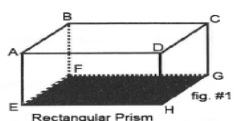
4. Name a pair of perpendicular lines.

5. Name a pair of skew lines.

6. Name the intersection of plane ABC and plane AEF.

7. Name the intersection of \overleftrightarrow{DH} and plane EFG.

8. Name the intersection of \overleftrightarrow{AB} and \overleftrightarrow{EA} .



9. Are E, R, and C coplanar? Explain.

10. Are A and G collinear? Explain.

Use figure #2 to answer each of the following. Write your answers using correct notation and in complete sentences.

11. Are AB and BE coplanar? Explain.

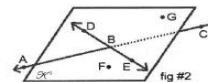
12. Are DB and F coplanar? Explain.

13. Are FE and BG coplanar? Explain.

14. How many lines contain points D and C? Explain.

15. How many planes contain points D, B and E? Explain.

16. How many planes contain points A, G and C? Explain.



Foundations Guided Practice Name _____ Date _____ period _____

1. Circle each set of coplanar points.

A, B, H, G A, C, G, E A, B, D, G
A, D, F, G E, H, G, C D, C, E, F

2. Name a plane parallel to plane ABD.

3. Circle each pair of parallel lines.

\overleftrightarrow{AB} and \overleftrightarrow{DC} \overleftrightarrow{CG} and \overleftrightarrow{FE} \overleftrightarrow{EH} and \overleftrightarrow{BF} \overleftrightarrow{AD} and \overleftrightarrow{FG} \overleftrightarrow{AB} and \overleftrightarrow{DH} \overleftrightarrow{BC} and \overleftrightarrow{CD}

4. Circle each pair of perpendicular lines.

\overleftrightarrow{AB} and \overleftrightarrow{DC} \overleftrightarrow{CG} and \overleftrightarrow{FE} \overleftrightarrow{EH} and \overleftrightarrow{BF} \overleftrightarrow{AD} and \overleftrightarrow{FG} \overleftrightarrow{AB} and \overleftrightarrow{DH} \overleftrightarrow{BC} and \overleftrightarrow{CD}

5. Circle each pair of skew lines.

\overleftrightarrow{AB} and \overleftrightarrow{DC} \overleftrightarrow{CG} and \overleftrightarrow{FE} \overleftrightarrow{EH} and \overleftrightarrow{BF} \overleftrightarrow{AD} and \overleftrightarrow{FG} \overleftrightarrow{AB} and \overleftrightarrow{DH} \overleftrightarrow{BC} and \overleftrightarrow{CD}

6. Circle each set of collinear points.

A, E, F A, K, P D, P, C K, C, B G, F, R

Use proper notation to name, identify each of the following:

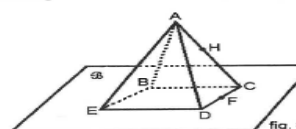
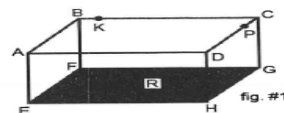
7. Four coplanar points

8. Two skew lines

9. Two parallel segments

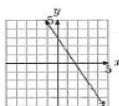
10. A line containing point C.

11. Three collinear points.

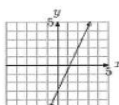


Appendix J Carlos' Lesson Plan

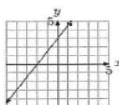
Draw the inverse of the following graphs. Hint: Pick three points in the graph, invert them and draw the new graph.



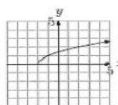
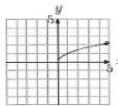
30.



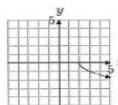
31.



34.



35.



Find the inverse of the following functions (don't work with the rational functions). Is the inverse a function? Sketch the graphs reflected across the line $y=x$.

$$f(x) = 5x^3 - 5$$

$$f(x) = 4x^7 - 3$$

$$f(x) = \frac{9x-3}{7x+6}$$

$$f(x) = 6x - 4$$

$$f(x) = 7x - 9$$

$$f(x) = 7x + 4$$

$$f(x) = 3x^5 - 9$$

$$f(x) = 6x + 7$$

$$f(x) = \frac{4x+2}{4x+3}$$

$$f(x) = 5x^7 + 4$$

$$f(x) = \frac{4x-1}{2x+2}$$

$$f(x) = \sqrt[3]{8x-3}$$

$$f(x) = \sqrt[3]{-6x-4}$$

$$f(x) = \frac{8x-7}{3x-6}$$

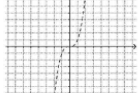
$$f(x) = \sqrt[3]{-3x-5}$$

Solving Equations (pp. 2 of 4)

Use your knowledge of transformations of parent functions to sketch the graph of each function below. Describe the transformations used.

1) Parent: $y = x^3$ (dotted)

Function: $f(x) = -(x+2)^3 + 4$



Transformations:

-
-
-

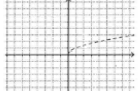
Equation: $f(x) = 12$

$$-(x+2)^3 + 4 = 12$$

Solution:

2) Parent: $y = \sqrt{x}$ (dotted)

Function: $f(x) = 2\sqrt{-x} - 5$



Transformations:

-
-
-

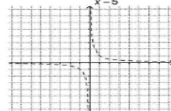
Equation: $f(x) = 3$

$$2\sqrt{-x} - 5 = 3$$

Solution:

3) Parent: $y = \frac{1}{x}$ (dotted)

Function: $f(x) = \frac{4}{x-5} + 2$



Transformations:

-
-
-

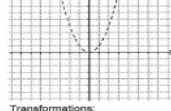
Equation: $f(x) = 10$

$$\frac{4}{x-5} + 2 = 10$$

Solution:

4) Parent: $y = x^2$ (dotted)

Function: $f(x) = \frac{1}{2}(x+3)^2 - 4$



Transformations:

-
-
-

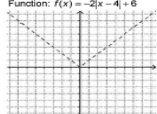
Equation: $f(x) = 12$

$$\frac{1}{2}(x+3)^2 - 4 = 12$$

Solution:

Solving Equations (pp. 3 of 4)

- 5) Parent: $y = |x|$ (dotted)
Function: $f(x) = -2|x - 4| + 6$



Transformations:

-
-
-
-

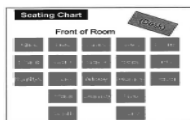
Equation: $f(x) = 1$
 $-2|x - 4| + 6 = 1$

Solution:

Questions:

- 6) Explain how the equation-solving processes for quadratic and absolute value functions (#4 and #5) are different from the techniques used on the other equations.
- 7) What property do the quadratic and absolute value functions possess (or lack) that would require the use of such different equation-solving processes?

Seating Chart (pp. 1 of 2)



Mrs. Ramsey has 20 students in her Precalculus class, as shown in the seating chart. Often, the students do activities where they must work with classmates who sit around them. To help with the instructions for these activities, Mrs. Ramsey developed the following function rules.

If x = a student in the class, then:

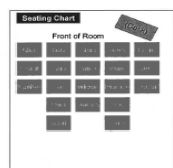
- $l(x)$ = the student who sits directly to the left of x .
- $r(x)$ = the student who sits directly to the right of x .
- $b(x)$ = the student who sits directly behind x .
- $f(x)$ = the student who sits directly in front of x .
- $y(x)$ = the student who sits at the front of x 's row.

Use the functions to evaluate each expression, answer each question, or solve the given equation.

- 1) $l(\text{Quincy}) =$ _____ 5) $r(\text{Clara}) =$ _____
2) $r(\text{Missy}) =$ _____ 6) $f(\text{Tara}) =$ _____
3) $b(\text{Bob}) =$ _____ 7) $f(\text{Bob}) =$ _____
4) $y(\text{Oscar}) =$ _____ 8) $y(\text{Dave}) =$ _____
9) Which students would not be in the domain of $f(x)$?
10) Which students would not be in the domain of $l(x)$?

- 11) If $l(x)$ = Harry, then $x =$ _____ 13) If $b(x)$ = Rob, then $x =$ _____
12) If $f(x)$ = Lori, then $x =$ _____ 14) If $y(x)$ = Alice, then what could x be?

Seating Chart (pp. 2 of 2)



If x = a student in the class, then:

- $l(x)$ = the student who sits directly to the left of x .
- $r(x)$ = the student who sits directly to the right of x .
- $b(x)$ = the student who sits directly behind x .
- $f(x)$ = the student who sits directly in front of x .
- $y(x)$ = the student who sits at the front of x 's row.

These functions can also be combined. For example:

- $l(b(x))$ This reads the student who sits to the left of the person who sits behind x which means the student who sits directly behind x and finding the student to the left of that person.
- $f(f(x))$ This reads the student who sits in front of the person who sits in front of x which means the student who sits in front of x and then finding the person who sits in front of that person.

...And so on.

- 15) $l(b(\text{Clara})) =$ _____ 18) $f(\text{Oscar}) =$ _____
16) $r(f(\text{Missy})) =$ _____ 19) $b(\text{Oscar}) =$ _____
17) $b(b(\text{Bob})) =$ _____ 20) $l(b(\text{Oscar})) =$ _____
21) Solve the equation $r(f(x)) = \text{Nancy}$, and explain how you found the solution.
22) Solve the equation $f(f(x)) = \text{Bob}$, and explain how you found the solution.
23) $f(b(\text{Paul})) =$ _____ 26) Make a general statement about the values of any expression of the form $f(b(x))$.
24) $f(b(\text{Jeff})) =$ _____
25) $f(b(\text{Dave})) =$ _____ 27) For which students would this generalization not be true? Why?

Verifying Inverses

If $f(x)$ and $f^{-1}(x)$ are inverse functions, then
 $f^{-1}(f(x)) = x$ for all x in the domain of $f(x)$
 $f(f^{-1}(x)) = x$ for all x in the domain of $f^{-1}(x)$
 $f(x)$ and $f^{-1}(x)$ are symmetrical to the line $y=x$

♦Verify algebraically which functions are inverses.

1. $f(x) = \sqrt{x+2}$ and $g(x) = x^2 - 2, x \geq 0$

2. $f(x) = \frac{x+1}{x}$ and $g(x) = \frac{1}{x+1}$

3. $f(x) = \frac{x-1}{x}$ and $g(x) = \frac{1}{1-x}$

Appendix K Angelica's Lesson Plan

Para el examen Factorización

Los principales casos de descomposición en factores

1. Cuando el factor común es un monomio.
2. Cuando el factor común es un polinomio.
3. Por agrupación de términos.
4. Trinomio cuadrado perfecto.
5. Diferencia de cuadrados.
6. Suma o diferencia de cubos.
7. Trinomios de la forma $x^2 + bx + c$.
8. Trinomios de la forma $ax^2 + bx + c$.

Esto para estudiar

84

En álgebra existen algunos casos de descomposición en factores que se pueden hacer rápidamente. Algunos de estos casos son:

Factor común:	$am + an = a(m + n)$
Agrupación:	$am + an + bm + bn = a(m + n) + b(m + n)$ $= (a + b)(m + n)$
Trinomio cuadrado perfecto:	$x^2 \pm 2xy + y^2 = (x \pm y)^2$
Diferencia de cuadrados:	$x^2 - y^2 = (x + y)(x - y)$
Suma o diferencia de cubos:	$x^3 \pm y^3 = (x \pm y)(x^2 \mp xy + y^2)$
Por término común:	$x^2 + (m + n)x + mn = (x + m)(x + n)$
Por términos semejantes:	$ax^2 + (ad + bc)xy + bd^2 = (ax + by)(cx + dy)$

Formulario

98

Cuando el factor común es un monomio

$$xy + xz = x(y + z)$$

Actividades de aprendizaje

Descompon en factores las siguientes expresiones.

1. $10x^2$
2. $30x^3$
3. $4x^2 + 20x$
4. $3x^2 - 3x^2$
5. $4x^3 - 20x^2 + 12x$
6. $3x^3 + 9x^2 - 24x$
7. $5x^2 + 10x - 20$
8. $mx^2 + mp - mb$
9. $xy + xz$
10. $8m^2 + 12mn$
11. $8ab - 20a$
12. $32xyz + 48mxy$
13. $m^3n - m^2p + mg$
14. $xy - x^2 + x$
15. $abcd - abc + acm$

UNIDAD 1 | lenguaje algebraico

DESCOMPOSICIÓN EN FACTORES

Descomponer en factores significa expresar un número como multiplicación de números primos. Este proceso ya lo has realizado muchas veces por ejemplo, cuando has tenido que buscar un denominador común en las operaciones con fracciones comunes. ~~Volvamos a pensar en los factores para factorizar los siguientes polinomios.~~

$$\begin{aligned} 8 &= 2 \times 2 \times 2 \\ 12 &= 2 \times 2 \times 3 \\ 40 &= 2 \times 2 \times 3 \times 5 \\ 6 &= 2 \times 3 \\ 20 &= 2 \times 2 \times 5 \end{aligned}$$

Ejemplos

1. Descompon en factores la siguiente adición.

$$15 + 6 = (3)(5) + (2)(3) = 3(5 + 2)$$

2. Descompon en factores la siguiente sustracción.

$$28 - 12 = (4)(7) - (4)(3) = 4(7 - 3)$$

En álgebra, aunque el proceso de descomposición de factores es parecido, existen casos notables que se pueden identificar para simplificar el proceso. Si se tiene que descomponer en factores un monomio, es fácil hacerlo.

$$1. m^3 = m \cdot m \cdot m$$

$$2. 3x^2 = 3(x)(x)$$

$$3. 15m^2n^3 = (3)(5)(m)(m)(n)(n)(n)$$

Se descompone en factores la parte numérica y la parte literal.

Uno de los objetivos de descomponer en factores un polinomio es buscar que no haya signos + o - fuera de los paréntesis.

4. Descompon en factores las siguientes adiciones.

$$\begin{aligned} 8 + 12 &= (2 \times 2 \times 2) + (2 \times 2 \times 3) \\ &= (2 \times 2)(2 + 3) \\ &= 4(2 + 3) \end{aligned}$$

El factor 2 se repite 2 veces en el 8 y 2 veces en el 12, por lo que a 2×2 se le llama factor común.

Cuando el factor común es un polinomio

$$x(a+b) - y(a+b) = (a+b)(x-y)$$

Descomposición en factores por agrupación de términos

$$mx + ms + nx + ns = (m+n)(x+n)$$

Actividades de aprendizaje

Descompon en factores los siguientes polinomios.

1. $9ac + 15ad + 12bc + 20bd$
2. $24x^2 + 40xy + 9y + 15y^2$
3. $12xw + 15xz + 8wy + 10yz$
4. $mx + nr + pn + pr$
5. $ad - ap - bd + bp$
6. $xy + gy + xp + gp$
7. $4xy + 8mx + 6y + 12mn$
8. $20am + 15an + 12bm + 9bn$
9. $15am + 10mb + 12an + 8bn$
10. $16mx + 40nx + 8xy + 20ny$
11. $6am + 15an + 8bm + 20bn$
12. $10mx + 5nx + 8xy + 4ny$
13. $9am + 6an + 15bm + 10bn$
14. $16x^3 + 56xy^2 + 12x^2y^2 + 42y^3$
15. $6am^2 + 9an + 10bm^2 + 15bn$

Appendix L Yolanda's Lesson Plan

Yolanda Duarte Ortega

Unidad #3

Productos Notables

Copiar lo de productos notables de la pág 65
Pasar copia de la pág 65 al final del cuaderno.

Cuadrado de un Binomio

$$(a+b)^2 = a^2 + 2ab + b^2 \quad (1)$$

$$(a-b)^2 = a^2 - 2ab + b^2 \quad (2)$$

De donde zona la fórmula:

$$(a+b)^2 = (a+b)(a+b) \quad (a-b)^2 = (a-b)(a-b)$$

$$\begin{array}{r} a+b \\ a+b \\ \hline a^2+ab \\ ab+b^2 \\ \hline a^2+2ab+b^2 \end{array} \quad (1)$$

$$\begin{array}{r} a-b \\ a-b \\ \hline a^2-ab \\ -ab+b^2 \\ \hline a^2-2ab+b^2 \end{array} \quad (2)$$

Ejemplos

$$1 \quad (x+y)^2 = x^2 + 2xy + y^2$$

$$2 \quad (p-q)^2 = p^2 - 2pq + q^2$$

$$3 \quad (3x+2y)^2 = (3x)^2 + 2(3x)(2y) + (2y)^2 \\ = 9x^2 + 12xy + 4y^2$$

$$4 \quad (a-b)^2 = a^2 - 2ab + b^2 \\ (5a-3b)^2 = (5a)^2 - 2(5a)(3b) + (3b)^2 \\ = 25a^2 - 30ab + 9b^2$$

$$5 \quad (a+b)^2 = a^2 + 2ab + b^2 \\ (2x^3+3y^4)^2 = (2x^3)^2 + 2(2x^3)(3y^4) + (3y^4)^2 \\ = 4x^6 + 12x^3y^4 + 9y^8$$

$$6 \quad (a-b)^2 = a^2 - 2ab + b^2 \\ (3ab^2-5m^3n)^2 = (3ab^2)^2 - 2(3ab^2)(5m^3n) + (5m^3n)^2 \\ = 9a^2b^4 - 30ab^2m^3n + 25m^6n^2$$

Actividad para el Alumno

resolver en el libro la pág 70 Actividad 4 toda la tabla

Binomios Conjugados

$$(a+b)(a-b) = a^2 - b^2 \quad (3)$$

$$\begin{array}{r} a+b \\ a-b \\ \hline a^2-ab \\ -ab-b^2 \\ \hline a^2-b^2 \end{array} \quad (3)$$

Ejemplos

$$1 \quad (a+b)(a-b) = a^2 - b^2 \\ (2a+5b)(2a-5b) = (2a)^2 - (5b)^2 \\ = 4a^2 - 25b^2$$

$$2 \quad (a+b)(a-b) = a^2 - b^2 \\ (2x^3+3y^4)(2x^3-3y^4) = (2x^3)^2 - (3y^4)^2 \\ = 4x^6 - 9y^8$$

Trabajo para el alumno

Resolver en el libro pag 72 y 73

Actividad 1 de la "a" a la "o"

$$3 \quad (a+b+c)(a+b-c) \\ ((a+b)+c)((a+b)-c) = (a+b)^2 - (c)^2 \\ = a^2 + 2ab + b^2 - c^2$$

$$4 \quad (2x+3y+z)(2x+3y-z) \\ ((2x+3y)+z)((2x+3y)-z) = (2x+3y)^2 - z^2 \\ = (2x)^2 + 2(2x)(3y) + (3y)^2 - z^2 \\ = 4x^2 + 12xy + 9y^2 - z^2$$

$$5 \quad [(x+y)+(m+n)][(x+y)-(m+n)] \\ (a+b)(a-b) = a^2 - b^2 \\ (a+b)^2 = a^2 + 2ab + b^2 \\ a^2 = (x+y)^2 = x^2 + 2xy + y^2 \\ b^2 = (m+n)^2 = m^2 + 2mn + n^2 \\ (x^2 + 2xy + y^2) - (m^2 + 2mn + n^2)$$

Trabajo para el Alumno

Resolver en el cuaderno la pag 73 Act 1 de la "e" a la "v"

Trabajo para el Alumno

En hojas de maquina resolver Pag 73 y 74

Actividad 2 de la "a" a la "s". Todo a mano, grapada, con portada, ejercicio, fórmula sustitución y resultado en cada ejercicio.

Cuadrado de un Trinomio

$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc \quad (4)$$

Ejemplos

$$1 \quad (a+b+c)^2 \\ (2x+3y+4z)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc \\ = (2x)^2 + (3y)^2 + (4z)^2 + 2(2x)(3y) + 2(2x)(4z) + 2(3y)(4z) \\ = 4x^2 + 9y^2 + 16z^2 + 12xy + 16xz + 24yz$$

$$2 \quad (a+b+c)^2 \rightarrow \text{Como solo tenemos la fórmula para números positivos haremos las operaciones con signos}$$

$$(2m-2n-4p)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc \\ = (2m)^2 + (-2n)^2 + (-4p)^2 + 2(2m)(-2n) + 2(2m)(-4p) + 2(-2n)(-4p) \\ = 4m^2 + 4n^2 + 16p^2 - 8mn - 16mp + 16np$$

Trabajo para el alumno

Resolver en el cuaderno pag 75 Actividad del "1" al "10"

Producto de un Binomio con un termino comun

$$(a+b)(a+c) = a^2 + a(b+c) + bc \quad \textcircled{5}$$

Ejemplos

$$\star 1 \quad \begin{array}{cc} (x+y)(x+z) & = & x^2 + x(y+z) + yz \\ \begin{array}{cc} \downarrow & \downarrow \\ a & b \end{array} & \begin{array}{cc} \downarrow & \downarrow \\ a & c \end{array} & & \end{array}$$

$$\star 2 \quad \begin{array}{cc} (x+3)(x+2) & = & x^2 + x(3+2) + (3)(2) \\ \begin{array}{cc} \downarrow & \downarrow \\ a & b \end{array} & \begin{array}{cc} \downarrow & \downarrow \\ a & c \end{array} & & \end{array}$$

$$\star 3 \quad \begin{array}{cc} (x+7)(x-2) & = & x^2 + x(7-2) + (7)(-2) \\ \begin{array}{cc} \downarrow & \downarrow \\ a & b \end{array} & \begin{array}{cc} \downarrow & \downarrow \\ a & -c \end{array} & & \end{array}$$

Trabajo para el Alumno

Resolver en el cuaderno Pág. 77 Actividades
Única del "1" al "23"

Appendix M IRB Proposal

Use this template to provide a description of your research proposal. All applications for review should contain the following information, presented in paragraphs prefaced by the number of the item and the underlined descriptive phrase. When not applicable, DO list the heading and then indicate N/A.

Please note that if this study is part of an NIH funded grant proposal, you will need to attach ONE copy of the complete grant proposal, in addition to the information requested below.

I. Title:

Borderland Pedagogy study of high school mathematics teacher's lesson plan development practices

II. Investigators:

Principia investigator. Rocio E. Gallardo

Co-PI Mouratt Tchoshanov

Team Member Monica Lizbeth Sifuentes Alfaro

III. Hypothesis, Research Questions, or Goals of the Project

The purpose of this study is about the practices that High School mathematics teachers use in El Paso and Mexico to develop their lesson plans. Also, the general educational concern is to understand what are the factors that influence the teacher's development of lessons plans in order to successfully teach it to their students. It is important to know and learn about these factors to identify if teachers are designing well thought-out and high quality lesson plans and to identify if teachers are building a solid base for classroom implementation (Li et al, 2009, p. 717). Furthermore, this qualitative research project will seek a deeper understanding of the teachers' preparations of their lesson plans in order to improve students' academic achievement. It will be a comparative study (El Paso and Mexico). I will identify two High School teachers in Mexico and four teachers in El Paso TX..

The following are the research question for the study:

(1) How does a transition from Mexico (Ciudad Juarez, Chihuahua) to the U.S. (El Paso, TX.) impacts high school mathematics teacher's lesson plan development and implementation practices incorporating the Borderland Pedagogy?

(2) What are the lesson plan development and implementation practices incorporating the Borderland Pedagogy used by U.S. high school mathematics teachers in El Paso, TX.? (3) What are the lesson plan development and implementation practices incorporating the Borderland Pedagogy used by Mexican high school mathematics teachers in Ciudad Juarez, Chihuahua?

IV. Background and Significance:

Educational researchers examine factors such as teacher's mathematics knowledge and teachers' view of effective teaching (Li et al, 2009). The development of lesson plan is conceptualized as a part of teachers' general pedagogical knowledge. This conceptualization suggests a pedagogical perspective to examine and understand lesson plan in an international context (Li et al, p. 718). As Shulman (1986) states pedagogical knowledge is an important to have successful in teaching. Teachers are critical players to consider while using a pedagogical model. Teaching requires an understanding of differences that may arise from culture, family experiences, developed intelligences, and approaches to learning. Additionally, the practice of lesson plan development would require further specifications of different systemic contexts and their nature. A curriculum perspective would also allow us to gain a better understanding of teachers' lesson planning practices in a specific context. The nature of different system contexts likely imposes different expectations for teachers to structure content for teaching (Li et al, p. 729). The present study will contribute to the identification of practices and challenges that Mexico and US mathematics teachers present to develop lesson plans. As well, this study will contribute about the Borderland Pedagogy study of high school mathematics teacher's lesson plan development practices. Cline and Necochea (2006) point out that border pedagogy posits pedagogical processes, in part, as a form of border crossing where existing social boundaries can be challenged and redefined. Borderland pedagogy has been taken up frequently, and convincingly, by scholars and educators living on the geopolitical border between the United States and Mexico (Cline & Necochea, 2006; Romo & Chavez, 2006), as a pedagogical practice that has resonances with their lived experience in borderland communities, we see a more far reaching ambit for the notion of border pedagogy.

The educational systems in the United States and Mexico are so distinct and disconnected from one another that they may as well be in two separate worlds (Cline and Necochea 2002; Reyes and Garza. 2005). Current teacher preparation and in-service training programs in the United States, even those close to la línea (the border), rarely mention the Mexican educational system, much less provide the professional development programs teachers need to be effective with borderland students especially in the areas of dispositions, attitudes, and motivation (Cline et al, 2003).

Reference:

- Cline, Z., J. Necochea, P. Prado-Olmos, and J. Halcon. (2003). The border pedagogy initiative. *Hispanic Outlook in HigherEducation* 13(9): 10–13.
- Cline, Z., & Necochea, J. (2006). Teacher Dispositions for Effective Education in the Borderlands. *Educational Forum*, 70(3), 268-282.
- Emerson, R. Fritz, R., and Shaw, L. (2011). Writing ethnographic fieldnotes. Chicago: Chicago University Press.
- Li, Y., Chen, X., and Kulm, G. (2009). Mathematics teachers' practices and thinking in lesson plan development: a case of teaching fraction division. *Mathematics Education*, (6), 41, 717-731.

- Reyes, M., and Garza, E.. (2005). Teachers on the border: In their own words. *Journal of Latinos and Education* 4(3), 153–70.
- Romo, J. J., & Chavez, C. (2006). Border Pedagogy: A Study of Pre-service Teacher Transformation. *Educational Forum*, 70(2), 142-153.
- Shulman, L. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, (2), 15, 4-14.

V. Research Method, Design, and Proposed Statistical Analysis:

The study will implement a qualitative comparative study design in order to develop a clearer understanding about the factors influencing practice the Mexico and US High School teachers use to develop lesson plans.

Data sources will include the following: (1) interviews with mathematics teachers; (2) classroom observations protocol; (3) field notes which will be made on the content of the lessons and teacher behaviors and interactions that occurred during each lesson observed (4) video recording of mathematics classes (5) survey. The data collected will be used to not only understand these factors but will also be used to inform teachers on lesson plan development practices.

First, once the high schools are selected in Mexico, an appointment will be set up with the principals of the schools. Team member Ms. Monica Lizbeth Sifuentes will be responsible to contact high schools in Mexico and interview the participants since she is Mexican Resident. Later, an interview will take place. Second, in El Paso TX. the PI will be responsible to contact high school in El Paso TX. and interview the participants. Additionally, this qualitative study will compare three groups of mathematics teachers. First group is formed for two Mexican mathematics' teachers who teach in Juarez City. Second group is formed for two mathematics teachers who teach in El Paso TX.. Third group is formed for two mathematics teachers who came from Juarez to El Paso to teach in El Paso TX.. Moreover, 6 mathematics teachers will be invited to participate in the study. As a mention before, the team member will be responsible to contact the mathematics teachers who teach in Juarez City.

The qualitative data will be fully transcribed and then analyzed in order to codify pattern and themes related to the research questions. Also, an open and closed approach to coding (Emerson et al, 2011) will be used in order to gain a broader picture of the factors that influence on the teachers' preparation on lesson plans. Data related will be compiled and analyzed using NVIVO program in rank to determine the success the open codes related to research questions.

VI. Human Subject Interactions

This project will involve 6 High School mathematics teachers. 2 teachers from Mexico and 4 El Paso. Participants will be recruited and selected according to the purpose of the study. The participants will be invited to participate on this project and it will be entirely voluntary. The selection of the sample will consider the experiences of teachers in teaching mathematics.

Some of the parameters that I will use for the observations are: the teachers' preparation of their lesson plan to teach geometry and trigonometry class. Some teachers participants may be fluent in English or Spanish language, so the consent forms will be presented in two languages.

B. Describe the procedures for the recruitment of the participants are the following:

Mathematics teachers will be surveyed about their perceptions and beliefs to develop their lesson plan in order that students have good academic achievement. The researcher will select 6 mathematics' teachers who teach High School Geometry. The researcher will conduct a visit to the High School to ask for permission to approach teachers and be able to talk to them about how they prepare their lesson plans to teach geometry class. In the Spring semester of school year (January-May, 2013) teachers will meet with the investigator and Co-Investigator to review and seem the consent form. The potential participants are not considered members of a group that may be construed as stigmatized, so my contact with participants will not be done through flyers or advertisements.

C. Describe the procedure for obtaining informed consent.

Within the first meeting in which the researcher will introduce herself to mathematics teachers, the consent form will be given to them and the researcher will explain each part of the consent form and will clarify any doubts or concerns that the teachers may have.

The consent form will include the following information: purpose of the research; time involved in completing the research; risks and benefits to participation; safeguards for protecting data; protection of confidentiality of participants'. Also the consent form will inform that the participation is voluntary.

D. Research Protocol. The qualitative comparative case study will take place over a period of (one year).

Participants in the study will engage in the following activities:

(1) Interviews

Individual teacher interviews will last approximately 30-40 minutes and will be conducted as a semi-structured interview. The interviews will take place at High School or at the convenience location for the participant. It is expected that the entire sample of participants will be interviewed. In the interview the participants will be asked for permission to audio record the interviews.

(2) Observations

The investigator will conduct classroom observations. The observations will be guided by the research questions of the project. There will be 2-3 observations per teacher.

(3) Field-notes

Field-notes will be made on the content of the lessons and teacher behaviors and interactions that occurred during each lesson observed.

(4) Video recording of class sections

Class sessions will be video recorded. However only, data from the participants who consent being videotaped will be utilized.

E. How will you protect the **privacy and confidentiality of participants**? The participation of the participants in this study is confidential. None of the information will be identified by teacher participant name. All records will be coded to maintain anonymity. The surveys will be accessible to the research team during the time of this study, as well as follow-up studies to be generated as a result of data analysis. The interviews, field notes will be permanently deleted or destroyed when all studies are completed. All records shall be maintained by assigning a number code for each teacher. Teachers' identities will remain anonymous.

F. Discuss the procedures that will be used to maintain the **confidentiality of the research data**.

All the recordings will be stored in a locker of Teacher Education Department at UTEP where officers of investigators are located. Electronic versions will be stored on the researcher's laptop and will be password protected. The researcher will be the only person with access to it. The researcher will use this information to perform data analysis and she will retain it after the study is complete for purposes of further publications. However, the confidentiality will not be affected since the teacher names will be changed.

G. Please describe your research resources. The researcher has a laptop, an office space in ED 800, with access to printing.

VII. Describe any **potential risks** (physical, psychological, social, legal, or other).

The possible risk associated with this research is minimal. The researcher will communicate to the participants that the information will be used only to gain a deeper understanding of lesson plan development practices and it will not be shared with schools and school staff to avoid conflict or confrontation. The researcher will do everything possible to keep only the information confidential.

The results of this research may be presented at meetings or in publications, however, the identity of participants will not be revealed.

VIII. Describe and assess the **potential benefits**.

There will be no direct benefits for taking part in this study. However, this research may help us to better understand the teachers' practices to develop their lesson plans. In addition, the comparison between mathematics teachers' practices in Mexico and US will contribute to the field of mathematics education research.

IX. Indicate the specific **sites or agencies involved in the research**

The US participants are High School mathematics teachers at local public school; the Mexican participants are employed as High School teachers in Ciudad Juarez.

X. The project has not had or will not receive **review by another IRB**. N/A

Appendix N Consent Form

University of Texas at El Paso (UTEP) Institutional Review Board Informed Consent Form for Research Involving Human Subjects

Protocol Title: Borderland Pedagogy study of high school mathematics teacher's lesson plan development practices

Principal Investigator: Rocio Gallardo

Co-Pi: Dr. Mourat Tchoshanov

Team Member: Monica Lizbeth Sifuentes

UTEP College of Education: Teacher Education

In this consent form, “you” always means the study subject. If you are a legally authorized representative (such as a parent or guardian), please remember that “you” refers to the study subject.

1. Introduction

You are being asked to take part voluntarily in the research project described below. Please take your time making a decision and feel free to discuss it with your friends and family. Before agreeing to take part in this research study, it is important that you read the consent form that describes the study. Please ask the study researcher or the study staff to explain any words or information that you do not clearly understand.

2. Why is this study being done?

You have been asked to participate in a research study on Borderland Pedagogy study of high school mathematics teacher's lesson plan development practices. This project will involve 6 mathematics teachers. 2 teachers enrolled at Mexico and 4 mathematics teachers enrolled at El

Paso. Participants will be recruited and selected according to the purpose of the study. If you decide to enroll in this study, your involvement for one year.

3. What is involved in the study?

If you agree to participate in this study, the research team will provide an analysis of the interviews and observations and field notes taken during class. The interviews will last approximately 30-45 minutes.

4. What are the risks and discomforts of the study?

There are no known risks associated with this research

5. What will happen if I am injured in this study?

Not applicable

6. Are there benefits to taking part in this study?

There will be no direct benefits to you for participating in this study. This research may help us understand what are the practices of High School mathematics teachers use in El Paso and Mexico to develop their lesson plans.

7. What other options are there?

You have the option not to take part in this study. There will be no penalties involved if you choose not to take part in this study.

8. Who is paying for this study?

Internal In-kind Funding: Funding for this study is provided by UTEP, Department of Education in the form of in-kind services, such as, copies.

9. What are my costs?

There are no direct costs.

10. Will I be paid to participate in this study?

You will not be paid for taking part in this study.

11. What if I want to withdraw, or am asked to withdraw from this study?

Taking part in this study is voluntary. You have the right to choose not to take part in this study.

If you do not take part in the study, there will be no penalty.

If you choose to take part, you have the right to stop at any time. However, we encourage you to talk to a member of the research group so that they know why you are leaving the study. If there are any new findings during the study that may affect whether you want to continue to take part, you will be told about them.

The researcher may decide to stop your participation without your permission, if he or she thinks that being in the study may cause you harm.

12. Who do I call if I have questions or problems?

You may ask any questions you have now. If you have questions later, you may call Rocio Gallardo regallardo@miners.utep.edu or (915)255-5261

If you have questions or concerns about your participation as a research subject, please contact the UTEP Institutional Review Board (IRB) at (915-747-8841) or irb.orsp@utep.edu.

13. What about confidentiality?

Your part in this study is confidential. None of the information will identify you by name. All records will be coded to maintain anonymity. Surveys will be accessible to the research team during the time of this study, as well as, follow-up studies that may be generated as a result of data analysis. Surveys will be permanently deleted or destroyed when all studies are completed.

All records will be kept by assigning a number to each teacher. Every teacher's identities will be kept anonymous.

14. Mandatory reporting

If information is revealed about child abuse or neglect, or potentially dangerous future behavior to others, the law requires that this information be reported to the proper authorities.

15. Authorization Statement

I have read each page of this paper about the study (or it was read to me). I know that being in this study is voluntary and I choose to be in this study. I know I can stop being in this study without penalty. I will get a copy of this consent form now and can get information on results of the study later if I wish.

Participant Name: _____ Date: _____

Participant Signature: _____ Time: _____

Consent form explained/witnessed by: _____

Printed name: _____

Date: _____ Time: _____

Universidad de Texas en El Paso (UTEP) Revisión Institucional

Formulario de Consentimiento para Investigación en Seres Humanos

Titulo del Protocolo: Estudio de las practicas Pedagógicas de la Frontera sobre el desarrollo de los planes de estudio a nivel preparatoria.

Principal Investigador: Rocio Gallardo

UTEP, Colegio de Educación

En este formulario de consentimiento “usted” es sometido al estudio. Si usted es un representante legalmente autorizado (por ejemplo, un padre o tutor), recuerde que "usted" es participante del estudio.

1. Introducción

Usted está siendo invitado a participar voluntariamente en el proyecto de investigación que se describe a continuación. Por favor, tómese su tiempo en tomar una decisión y no dude en comentarlo con sus amigos y familiares. Antes de aceptar en participar en este estudio, es importante que usted lea el formulario de consentimiento que describe el estudio. Por favor, pregunte al investigador principal del estudio o al personal del estudio para que le explique cualquier palabra o información que no se entienda claramente.

2. Porque se está realizando este estudio?

Se le ha pedido a participar en un estudio de investigación sobre un estudio sobre las prácticas que utilizan los maestros de matemáticas para la elaboración del plan de estudio.

“Aproximadamente, 6 maestros de matemáticas participaran en este estudio: 2 en Ciudad Juárez y 4 en El Paso Texas. Los participantes tendrán que responder una serie de preguntas sobre las prácticas que utilizan para la elaboración de los planes de estudio en la enseñanza de la geometría. Si usted decide inscribirse en este estudio, su participación tendrá una duración de 1 año.

3. En qué consiste el estudio?

Si usted acepta participar en este estudio, el equipo de investigación dará un análisis de las entrevistas y de las observaciones y de las notas de campo tomadas durante la clase. Las entrevistas tendrán una duración aproximada de 30-45 minutos.

4. Cuáles son los riesgos y molestias del estudio?

No hay riesgos relacionados con la investigación

5. Que sucederá si me lesión durante el estudio?

No aplica

6. Hay beneficios por participar en el estudio?

No habrá beneficios directos para usted por participar en este estudio. Esta investigación puede ayudarnos a entender cuáles son las prácticas que utilizan los maestros de matemáticas para la elaboración de los planes de estudio para la enseñanza de la geometría

7. Que otras opciones hay?

Usted tiene la opción de no ser parte del estudio. No habrá sanciones involucrados si decide no tomar parte del estudio

8. Quien paga por este trabajo?

Internos en especie Financiación: La financiación de este estudio es proporcionado por UTEP, el Departamento de Educación en forma de servicios en especie, tales como copias.

9. Cuáles son mis costos?

No hay costos directos

10. Me van a pagar para participar en este estudio?

No se le pagará por participar en este estudio.

11. ¿Qué pasa si quiero retirarme, o si me piden que me retire de este estudio?

La participación en este estudio es voluntaria. Usted tiene el derecho de optar por no participar en este estudio. Si usted no toma parte en el estudio, no habrá penalización.

Si usted decide participar, usted tiene el derecho de detenerse en cualquier momento. Sin embargo, le animamos a que hable con un miembro del grupo de investigación para que sepan por qué usted está dejando el estudio. Si hay algunos nuevos hallazgos en el estudio que pueden afectar puede dejar de seguir participando en el estudio

El investigador puede decidir terminar su participación sin su permiso, si él o ella piensan que en el estudio puede causar daño

12. A quien llamo si tengo preguntas o problemas?

Usted puede hacer cualquier pregunta que usted tenga ahora. Si usted tiene preguntas posteriores, puede llamar a Rocio Gallardo. regallardo@miners.utep.edu o al teléfono: (915) 255-5261

Si usted tiene preguntas o inquietudes acerca de su participación como participante en la investigación, por favor póngase en contacto con la Junta de UTEP Revisión Institucional (IRB) al (915-747-8841) o irborsp@utep.edu.

13. ¿Qué pasa con la confidencialidad?

Su participación en este estudio es confidencial. Ninguna de la información se le identifica por nombre. Todos los registros serán codificados para mantener el anonimato. Las encuestas serán accesibles para el equipo de investigación durante el tiempo de este estudio, así como, los estudios de seguimiento que se generen como consecuencia del análisis de datos. Las encuestas serán permanentemente borradas o destruidas cuando todos los estudios se hayan completado. Todos los registros se mantendrán asignando a un número por cada maestro. Las identidades de los maestros se mantendrán anónimas.

14. La notificación mandatorio

Si la información se revela sobre el abuso infantil o negligencia, o el comportamiento futuro potencialmente peligroso para otros, la ley exige que esta información sea informada a las autoridades competentes.

15. Declaración de autorización

He leído cada página de este documento sobre el estudio (o se lee para mí). Yo sé que este estudio es voluntario y puedo optar por participar en este estudio. Sé que puedo dejar de estar en este estudio sin penalización. Voy a obtener una copia de este formulario de consentimiento y ahora sé que puedo obtener información sobre los resultados del estudio más adelante si lo deseo

Nombre del Participante: _____ Fecha: _____

Firma del Participante: _____ Tiempo: _____

Formulario de consentimiento explica / presenciada por_____

Signature

Nombre impreso: _____

Date: _____ Time: _____

Vita

Rocio E. Gallardo earned his Bachelor degree with a Major in Mathematics and a minor in Physics from the University of Texas at El Paso. She received her Master in Mathematics Education from the Universidad Autónoma de Ciudad Juárez (UACJ). In 2010, she joined the doctoral program in Teaching Learning in Culture at the University of Texas at El Paso.

While pursuing her degree, Dr. Gallardo worked as a research assistant and associate in the Department of Education. She had developed a deep understanding of Math in Education. Also, she had learned to develop qualitative and quantitative research and how to analyze data through some software like SPSS, and NVIVO. Her focus interest narrowed as a result of extensive exposure to a variety of ideas linked with teaching mathematics and STEM. Under the guidance of some teachers, she has identified an interest in exploring and understanding the factors that influence the mathematics teachers to develop their lesson plan to teach geometry class.

Dr. Gallardo has been presented her research in some conference meetings such as AERA, PMENA and WERA conference. Additionally, Dr. Gallardo's has published her research in some important *Journals of mathematics education*.

Dr. Gallardo's dissertation, *Borderland Pedagogy Study of High School Mathematics Teachers' Lesson Plan Development Practices*, was supervised by Dr. Mourat Tchoshanov.