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10-2015

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Technical Report: UTEP-CS-15-70

Published in *Proceedings of the 5th International Conference "Mathematics Education: Theory and Practice" MATHEDU'2015*, Kazan, Russia, November 27-28, 2015, pp. 14-16.

Recommended Citation

Kosheleva, Olga and Kreinovich, Vladik, "Oscillating Exam Averages and Their Control-Theory Explanation" (2015). *Departmental Technical Reports (CS)*. 960.
https://scholarworks.utep.edu/cs_techrep/960

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OSCILLATING EXAM AVERAGES AND THEIR CONTROL-THEORY EXPLANATION

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Abstract: When a student misses one of the exams, his overall grade for the class is often interpolated based on his available grades. This would have been a fair procedure if the grades for different tests were equally distributed. In practice, often, the average grades for different tests are oscillating. As a result, the usual interpolation techniques may inadvertently bias the student grade for the class. In this paper, we explain this oscillation, and analyze how to avoid the corresponding bias.

Keywords: grading, missing test grades, oscillating test averages

Grading: a brief reminder. In the US education system, the student's grade for a class is usually calculated as a weighted average of his or her grades over different tests and different homeworks, labs, etc.

How to deal with missing exams: a usual procedure. Sometimes, a student cannot attend one of the exams. For example, the student may be ill on the day of the exam. In this case, for this student, the grade for one of the exams is missing. How can we then calculate the student's overall grade for the class?

A usual practice is to take the weighted average of available grades. In effect, this is equivalent to using the average of the available exam grades as an estimate for the student's grade on a missing exam.

Limitation of the usual procedure. If the student grades for all the exams were equally distributed, so that, in particular, for different exams, we have similar means and similar standard deviations, then the above usual practice would indeed be a fair procedure. However, our empirical data shows that the means corresponding to different exams may differ – and this difference goes beyond the expected random deviations of the sample average from the mean.

In this case, the usual interpolation procedure may lead to a biased result. For example, if a student missed an exam on which an average grade was lower than on the other exams, this means that, on average, students did worse on this exam than on the other exams. Most probably the current student would have also done worse on this exam than on the other exams. As a result, the average of the student's grades on all other exams is probably higher than what he would have gotten on this missing exam. So, if we use this average as an estimate of how he/she would have performed on the missing exam, it will result to an overall grade which is higher than necessary – i.e., that overestimates the student's knowledge.

Similarly, if a student missed an exam on which an average grade was higher than on the other exams, this means that, on average, students did better on this exam than on the other exams. Most probably the current student would have also done better on this exam than on the other exams. As a result, the average of the student's grades on all other exams is probably lower than what he would have gotten on this missing exam. So, if we use this average as an estimate of how he/she would have performed on the missing exam, it will result to an overall grade which is lower than necessary – i.e., that underestimates the student's knowledge.

And indeed, the usual interpolation procedure often results in bias; see, e.g., [1].

What we do in this paper. In this paper, we provide examples of the oscillating average, provide a qualitative explanation of this phenomenon, and analyze how to avoid the bias when interpolating the missing grade.

Empirical data. Our data comes from the Introduction to Computer Science class taught at the University of Texas at El Paso in different years. In some years, in this class, we had 3 exams E1, E2, and E3, followed by the final exam FE. In other years, we also had a fourth exam E4. The average grades for all these exams are given below.

Year	E1	E2	E3	E4	FE
2014	79	77	71	n/a	79
2013	77	71	67	73	74
2012	60	49	55	n/a	75
2008	97	74	88	83	78

One can see that instead of – as may be expected – random deviations, we observe systematic oscillations. For example, in 2014, the grade monotonically decreases from Exam 1 to Exam 3 and then rises for the final exam. In 2013, the grade similarly decreases from Exam 1 to Exam 3, and then starts increasing, so that the average grade for Exam 4 is higher than for Exam 3, and the average grade for the final exam is still higher. In 2012, the increase started after Exam 2. In 2008, we have two oscillations.

The behavior differs year from year, what is common is oscillations. How can we explain them?

Control-theoretic explanation of the oscillations. Let us consider the following simple description of the students' learning. In this description, the level of student knowledge is described by a single number r – which is in perfect accordance with the fact that on all the exams, we gauge the student's level of knowledge by a single number. In accordance with this grades interpretation, the number r describes the relative student knowledge, so that on each stage of the learning process, $r = 1$ corresponds to the perfect knowledge, while the values $r < 1$ correspond to partial knowledge.

The change in knowledge level depends on the student's learning effort. The simple possible model of such a dependence is to assume that this dependence is linear, i.e., that the increase in knowledge is proportional to the student's learning effort e:

$$dr/dt = k * e, \tag{1}$$

for some coefficient k.

The amount of learning effort e, in its term, is determined by the student's desired level d: the further away the current level of knowledge is from d, i.e., the larger the difference $d – r$, the more effort the student will apply. In the ideal world, a student may aspire to achieve perfect knowledge in all the classes that he or she takes, but in practice, the amount of student's effort is limited, so a student needs to decide how much effort to spend on each class.

Similarly to the previous formula, the simplest model is to assume that the increase in effort is proportional to the difference $d – r$:

$$de/dt = c * (d – r), \tag{2}$$

for some coefficient c.

Now, we have a system of two differential equations that, in this simplified model, describe how the student's knowledge changes with time. To solve this equation, let us differentiate both sides of the equation (1), then we get

$$d^2r/dt^2 = k * de/dt. \quad (3)$$

Substituting the expression (2) for de/dt into this formula, we conclude that

$$d^2r/dt^2 = c * (d – r). \quad (4)$$

As a result, for x = d – r, we get

$$d^2x/dt^2 = -- c * x. \quad (5)$$

It is well known that the solution to this equation is a sinusoid x = A * sin(w * t + a), where w is the square root of c, and A and a are constants. A sinusoid oscillates.

Thus, the natural simple model of student learning indeed explains the observed oscillation.

What is commonsense meaning of this explanation? The above explanation is somewhat too mathematical, it does not provide us with a clear commonsense explanation of the oscillation phenomenon. However, it is possible to extract such an explanation from the above mathematics – especially if we take into account that our data comes from the Introduction to Computer Science class. For the students, this is the first Computer Science class, this is the first time that they encounter Computer Science faculty.

Introduction of Computer Science is a class which is mostly taught to Computer Science majors. These students are interested in the topic, they are highly motivated to succeed, and so, during the time leading to the first exam, they try their best to succeed on this exam.

Since the students spend a lot of efforts on this class, their results are, on average, good – often at the expense of other classes that these student take. So, when the students see that their grade in Computer Science is high but their grades in other classes is lower than expected, their natural reaction is to spend less effort on Computer Science and more effort on other classes.

However, it is difficult for the incoming students to accuracy predict the results of their learning efforts. Just like originally, they probably spent too much effort on learning Computer Science, when they decrease the amount of effort, they often err in a different direction – as a result, their grade in Computer Science swings below what they wanted. At this time, they put their effort back – which leads to an increasing trend.

In a nutshell, this oscillatory behavior can be explained as follows. First, students are scared, they study hard and get a very good grade. The fact that they succeed indicates, to them, that this material is not that difficult, so they can cut down on the learning effort, As they cut down this effort more and more, their grade slips below what they want – so then they again start studying more, and their grades improve.

So what we do with the missing grades? Now that we explained the oscillation phenomenon, the natural next question is: how to avoid related bias when calculating the overall class grade for students who missed one of the exams?

For this, as [1] shows, a reasonable idea is to average not the exam grades, but the normalized exam grades – e.g., the differences between the student's grade on an exam and the average class grade on this exam. (We can also divide by the standard deviation.) This indeed eliminates the effect of the bias.

Acknowledgments. This work was supported in part by the US National Science Foundation grants HRD-0734825, HRD-1242122, and DUE-0926721.

The authors are thankful to Dr. Mourat Tchoshanov for his encouragement and to all the participants of the 2015 International IEEE Frontiers in Education Conference (El Paso, Texas, October 21-24, 2015), especially to Dr. Michael C. Loui (Purdue University) for valuable discussions.

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