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Why in Mayan Mathematics, Zero and Infinity Are the Same: A Possible Explanation

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Why in Mayan Mathematics, Zero and Infinity Are the Same: A Possible Explanation

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Abstract

In Mayan mathematics, zero is supposed to be, in some sense, equal to infinity. At first glance, while this statement may have a deep philosophical meaning, it does not seem to make much mathematical sense. In this paper, we show, that this statement may be made mathematically reasonable. Specifically, on a real line, it is often useful to consider both $-\infty$ and $+\infty$ as a single infinity. When we deal with very small and very large numbers, it makes sense to use floating point representation, i.e., in effect, consider logarithms of the original values. In terms of logarithms, the original value 0 corresponds to $-\infty$, while the original infinite value corresponds to $+\infty$. When we treat both possible values $-\infty$ and $+\infty$ as a single infinity, we thus treat the original values 0 and ∞ as similar.

1 Formulation of the Problem

Zero is infinity in Mayan math. According to the traditional Mayan teachings, zero and infinite are one. These teaching are described in [4, 5, 6] and summarized in [1].

There are similar philosophical statements in other religious teachings. Similar statements about the similarity between nothing (zero) and everything (infinity) abound in many philosophical and religious teachings. For example, many Jewish thinkers cite a 19 century Rabbi Simhah Bunim who said that "A person should have two pieces of paper, one in each pocket, to be used as necessary. On one of them [is written] 'The world was created for me,' and on the other, 'I am dust and ashes'." [7]; according to Rabbi Bunim, the key to living a successful life is to be guided by both of those statements and keep those two opposing truths in balance.

While philosophically profound, the Mayan statement about zero and infinity does not seem to make much mathematical sense. While the above Mayan statement – that zero and infinity is one – may have deep philosophical roots, from the purely mathematical viewpoint, it does not seem to make sense: a number zero is clearly different from infinity.

What we do in this paper. In this paper, we provide a mathematical explanation in which the "equality" between zero and infinity starts making sense.

2 Our Idea

Infinity in mathematical description of numbers: a brief reminder. The above Mayan statement equates zero and infinity. So, to analyze this statement, let us briefly recall the use of infinity in describing real numbers.

A consistent use of infinity in describing real numbers emerged with the development of calculus, where infinity appears as a limit. Some sequences of real numbers, such as $x_n = \frac{1}{n}$, have finite limits: e.g., for the above sequence, we have $x_n \to 0$. Other sequences, such as $y_n = n$, increase indefinitely, they do not have a finite limit, so we say that $y_n \to +\infty$. Similarly, for a sequence $z_n = -n$, we have $z_n \to -\infty$.

For such sequences, we have two different infinities: $+\infty$ and $-\infty$, located at two different sides of the real line.

Sometimes, we have a singly infinity. If we know the limit x of a sequence x_n, i.e., if we know that $x_n \to x$, then for $x \neq 0$, we can conclude that the sequence $\frac{1}{x_n}$ tends to $\frac{1}{x}$.

What if $x = 0$?

- For a sequence $x_n = \frac{1}{n} \to 0$, we have $\frac{1}{x_n} = n \to +\infty$.
- For a sequence $x_n = -\frac{1}{n} \to 0$, we have $\frac{1}{x_n} = -n \to -\infty$.
- For an oscillating sequence $x_n = \frac{(-1)^n}{n} \to 0$, we have $\frac{1}{x_n} = (-1)^n \cdot n$, and this sequence converges neither to $+\infty$ nor to $-\infty$.

To describe the behavior of such oscillating sequences, we can "merge" the two previously separate infinities $-\infty$ and $+\infty$ into a single infinity, and say that these sequence "converge to ∞".

This merging enables us to extend the above rule about the limit of the sequence $\frac{1}{x_n}$ to the case when $x_n \to x = 0$: in this case, we have $\frac{1}{x_n} \to \frac{1}{x}$, where we defined $\frac{1}{0} \stackrel{\text{def}}{=} \infty$.

Comment. It should be mentioned that while for *real* numbers, having a single infinity is a (reasonably minor) convenience, for *complex* numbers, the existence of a single infinite point is a must for many methods and results; see, e.g., [2].

Unsigned infinities are also useful for computations. Infinities are useful no only in pure math, they are also unseful for computations. For example, infinities are often useful to make sure that seemingly equivalent algebraic transformations of an expression do not change its computed value. For example, an expression $\dfrac{a}{a+b}$ can be represented in the equivalent form $\dfrac{1}{1+\dfrac{b}{a}}$. A possible problem with this transformation occurs when $a=0$ and $b\ne 0$; in this case:

- the original expression is simply equal to 0, while
- the second expression requires division by zero and thus, does not have a direct mathematical sense (at least if we only consider usual (finite) numbers).

To avoid this problem, most computers assume that $\dfrac{b}{0}=\infty$ for $b\ne 0$. In this case, $1+\dfrac{b}{a}=1+\infty=\infty$, and $\dfrac{1}{1+\dfrac{b}{a}}=\dfrac{1}{\infty}=0$.

How to represent numbers ranging from very small to very large: floating point representation is needed. To represent usual-size numbers, we can use a usual fixed point format, and represent, e.g., $\dfrac{1}{4}$ as 0.25. However, if we want to represent all the numbers describing the Universe, from very small numbers describing the size of elementary particles to very large numbers describing the size of galaxies and of the Universe itself, then we cannot avoid using floating point numbers, i.e., numbers of the type $1.2\cdot 10^{-23}$. This is such numbers are described and processed in physics (see, e.g., [3]), this is how computers represent such numbers.

Resulting explanation of Mayan identification of zero and infinity. In mathematical terms, when we represent a real number x in the form $a\cdot 10^b$ with $a\approx 1$, then $b\approx \log_{10}(x)$. From this viewpoint:

- values $x\approx 0$ correspond to $e\approx -\infty$, while
- values $x\approx +\infty$ correspond to $e\approx +\infty$.

When we apply the usual mathematical idea of treating both limit values $e=-\infty$ and $e=+\infty$ as a single infinity, we thus treat the original values $x=0$ and $x=\infty$ as similar.

This provides an explanation for Mayan identification of zero and infinity.

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