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How Design Quality Improves with Increasing Computational Abilities: General Formulas and Case Study of Aircraft Fuel Efficiency

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It is known that the problems of optimal design are NP-hard – meaning that, in general, a feasible algorithm can only produce close-to-optimal designs. The more computations we perform, the better design we can produce. In this paper, we theoretically derive quantitative formulas describing how the design qualities improves with the increasing computational abilities. We then empirically confirm the resulting theoretical formula by applying it to the problem of aircraft fuel efficiency.

Keywords: Design quality, computational abilities, aircraft fuel efficiency

1. Formulation of the Problem

Design objective is to produce an optimal design. Starting from 1980s, computers have become ubiquitous in engineering design; see, e.g., [1, 8, 10, 12]. An important breakthrough in computer-aided design was Boeing 777, the first commercial airplane which was designed exclusively by using computers; see, e.g., [18].

The main objective of a computer-aided design is to come up with a design which optimizes the corresponding objective function – e.g., fuel efficiency of an aircraft.

Optimization is, in general, NP-hard. The corresponding optimization problems are non-linear, and non-linear optimization problems are, in general, NP-hard; see, e.g., [9, 15]. This means that – under the belief of most computer scientists that P$\neq$NP – a feasible algorithm cannot always find the exact optimum; see, e.g., [5, 14]. In general, we can only find an approximate optimum.

Problem. The more computations we perform, the better the design. It is desirable to come up with a quantitative description of how increasing computational abilities improve the design quality.

2. Analysis of the Problem and the Derivation of the Resulting Formula

Because of NP-hardness, more computations simply means more test cases. In principle, each design optimization problem can be solved by exhaustive search – we can try all possible combinations of parameters, and see which combination leads to the optimal design. This approach may work if we have a small number of parameters, then we can indeed try all possible combinations. If, on average, we have C possible values of each parameter, then:

- we need to compare C test cases when we have only one parameter,
- we need C^2 test cases when we have two parameters,
- and we need C^3 test cases when we have three parameters.

In general, when we have d parameters, we need to analyze C^d test cases. For large systems (e.g., for an aircraft), we have thousands of possible parameters, and for $d \approx 10^3$, the exponential value C^d exceeds the lifetime of the Universe. As a result, for realistic d, instead of the exhaustive search of *all* possible combinations of parameters, we can only test *some* combinations.

NP-hardness means, crudely speaking, that we cannot expect optimization algorithms to be significantly faster than this exponential time C^d. This means that, in effect, all possible optimization algorithm boil down to trying many possible test cases.

When computational abilities increase, we can test more cases. From this viewpoint, increasing computational abilities mean that we can test more cases. Thus, by increasing the scope of our search, we will hopefully find a better design.

How can we describe this in quantitative terms?

How to describe quality of an individual design. Since we cannot do significantly better than with a simple search, the resulting search is not well directed, we cannot meaningfully predict whether the next test case will be better or worse – because if we could, we would be able to significantly decrease the search time.

The quality of the next test case – i.e., in precise terms, the value of the objective function corresponding to the next test case – cannot be predicted and is, in this sense, a random variable.

Many different factors affect the quality of each individual design. It is known that, under reasonable conditions, the distribution of the resulting effect of several independent random factors is close to Gaussian; this fact is known as the *Central Limit Theorem*; see, e.g., [19]. Thus, we can conclude that the quality of a (randomly selected) individual design is normally distributed, with some mean μ and standard deviation σ.

What if we test n possible designs. After computation, we select the design with the largest value of the objective function. Let n denote the number of designs that our program tests. If x_i denotes the quality of the i-th design, then the resulting quality is equal to $x = \max(x_1, \ldots, x_n)$. We know that the variables x_i are independent and identically normally distributed with some mean μ and standard deviation μ. What is the resulting probability distribution for the quality x? What is the expected value of this quality?

To answer this question, let us first reduce this question to its simplest case of a standard normal distribution, with $\mu = 0$ and $\sigma = 1$. It is know that a general normally distributed random variable x_i can be represented as $x_i = \mu + \sigma \cdot y_i$. Since adding μ and multiplying by a positive constant $\sigma > 0$ does not change which of the values are larger and which are smaller, we have

$$x = \max(x_1, \ldots, x_n) =$$

$$\max(\mu + \sigma \cdot y_1, \ldots, \mu + \sigma \cdot y_n) = \mu + \sigma \cdot y,$$

where $y \stackrel{\text{def}}{=} \max(y_1, \ldots, y_n)$.

For large n, the *max-central limit theorem* [2–4, 6] (also known as Fisher-Tippet-Gnedenko Theorem) says that the cumulative distributive function $F(y)$ for y is approximately equal to

$$F(y) \approx F_{EV}\left(\frac{y - \mu_n}{\sigma_n}\right),$$

where:

$$F_{EV}(y) \stackrel{\text{def}}{=} \exp(-\exp(-y))$$

is known as the *Gumbel distribution*,

$$\mu_n \stackrel{\text{def}}{=} \Phi^{-1}\left(1 - \frac{1}{n}\right),$$

$$\sigma_n \stackrel{\text{def}}{=} \Phi^{-1}\left(1 - \frac{1}{n} \cdot e^{-1}\right) - \Phi^{-1}\left(1 - \frac{1}{n}\right),$$

and $\Phi^{-1}(t)$ is the inverse function to the cumulative distribution function $\Phi(y)$ of the standard normal distribution (with mean 0 and standard deviation 1). In other words, the distribution of the random variable y is approximately equal to the distribution of the variable $\mu_n + \sigma_n \cdot \xi$, where ξ is distributed according to the Gumbel distribution.

It is known that the mean of the Gumbel distribution is equal to the Euler's constant $\gamma \approx 0.5772$. Thus, the mean value m_n of y is equal to $\mu_n + \gamma \cdot \sigma_n$. For large n, we get asymptotically

$$m_n \sim \gamma \cdot \sqrt{2\ln(n)},$$

hence the mean value e_n of $x = \mu + \sigma \cdot x$ is asymptotically equal to

$$e_n \sim \mu + \sigma \cdot \gamma \cdot \sqrt{2\ln(n)}.$$

Resulting formula. When we test n different cases to find the optimal design, the quality e_n of the resulting design increases with n as

$$e_n \sim \mu + \sigma \cdot \gamma \cdot \sqrt{2\ln(n)}.$$

3. Case Study of Aircraft Fuel Efficiency Confirms the Theoretical Formula

Case study: brief description. As a case study, let us take the fuel efficiency of commercial aircraft; see, e.g., [11, 16, 17]. It is known that the average energy efficiency E changes with time T as

$$E = \exp(a + b \cdot \ln(T)) = C \cdot T^b,$$

for $b \approx 0.5$.

How to apply our theoretical formula to this case? The above theoretical formulas describes how the quality changes with the number of computational steps n. In the case study, we know how it changes with time T. So, to compare these two formulas, we need to know how the number of computational steps which can be applied to solve the design problem changes with time T. In other words, we need to know how the computer's computational speed – i.e., the number of computational steps that a computer can perform in a fixed time period – changes with time T.

This dependence follows the known *Moore's law*, according to which the computational speed grows exponentially with time T: $n \approx \exp(c \cdot T)$ for some constant c. Crudely speaking, the computational speed doubles every two years; [7, 13].

Applying the theoretical formula to this case study. When $n \approx \exp(c \cdot T)$, we have $\ln(n) \sim T$. Thus, the dependence

$$e_n \sim \mu + \sigma \cdot \gamma \cdot \sqrt{2\ln(n)}$$

of quality $q \stackrel{\text{def}}{=} e_n$ on time takes the form

$$q \approx a + b \cdot \sqrt{T}.$$

This is exactly the empirical dependence that we actually observe.

Thus, *the empirical data confirm the above theoretical formula.*

Comment. It is important to be cautious when testing the formula. For example, in a seemingly similar case of cars, the driving force for their fuel efficiency is not computer design but rather federal and state regulations which prescribe what fuel efficiency should be. Because of this, for cars, the dependence of fuel efficiency on time T is determined by the political will and is, thus, not as regular as for the aircraft.

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