1-2015

When an Idea Comes, Write It Down Right Away: Mathematical Justification of Vladimir Smirnov's Advice

Olga Kosheleva
The University of Texas at El Paso, olgak@utep.edu

Vladik Kreinovich
The University of Texas at El Paso, vladik@utep.edu

Comments:

Technical Report: UTEP-CS-15-04

When an Idea Comes, Write It Down Right Away: Mathematical Justification of Vladimir Smirnov's Advice

Olga Kosheleva and Vladik Kreinovich
University of Texas at El Paso
500 W. University
El Paso, TX 79968, USA
olgak@utep.edu, vladik@utep.edu

Abstract

Among several advices to students, Vladimir Smirnov, a renowned Russian mathematician, suggested that when an idea comes, it is better to write it down right away. In this paper, we provide a quantitative justification for this advice.

1 Formulation of the Problem

Advice of Vladimir Smirnov. When one of us (VK) became a student at the Mathematics Department of St. Petersburg University, the department had a special poster for incoming students with advice from different professors. One of these advices was from Professor Vladimir Smirnov, the author of a widely used course in higher mathematics [2]: when an idea comes, write it down right away, do not delay.

Qualitative explanation. If one does not write down his/her ideas right away, he/she will forget them, and it will require an additional time to recall it. From this viewpoint, to avoid wasting time, it is better to write down the idea right away.

What we plan to do. The objective of this paper is to provide a quantitative explanation for Smirnov's advice.

2 Analysis of the Problem

An idea comes to mind: two possible reactions. Suppose that an idea comes to mind when a person is in the middle of some activity. Then, the person has two options:

- the first option is to interrupt the current activity, write down the idea, and then resume the current activity;
- the second option is to wait until the end of the current activity, and then write down the idea.

Let us analyze when the first reaction is better, and when the second reaction is better.

First option. If we select the first option, then we need extra time to write down the idea, and we also need some additional time to interrupt (and later resume) the current activity. Let:

- t_w denote the time that is needed to write down the idea right away; and
- t_i denote the additional time needed to interrupt the current activity and to resume it again later.

In these terms, in the first option, we spend an additional time $t_w + t_i$.

Second option: analysis of the problem. In the second option, we do not interrupt the current activity. Instead, we wait until the end of this activity. In this case, we do not spend time on the interruption, but we do need to spend time trying to recover the idea.

Let us estimate this recovery time. Human forgetting is well described by the so-called *Ebbinghaus forgetting curve* [1], according to which the amount $a(t)$ of material that we remember decreases with time as

$$\frac{da}{dt} = -k_f \cdot a,$$

where k_f is a parameter describing the forgetting.

This equation makes perfect sense: in general, we can write that $\frac{da}{dt} = f(a)$ for some function $f(a)$. In the first approximation, we can approximate the function $f(a)$ with the first two terms in its Taylor expansion: $\frac{da}{dt} = c_0 + c_1 \cdot a$. When we have no knowledge, i.e., when originally $a(0) = 0$, then of course there is nothing to forget, thus $\frac{da}{dt} = 0$ as well. The condition that $f(0) = c_0 + c_1 \cdot 0 = 0$ implies that $c_0 = 0$ and thus, $\frac{da}{dt} = c_1 \cdot a$. Forgetting means that the amount of remembered material decreases with time, so $c_1 < 0$, and thus, $c_1 = -k_f$ for some $k_f > 0$. This is exactly the Ebbinghaus law.

Because of this equation, the amount of material remembers after time t is equal to $a(0) \cdot \exp(-k_f \cdot t)$.

Let t_e denote the time needed to finish the current activity. Then, by the time t_e, instead of the original amount of information $a(0)$ about our idea, we remember only the amount $a(0) \cdot \exp(-k_f \cdot t_e)$.

Before we write down the idea, we need to recall it. How can we describe a recall? In general, the amount $a(t)$ recalled by time t can also be described by a differential equation $\frac{da}{dt} = g(a)$ for some function $g(a)$. In the first approximation, we can approximate the function $g(a)$ with the first two terms in its Taylor expansion: $\frac{da}{dt} = c'_0 + c'_1 \cdot a$. Let us consider an ideal situation when eventually, i.e., when $t \to \infty$, we can recall everything, i.e., we have $a(t) \to a(0)$. For $t \to \infty$, we get $\frac{da}{dt} = 0$ and $a = a(0)$. Thus, $c'_0 + c'_1 \cdot a(0) = 0$ and therefore, $c'_0 = -c'_1 \cdot a(0)$. So, the recall equation $\frac{da}{dt} = c'_0 + c'_1 \cdot a$ can be described as $\frac{da}{dt} = c'_1 \cdot (a - a(0))$. When $a < a(0)$, the amount of recalled material increases with time, so $\frac{da}{dt} > 0$ and thus, $c'_1 < 0$. Thus, we can write that $c'_1 = -k_r$ for some parameter k_r that describes a person's recall rate. In terms of this parameter, the recall equation takes the form

$$\frac{da}{dt} = -k_r \cdot (a - a(0)).$$

This equation can be rewritten as $\frac{d(a(0) - a(t))}{dt} = k_r \cdot (a(0) - a(t))$. At moment t_e, when we start the recall process, we have $a(t_e) = a(0) \cdot \exp(-k_f \cdot t_e)$ and thus, $a(0) - a(t_e) = a(0) \cdot (1 - \exp(-k_f \cdot t_e))$. The corresponding solution to the recall differential equation has the form

$$a(0) - a(t) = (a(0) - a(t_e)) \cdot \exp(-k_r \cdot (t - t_e)) =$$

$$a(0) \cdot (1 - \exp(-k_f \cdot t_e)) \cdot \exp(-k_r \cdot (t - t_e)).$$

Ideally, we should stop recalling at the moment t_s at which we have recovered everything, i.e., at which $a(t_s) = a(0)$ and $a(0) - a(t_s) = 0$. However, the above expression never reaches 0, so we stop when we have recovered the overwhelming part of the original idea, i.e., when $a(t_s) = a(0) \cdot (1 - \varepsilon)$ for some small value $\varepsilon > 0$. In this case, $a(0) - a(t_s) = a(0) \cdot \varepsilon$. By equating

$$a(0) - a(t_s) = a(0) \cdot (1 - \exp(-k_f \cdot t_e)) \cdot \exp(-k_r \cdot (t_s - t_e))$$

with $a(0) \cdot \varepsilon$, we can deduce the time $t_s - t_e$ needed for this recall: namely, by dividing both sides of the equality by $a(0) \cdot (1 - \exp(-k_f \cdot t_e))$, we conclude that

$$\exp(-k_r \cdot (t_s - t_e)) = \frac{\varepsilon}{1 - \exp(-k_f \cdot t_e)}.$$

By taking logarithms of both sides and changing signs, we get

$$k_r \cdot (t_s - t_e) = \ln(1 - \exp(-k_f \cdot t_e)) - \ln(\varepsilon),$$

and thus,

$$t_s - t_e = \frac{1}{k_r} \cdot \ln(1 - \exp(-k_f \cdot t_e)) - \frac{1}{k_r} \cdot \ln(\varepsilon).$$

In this option, the overall additional time needed to record the idea is equal to $t_w + (t - t_e)$.

Conclusion: when is it better to write down the idea right away. It is beneficial to write down the idea right away if the first alternative leads to smaller amount of additional time, i.e., when $t_i + t_w < t_w + (t_s - t_e)$. This inequality is equivalent to $t_s - t_e > t_i$. In view of the above formula for $t_s - t_e$, Thus, it is better to write down the idea if

$$\frac{1}{k_r} \cdot \ln(1 - \exp(-k_f \cdot t_e)) - \frac{1}{k_r} \cdot \ln(\varepsilon) > t_i,$$

i.e., equivalently, when

$$\ln(1 - \exp(-k_f \cdot t_e)) - \ln(\varepsilon) > k_r \cdot t_i,$$

where:

- k_f is the rate which which the person forgets,
- ε is a portion of the original idea that we are willing to ignore,
- k_r is the rate with which a person recalls a forgotten information, and
- t_i is the time needed to interrupt and then resume the current activity.

Acknowledgments. This work was supported in part by the National Science Foundation grants HRD-0734825 and HRD-1242122 (Cyber-ShARE Center of Excellence) and DUE-0926721.

References

[1] D. L. Schacter, *Psychology*, Worth Publishers, New York, 2009.

[2] V. I. Smirnov, *A Course in Higher Mathematics*, Pergamon Press, Oxford, London, Edinburgh, New York, 1964.