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How to Reconstruct the System's Dynamics by Differentiating Interval-Valued and Set-Valued Functions

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Abstract. To predict the future state of a physical system, we must know the differential equations $\dot{x} = f(x)$ that describe how this state changes with time. In many practical situations, we can observe individual trajectories $x(t)$. By differentiating these trajectories with respect to time, we can determine the values of $f(x)$ for different states x; if we observe many such trajectories, we can reconstruct the function $f(x)$. However, in many other cases, we do not observe individual systems, we observe a set X of such systems. We can observe how this set X changes, but not how individual states change. In such situations, we need to reconstruct the function $f(x)$ based on the observations of such "set trajectories" $X(t)$. In this paper, we show how to extend the standard differentiation techniques of reconstructing $f(x)$ from vector-valued trajectories $x(t)$ to general set-valued trajectories $X(t)$

Keywords: prediction under uncertainty, differentiation of interval-valued and set-valued functions.

1 Formulation of the Problem

One of the main objectives of science and engineering: a brief reminder. One of the main objectives of *science* is to predict the future state of different systems. We want to predict the future weather, we want to predict the future trajectories of celestial bodies, etc. To make these predictions, we need to know the current state of the system, and we need to know how the state evolves with time.

For *engineering*, the main objective is to produce a design that satisfies the given properties, a control that leads the object into the given location, etc. In all such problems, we also need to be able to predict the future behavior of the designed and/or controlled system. The state of a physical object (system) can be characterized by the values $x = (x_1, \ldots, x_n)$ of different physical characteristics $x_1, \ldots, x_n$ of this object. For a celestial object, these characteristics include its mass, its location, its velocity, its angular velocity relative to different axes, its brightness and reflectivity at different places, etc. For the atmosphere, these

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characteristics include temperature, atmospheric pressure, wind speed, etc., at different locations. The evolution of macro-objects is usually reasonably well described by a (deterministic) ordinary differential equation $\dot{x} = f(x)$, where $\dot{x} \stackrel{\text{def}}{=} \dfrac{dx}{dt}$ is the time derivative.

Need for empirical differentiation. In many cases, the mapping $f(x)$ that describes the system's dynamics is known. For example, a point object can be characterized by its location r and its velocity v: $x = (r, v)$. Newton's equations $m\ddot{r} = F(r)$ describe the dynamics of this object in the force field $F(r)$. These equations can be described in the desired form $\dot{x} = f(x)$ as follows: $\dot{x} = v$, $\dot{v} = F(r)$, i.e., $f(r, v) = (v, F(r))$.

However, often, we do not know the exact dynamics $f(x)$. In such situations, we need to reconstruct the values $f(x)$ based on the observed trajectories of a system, i.e., on the values $x(t_i)$ measured for different values $t_1 < t_2 < \ldots < t_m$. When the observations are close in time, we can approximately describe the corresponding time derivatives as $\dot{x}(t_i) \approx \dfrac{x(t_i) - x(t_{i-1})}{t_i - t_{i-1}}$, and then reconstruct $f(x)$ from the condition that $\dot{x}(t_i) = f(x(t_i))$ for all observation moments t_i – and for all observed objects $x(t)$.

Need for interval-valued and set-valued functions: case of uncertainty. The above description is based on the ideal case when we observe a single object and its trajectory. In practice, often, instead of a *single* object, we observe the whole *group* of objects, a group in which it is very difficult to distinguish between individual objects.

For example, in biology, we can analyze the spread of the bacteria or viruses by tracing the corresponding epidemics, but it is practically impossible to trace individual bacteria or viruses. In meteorology, we can trace, e.g., how water goes from one state into another, from clouds to rain to rivers to evaporation etc., but it is impossible to trace individual molecules. In such cases, at any given moment of time t, instead of a single state $x(t)$, we observe the *collection* (*set*) $X(t)$ of different states.

Need to extend differentiation techniques to interval-valued and set-valued functions. In order to make predictions, we need to know the dynamics $f(x)$. Thus, we need to be able to reconstruct the dynamics from the observed sets $X(t_1)$, $X(t_2)$, ..., $X(t_n)$. For the case of exactly known states, when each set $X(t_i)$ consists of a single state $X(t_i) = \{x(t_i)\}$, this reconstruction is based on the differentiation. Thus, it is reasonable to call the process of reconstructing the dynamics $f(x)$ from the observed sets "differentiations" of the corresponding set-valued function $X(t)$.

Differentiation of set-valued functions: what is known. There have been many generalizations of differentiation to set-valued functions. Many such generalizations appeared in *rough set* theory; see, e.g., [19]. The main idea behind rough sets is that instead of the exact set X of possible states, we only store its lower

and upper approximations $\underline{X}$ and $\overline{X}$: the set $\underline{X} \subseteq X$ is formed by all the granules that are fully contained in X, and the set $\overline{X} \supseteq X$ is formed by all the granules that may have common elements with X. When the situation changes, both sets change. To describe the rate of such change, Pawlak described differentiation of rough sets [19,20]; see also [14,15,18,22,23,24,25].

Similar set-valued differentiation procedures [1,4,8,9,10,11,12,21] have been defined in within a *set-valued analysis* [2,3], when we need, e.g., to find the optimal shape (set) of a design (see, e.g. [16]). Several papers describe the application of these techniques to the important problem of find the range of the solution to a differential equation under uncertain initial conditions and uncertain values of the parameters; see, e.g., [5,6,7].

What we do. None of the existing set differentiation procedures directly solves our problem – of reconstructing the function $f(x)$ from the observed set trajectories $X(t)$. We show, however, that by properly modifying the known differentiation techniques, we can extract the dynamics $f(x)$ from the observed behavior $X(t_i)$. This extraction uses techniques that generalize the standard differentiation techniques from the case of vector-valued trajectories $x(t)$ to the more general case of set-valued trajectories $X(t)$.

In this paper, we first consider a 1-D (interval-valued) case in Section 2, then a fuzzy case in Section 3, and finally, the general multi-D case in Section 4.

2 Case of Interval-Valued Functions

Formulation of the case. Let us start with the simplest case in which the state of a system is characterized by a single variable x, i.e., when $x = x_1$ and $n = 1$. In this case, each state is a point on a real line, and thus, for each moment of time t, the set $X(t)$ of all observed states is a subset of the real line. In general, the set $X(t)$ of observed states can be disconnected (in the standard topological sense). However, in this case, we would be able to individually trace every connected component separately. So, for our purpose, it makes sense to consider the case when the set $X(t)$ of possible states is connected. It also makes sense to consider the case when this set is bounded – since in practice, most observed collections are bounded. On the real line, the only bounded connected sets are intervals. Thus, we can conclude that for every t, we observe the corresponding interval $X(t) = [\underline{x}(t), \overline{x}(t)]$.

Analysis of the problem. Let $f(x)$ be a function that describes the system's dynamics. This means that once at some moment t, we have a state $x(t)$, then at the next moment of time $t+\Delta t$, we have the state $x(t+\Delta t) \approx x(t)+f(x(t))\cdot\Delta t$. Different values $x \in [\underline{x}(t), \overline{x}(t)]$ lead, in general, to different values $x + f(x) \cdot \Delta t$. Thus, to find the upper endpoint $\overline{x}(t + \Delta t)$ of the interval

$$X(t + \Delta t) = [\underline{x}(t + \Delta t), \overline{x}(t + \Delta t)],$$

we need to find the largest possible value of the expression $x + f(x) \cdot \Delta t$ when $x \in [\underline{x}(t), \overline{x}(t)]$.

In physics, in most dynamical equations, the transformation $f(x)$ is usually smooth (differentiable). Thus, it is reasonable to assume that $f(x)$ is differentiable and thus, that the mapping $x \to x + f(x) \cdot \Delta t$ is also differentiable. The derivative of this mapping with respect to x is equal to $1 + f'(x) \cdot \Delta t$. When the time step Δt is sufficiently small, we have $|f'(x) \cdot \Delta t| \ll 1$ and thus, $1 + f'(x) \cdot \Delta t > 0$. Hence, on this interval, the transformation $x \to x + f(x) \cdot \Delta t$ is strictly increasing. Thus, the largest value $\overline{x}(t+\Delta t)$ of the expression $x(t+\Delta t) = x(t) + f(x(t)) \cdot \Delta t$ is attained when $x(t)$ attains its largest value, i.e., when $x(t) = \overline{x}(t)$. In other words, $\overline{x}(t + \Delta t) \approx \overline{x}(t) + f(\overline{x}(t)) \cdot \Delta t$. Thus, we have $\dot{\overline{x}} = f(\overline{x})$.

Similarly, the smallest possible value $\underline{x}(t + \Delta t)$ of the expression $x(t + \Delta t) = x(t) + f(x(t)) \cdot \Delta t$ is attained when $x(t)$ attains its smallest value, i.e., when $x(t) = \underline{x}(t)$. In other words, $\underline{x}(t + \Delta t) \approx \underline{x}(t) + f(\underline{x}(t)) \cdot \Delta t$. Thus, we have $\dot{\underline{x}} = f(\underline{x})$. Hence, we arrive at the following conclusion.

Conclusion: how to reconstruct the dynamics from the interval-valued observations. If, for each moment of time t_i, we know the interval $X(t_i) = [\underline{x}(t_i), \overline{x}(t_i)]$, then we can reconstruct the dynamics $f(x)$ as follows. First, we estimate

$$\dot{\overline{x}}(t_i) \approx \frac{\overline{x}(t_i) - \overline{x}(t_{i-1})}{t_i - t_{i-1}}; \quad \dot{\underline{x}}(t_i) \approx \frac{\overline{x}(t_i) - \overline{x}(t_{i-1})}{t_i - t_{i-1}}.$$

Then, we reconstruct $f(x)$ from the conditions that $\dot{\overline{x}}(t_i) = f(\overline{x}(t_i))$ and $\dot{\underline{x}}(t_i) = f(\underline{x}(t_i))$ for all observation moments t_i – and for all observed interval-valued trajectories $X(t)$.

Example. For radioactive decay, $\dot{x} = -k \cdot x$, so $x(t) = x(0) \cdot \exp(-k \cdot t)$. Thus, if we start with an interval $X(0) = [1, 2]$, we get $X(t) = [\exp(-k \cdot t), 2 \cdot \exp(-k \cdot t)]$. By differentiating the lower endpoint, we conclude that for every t, we have $f(\exp(-k \cdot t)) = -k \cdot \exp(-k \cdot t)$, i.e., that indeed $f(x) = -k \cdot x$.

3 Fuzzy Case: Observation

Formulation of the problem. Instead of observing the crisp interval $X(t)$, we can be observing a *fuzzy* interval. In other words, in addition to the interval $[\underline{x}(t), \overline{x}(t)]$ that is guaranteed to contain all the observed objects, for every degree α from the interval $(0, 1)$, we also have narrower intervals $[\underline{x}_\alpha(t), \overline{x}_\alpha(t)]$ (alpha-cuts of the corresponding fuzzy sets) that contain $x(t)$ with certainty α.

Analysis of the problem. It is known that for every bounded continuous transformation, the alpha-cut of the result is equal to the result of applying this transformation to the original alpha-cut; see, e.g., [13,17]. Thus, for each α, the corresponding α-cut intervals $[\underline{x}_\alpha(t_i), \overline{x}_\alpha(t_i)]$ form a sequence from which we can extract $f(x)$ – by using the interval-based techniques described in the previous section. As a result, we arrive at the following technique.

Conclusion: how to reconstruct the dynamics from the fuzzy-valued observations. Let us assume that for each moment of time t_i, we know the fuzzy value $X(t_i)$.

In other words, we assume that for every moment of time t_i and for every degree $\alpha \in (0,1]$, we know the interval $[\underline{x}_\alpha(t_i), \overline{x}_\alpha(t_i)]$. Then we can reconstruct the dynamics $f(x)$ as follows. First, we estimate

$$\dot{\overline{x}}_\alpha(t_i) \approx \frac{\overline{x}_\alpha(t_i) - \overline{x}_\alpha(t_{i-1})}{t_i - t_{i-1}}; \quad \dot{\underline{x}}_\alpha(t_i) \approx \frac{\overline{x}_\alpha(t_i) - \overline{x}_\alpha(t_{i-1})}{t_i - t_{i-1}}.$$

Then, we reconstruct $f(x)$ from the conditions that $\dot{\overline{x}}_\alpha(t_i) = f(\overline{x}_\alpha(t_i))$ and $\dot{\underline{x}}_\alpha(t_i) = f(\underline{x}_\alpha(t_i))$ for all observation moments t_i, for all degrees α, and for all observed interval-valued trajectories $X(t)$.

4 General Multi-D Case

Formulation of the problem. In the multi-dimensional case, at different moments of time t, we observe the set $X(t)$ of states. For an individual state $x(t) \in X(t)$, we do not know what will be the corresponding state $x(t + \Delta t)$ at the next moment of time $t + \Delta t$, we only know that this unknown state $x(t + \Delta t)$ belongs to the observed set $X(t + \Delta t)$. We also know that all the states from the set $X(t + \Delta t)$ are obtained from the states of the set $X(t)$ by the corresponding evolution $\dot{x} = f(x)$. We may observe several different evolving sets $X^{(1)}(t)$, $X^{(2)}(t)$, ... Our objective is, based on this information, to reconstruct the dynamics $f(x)$.

Definitions and the main result. Let us formulate our main result in precise terms. By a *dynamical system*, we mean a smooth function $f : \mathbb{R}^n \to \mathbb{R}^n$. By a *smooth set* X, we mean a simply connected open set whose boundary ∂X is a smooth surface. For every dynamical system f and for every smooth set X, by a *set trajectory*, we mean a function that maps each positive real number t into the set $X(t) = \{x(t) : x(0) \in X \,\&\, \dot{x} = f(x)\}$. Let us denote the class of all set trajectories corresponding to a system f by $T(f)$.

Our main result is that is that a dynamical system is uniquely determined by the class of its set trajectories, i.e., if $T(f) = T(f')$ then $f = f'$.

Comment. As we will see from the proof, in order to uniquely determine f, it is not necessary to know *all* set trajectories, it is sufficient to have a class of set trajectories for which, for every point $x \in \mathbb{R}^n$, we have n set trajectories $X^{(i)}(t)$ and moments of time t_i at which $x \in \partial X^{(i)}(t_i)$ and at which the n normal vectors $N^{(i)} \perp \partial X^{(i)}(t_i)$ are linearly independent.

Analysis of the problem. Let x_0 be any point on the border $\partial X(t)$, and let N be a normal vector, i.e., the unit vector orthogonal to $\partial X(t)$ at the point x_0 (i.e., orthogonal to the tangent plane to $X(t)$).

As the states evolve, each state $x \in X(t)$ changes into the next state $x + f(x) \cdot \Delta t$. Locally, when x is close to x_0, the value $f(x)$ is close to $f(x_0)$ and thus, the whole plane shifts by the vector $\Delta x \stackrel{\text{def}}{=} f(x_0) \cdot \Delta t$. Let us first consider the situation when the vector $f(x_0)$ is in the tangent plane. In this

situation, while each state changes, the plane itself does not change. Thus, since we only observe the set $X(t)$ – i.e., in effect, its boundary $\partial X(t)$ – locally, we will not observe any difference. A general shift Δx can be represented as a linear combination of two shifts:

- a shift in the direction from the plane – which we cannot observe, and
- a shift in the direction orthogonal to the plane, i.e., in the direction parallel to the normal vector; this shift we can observe.

In geometric terms, the shift in the direction of N can be represented as $(\Delta x, N) \cdot N$, where $(a, b) \stackrel{\text{def}}{=} \sum\limits_i a_i \cdot b_i$ is a scalar (dot) product of two vectors, and the shift in the direction from the plane has the form $\Delta x - (\Delta x, N) \cdot N$. The value $(\Delta x, N)$ is actually equal to the distance between the two tangent planes:

- the tangent plane to the border set $\partial X(t)$ at the point x_0, and
- the tangent plane to the border set $\partial X(t + \Delta t)$ at the point closest to x_0.

So, by measuring this distance, we can find the scalar product $(\Delta x, N) = (f(x_0), N) \cdot \Delta t$, and thus, we can find the scalar product $(f(x_0), N)$. If we have several families of sets $X^{(1)}(t)$, ..., $X^{(k)}(t)$, then, in general, the normal vectors $N^{(j)}$ corresponding to the moment when the boundaries of the corresponding sets pass through (or close to) the point x_0, are different. Thus, we can get the scalar products $(f(x_0), N^{(j)})$ corresponding to different vectors $N^{(j)}$. Once we know a sufficient number of such products, we can thus uniquely reconstruct the vector $f(x_0)$ – i.e., all n coordinates $f_i(x_0)$ of this vector – from the corresponding system of linear equations $\sum\limits_{i=1}^{n} f_i(x_0) \cdot N_i^{(j)} = (f(x_0), N^{(j)})$. As a result, we arrive at the following conclusion.

Conclusion: how to reconstruct the dynamics from the set-valued observations. Let us assume that we have k dynamically changing sets. For each such set $X^{(j)}$, $j = 1, \ldots, k$, for different moments of times $t_1^{(j)} < t_2^{(j)} < \ldots < t_k^{(j)} < \ldots$ we observe the sets $X^{(j)}(t_k^{(j)})$ of possible states. As before, we assume that the consecutive moments are close to each other, i.e., $t_{k+1}^{(j)} \approx t_k^{(j)}$. Then, we can reconstruct the function $f(x)$ as follows.

For each family j, for each moment $t_k^{(j)}$, and for each point x from the boundary $\partial X^{(j)}(t_k^{(j)})$ of the set $X^{(j)}(t_k^{(j)})$, we compute the distance $\Delta\rho_k^{(j)}(x)$ between the following two planes:

- the plane P tangent to $\partial X^{(j)}(t_k^{(j)})$ at the point x, and
- the plane tangent to $\partial X^{(j)}(t_{k+1}^{(j)})$ at a point which is the closest to x.

Then, we compute the ratio $\dfrac{\Delta\rho_k^{(j)}(x)}{t_{k+1}^{(j)} - t_k^{(j)}}$. We also compute the unit vector $N_k^{(j)}(x)$ which is orthogonal to the plane P. Then, we conclude that the value $f(x)$ of the desired dynamical function $f(x)$ satisfies the equation $(f(x), N_k^{(j)}(x)) =$

$\dfrac{\Delta\rho_k^{(j)}(x)}{t_{k+1}^{(j)}-t_k^{(j)}}$. After that, we reconstruct the desired function $f(x)$ from the fact that these equations – with known values of the right-hand side – must be satisfied for all the points x on all the boundaries $\partial X^{(j)}(t_k^{(j)})$. Specifically, to find the value $f(x_0)$ for a given x_0, we collect all such equations for close values $x \approx x_0$. Since $x_0 \approx x$, we conclude that $f(x_0) \approx f(x)$ and thus, that $(f(x_0), N_k^{(j)}(x)) \approx (f(x), N_k^{(j)}(x)) = \dfrac{\Delta\rho_k^{(j)}(x)}{t_{k+1}^{(j)}-t_k^{(j)}}$. In general, for these equations, the normal vectors $N_k^{(j)}(x)$ will be different, so we have sufficiently many linear equations of the type $(f(x_0), N_k^{(j)}(x)) \approx \dfrac{\Delta\rho_k^{(j)}(x)}{t_{k+1}^{(j)}-t_k^{(j)}}$. from which we can uniquely reconstruct the vector $f(x_0)$.

5 Conclusions

In order to predict the evolution of a system, we need to know the differential equations that describe how its state changes with time. These equations can be determined from observations when we observe several trajectories of individual systems. However, in many practical situations, we do not observe individual trajectories, we observe the whole set of systems that evolve together, we observe how this set changes, but not how individual trajectories change. In this paper, we show that based on several such set observations, we can also uniquely reconstruct the differential equations that describe the system's dynamics.

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