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How to Take Into Account Student's Degree of Confidence When Grading Exams

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HOW TO TAKE INTO ACCOUNT STUDENT'S DEGREE OF CONFIDENCE WHEN GRADING EXAMS

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Abstract: When grading exams, it is important to take into account how confident the student is in the answer. If the answer is correct, then it is better – and thus, deserves a better grade -- if the student is absolutely confident in this correct answer. On the other hand, if the answer is wrong, then, the more confident the student is in this wrong answer, the worse. The grading scheme should be such that provides an incentive for the students to report their true degree of confidence. In this paper, we explain how to design such a grading scheme.

Keywords: grading, degree of confidence, gaming the system

Need to take student's degree of confidence into account when grading. Traditionally, a grade on an exam only takes into account whether the student's answers are correct or not. From the viewpoint of real-life applications, however, it is important to also take into account how confident the student is in his/her answer.

For example, if a medical doctor suggests a wrong diagnosis, it is not so bad if the doctor is not very confident in this diagnosis: in this case, the doctor will request additional tests or consult the colleagues before starting a treatment. The situation is much worse if the doctor is absolutely confident in the wrong answer. From this viewpoint, the less confident the doctor is in the wrong answer, the better.

Similarly, if a doctor suggest a correct diagnosis, and he/she is absolutely confident in the correct diagnosis, then the doctor can start the right treatment right away. On the other hand, if a doctor is not confident in the right diagnosis, he or she will probably request additional tests or consult with the colleagues before starting a treatment – thus introducing an unnecessary delay. From this viewpoint, the more confident the doctor is in the correct diagnosis, the better.

Similarly arguments can be made about an engineer designing a construction, etc.

How to make sure that the students supply correct degrees of confidence. The only way to get a student's degree of confidence is to ask the student for this information. Herein lies a danger: if a student is reasonably – but not fully – confident that the answer is correct, and he or she knows that the higher degree of confidence he or she reports, the higher the grade, will not the student be motivated to game the system and to report absolute confidence?

This would defeat the purpose of taking degrees of confidence into account. It is therefore desirable to design a grading scheme that would avoid such gaming and encourage students to submit correct degrees of confidence.

Formulation of the problem in precise terms. Let us formulate the problem of designing such a grading scheme in precise terms.

Let us assume that we have a question with n possible answers of which only one is correct. Instead of simply picking one of n possible answers, a student reports his/her degrees of confidence $q_1, \ldots, q_n$ in each of the answers. These degrees may be, e.g., subjective probabilities that the corresponding answer is correct, in which case these probabilities should add up to 1:

$$q_1 + \ldots + q_n = 1 \quad (1)$$

We want to make the number of points awarded to the student dependent on the degree q that this student assigned to the correct answer: the larger this degree, the more points the student gets. Let us denote the number of points assigned to the students by f(q).

Our objective is to select the function f(q) in such a way that the student is encouraged to report his or her true degrees of confidence. Let p_1, …, p_n be actual student's degrees of confidence, for which

$$p_1 + \ldots + p_n = 1 \tag{2}$$

If the student reports his or her actual degrees of confidence, then for each i, with probability p_i, the i-th answer is correct and the student gets $f(p_i)$ points. In this case, the expected number of points awarded to the student is equal to the sum

$$p_1 * f(p_1) + \ldots + p_n * f(p_n)\ . \tag{3}$$

If instead, the student reports different degrees q_1, …, q_n, then for each i, with probability p_i, the i-th answer is correct and the student gets $f(q_i)$ points. In this case the expected number of points will be equal to

$$p_1 * f(q_1) + \ldots + p_n * f(q_n)\ . \tag{4}$$

We need to select the function f(q) in such a way that for fixed values p_1, …, p_n, the largest possible expected value (4) is attained when the students reports the actual degrees, i.e., when $q_i = p_i$ for all i.

Analysis of the problem. Our goal is to find the function f(q) for which the values $q_i = p_i$ for all I solve the problem of maximizing the sum (4) under the constraint (1). By using the Lagrange multiplier method, we can reduce this constraint optimization problem to an unconstrained problem of maximizing the expression

$$L = p_1 * f(q_1) + \ldots + p_n * f(q_n) + w * (q_1 + \ldots + q_n - 1), \tag{5}$$

where w is the Lagrange multiplier. At the maximum, the partial derivatives of the function L are equal to 0. Differentiating the expression (5) with respect to q_i and equating the derivatives to 0, we get

$$p_i * f'(q_i) + w = 0 \quad \text{when } q_i = p_i, \tag{6}$$

where f' denotes the derivative of the function f(q). Thus,

$$p_i * f'(p_i) + w = 0. \tag{7}$$

In other words, the product $p_i * f'(p_i)$ is equal to –w and so, is the same for all i. This means that the product p * f'(p) is a constant which does not depend on p:

$$p * (df/dp) = \text{const.} \tag{8}$$

If we multiply both sides of this formula by dp and divide both sides by p, we separate the variables f and p and get the following equality:

$$df = \text{const} * (dp/p). \tag{9}$$

Integrating both sides of this equality, we conclude that

$$f(p) = const * \ln(p) + C \tag{10}$$

for some constant C.

Thus, we arrive at the following conclusion.

Conclusion. We want to take into account, when grading the student's answers, the student's degree of confidence in different answers. We want to take these degrees of certainty into account in such a way that the students are incentivized to report correct degrees of confidence.

Our result is that this is possible only in one case: when the number of points f(p) awarded to an answer is proportional to the logarithm ln(p) of the student's degree of confidence in the correct answer.

Discussion. If the student is absolutely confident in the correct answer, this students receives the largest possible number of points for this question – namely, C points. On the other hand, if the student is absolutely confident in the wrong answer, then this student received minus infinity points.

To avoid infinities, we can, e.g., take a small value d instead of 0 probability, and then tend d to 0 if we want to compare grades for two different sets of questions. This way, while in the limit, two absolutely confident wrong answers add up to the same minus infinity, for each d, we give more points to a student with only one wrong answer, and fewer point with two wrong answers.

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