Kansei Engineering: Towards Optimal Set of Designs

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Kansei Engineering: Towards Optimal Set of Designs

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Abstract

In many engineering situations, we need to take into account subjective user preferences; taking such preference into account is known as *Kansei Engineering*. In this paper, we formulate the problem of selecting optimal set of designs in Kansei engineering as a mathematical optimization problem, and we provide an explicit solution to this optimization problem.

Need for Kansei Engineering. Traditional engineering deals with objective characteristics of a design: we want a bridge which can withstand a given load, we want a car with a given fuel efficiency, etc. There may be several different designs with the given ranges on characteristics, e.g., we may have different car designs within the given price range, efficiency range, size restrictions, etc. Different people make different choices between these designs based on their subjective preferences.

This is how people select cars, this is how people select chairs, etc. Engineering that takes such subjective preference into account is known as *Kansei Engineering*; see, e.g., [1, 4, 8, 9, 10].

Need to select designs. Different people have different preferences. Thus, to satisfy customers, we must produce several different designs: a car company

produces cars of several different designs, a furniture company produces chairs of several different designs, etc.

The creation of each new design is often very expensive and time-consuming. As a result, the number of new designs is usually limited. The question is: once we know what customers want, and once we know how many different designs we can afford, how should we select these designs?

What we do in this paper. In this paper, we describe a reasonable mathematical model within which we can find an optimal collection of design.

Towards a mathematical model. Let us denote the number of parameters needed to describe different designs by n. Then, each design can be characterized by an n-dimensional vector $x = (x_1, \ldots, x_n)$. Let us assume that the unit of different parameters are selected in such a way that a unit of each parameter represents the same difference for the user. Under this selection, it is reasonable to assume that the user's difference between two designs can be described by the Euclidean distance $d(x, y)$ between the corresponding vectors $x = (x_1, \ldots, x_n)$ and $y = (y_1, \ldots, y_n)$:

$$d(x, x') = \sqrt{\sum_{i=1}^{n} (x_i - y_i)^2}.$$

We have a large number of potential users. For each user, some design is ideal, and the farther we are from this ideal design, the less desirable this design is. For our purposes, we can simply identify each user with this ideal vector x.

There are usually very many users, each of these users can be characterized by a vector x. Ideally, we should record all these vectors, but in practice, it is reasonable to describe how many users are in different zones. In other words, a reasonable way to describe the users is to provide the distribution on the set of all possible designs that characterizes how popular different designs are. A natural way to describe a distribution of customers is to provide the population density $\rho_u(x)$ at different points x from the corresponding n-dimensional region. For this function, $\rho_u(x) \geq 0$ and the integral $\int \rho_u(x)\, dx$ is equal to the total number of potential customers.

Similarly, we can have a large number of engineered designs. So, instead of explicitly listing these designs, we can simply describe how many different designs are manufactured in different zones. Let us describe the corresponding design density by $\rho_m(x)$. Here, $\rho_m(x) \geq 0$ and

$$\int \rho_m(x)\, dx = D, \tag{1}$$

where D denotes the total number of designs.

If a manufacturer produces an ideal design, then the potential customer will buy it for sure. The larger the distance between the ideal and the actual designs, the less probable it is that the customer will purchase this design. Let $p(r)$ be

the probability that a customer will purchase a design at distance r from the ideal one.

When the average density of the actual designs is $\rho_m(x)$, this means that in an area of linear size r and volume $V = r^n$, we have $\rho_m(x) \cdot r^n$ designs. So, we have one design in the area of size r for which $\rho_m(x) \cdot r^n = 1$. This equality leads to $r = \dfrac{1}{\sqrt[n]{\rho_m(x)}}$. So, around the point x, the probability that a customer buys a design is equal to $p(r) = p\left(\dfrac{1}{\sqrt[n]{\rho_m(x)}}\right)$. In the area of volume dx around the point x, there are $\rho_u(x)\,dx$ customers. Since the proportion $p(r)$ of them buys the design, the total number of customers in this area who purchased some design is equal to

$$\rho_u(x) \cdot p(r)\,dx = \rho_u(x) \cdot p\left(\frac{1}{\sqrt[n]{\rho_m(x)}}\right) dx.$$

Thus, the total number C of customers who bought our designs is equal to

$$C = \int \rho_u(x) \cdot p\left(\frac{1}{\sqrt[n]{\rho_m(x)}}\right) dx. \tag{2}$$

Our objective is to maximize the overall profit. Let s be our gain from selling a single unit. Then, by selling units to C customers, we gain the amount $C \cdot s$. Let d be the cost of generating one design; then, by producing D designs, we spend the amount $D \cdot d$. If we subtract the expenses from the gain, we get the profit

$$M = C \cdot s - D \cdot d. \tag{3}$$

Resulting optimization problem. We are given the functions $\rho_u(x)$ and $p(r)$ and the values s and d. We need to select a function $\rho_m(x)$ for which the profit (3) is the largest possible, where the values C and D by using formulas (1) and (2). In other words, we need to optimize the following expression:

$$M = s \cdot \int \rho_u(x) \cdot p\left(\frac{1}{\sqrt[n]{\rho_m(x)}}\right) dx - d \cdot \int \rho_m(x)\,dx. \tag{4}$$

Towards a solution. To solve the above optimization problem, we differentiate the objective function M by each unknown $\rho_m(x)$ and equate the resulting derivative to 0. Thus, we get

$$s \cdot \frac{1}{n} \cdot p'\left(\frac{1}{\sqrt[n]{\rho_m(x)}}\right) \cdot \frac{1}{\sqrt[n]{\rho_m(x)} \cdot \rho_m(x)} \cdot \rho_u(x) - d = 0, \tag{5}$$

where $p'(r)$ is the derivative of $p(r)$. By moving d to the right-hand side, we get an equivalent formula

$$s \cdot \frac{1}{n} \cdot p'\left(\frac{1}{\sqrt[n]{\rho_m(x)}}\right) \cdot \frac{1}{\sqrt[n]{\rho_m(x)} \cdot \rho_m(x)} \cdot \rho_u(x) = d. \tag{6}$$

By dividing both sides by $s \cdot \dfrac{1}{n} \cdot \rho_u(x)$, we keep all the terms depending on the unknowns in the left-hand side and move all the known terms to the right-hand side:

$$p'\left(\frac{1}{\sqrt[n]{\rho_m(x)}}\right) \cdot \frac{1}{\sqrt[n]{\rho_m(x)} \cdot \rho_m(x)} = \frac{d \cdot n}{s \cdot \rho_u(x)}. \tag{7}$$

Thus, for $z \stackrel{\text{def}}{=} \dfrac{1}{\sqrt[n]{\rho_m(x)}}$, we get an equation

$$p'(z) \cdot z^{n+1} = \frac{d \cdot n}{s \cdot \rho_u(x)}. \tag{8}$$

Thus, if we denote by i the function which is inverse to $p'(z) \cdot z^{n+1}$, we get, for z, an explicit formula

$$z = i\left(\frac{d \cdot n}{s \cdot \rho_u(x)}\right) \tag{9}$$

Once we know $z = \dfrac{1}{\sqrt[n]{\rho_m(x)}}$, we can the reconstruct the desired density $\rho_m(x)$ as $\rho_m(x) = \dfrac{1}{z^n}$, i.e., as

$$\rho_m(x) = \frac{1}{\left(i\left(\dfrac{d \cdot n}{s \cdot \rho_u(x)}\right)\right)^n}. \tag{10}$$

So, we arrive at the following solution to our original problem.

Solution. Let us form an auxiliary function $p'(z) \cdot z^{n+1}$, where $p'(z)$ denotes a derivative, and then form an inverse function $i(z)$ to this auxiliary function. In other words, we define $i(z)$ in such a way that $i(p'(z) \cdot z^{n+1}) = z$ for all z. Then, the optimal distribution $\rho_m(x)$ of designs can be described by the formula (10).

Comment. Similar arguments are used to select optimal sensor placements [2, 5, 6, 7], in optimal setting of cloud computing [3], etc.

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References

[1] V.-N. Huynh, Y. Nakamori, and H. Yan, "A comparative study of target-based evaluation of traditional craft patterns using Kansei data", In: Y. Bi and M.-A. Williams (Eds.), *Proceedings of the 4th International Conference on Knowledge Science, Engineering and Management, KSEM'2010*, Belfast, Northern Ireland, UK, September 1–3, 2010, Springer Lecture Notes in Computer Science, Vol. 6291, 2010, pp. 160–173.

[2] O. Lerma, E. Gutierrez, C. Kiekintveld, and V. Kreinovich, "Towards optimal knowledge processing: from centralization through cyberinsfrastructure to cloud computing", *International Journal of Innovative Management, Information & Production (IJIMIP)*, 2011, Vol. 2, No. 2, pp. 67–72.

[3] O. Lerma, C. Tweedie, and V. Kreinovich, "Towards optimal sensor placement in multi-zone measurements", *Proceedings of the 30th Annual Conference of the North American Fuzzy Information Processing Society NAFIPS'2011*, El Paso, Texas, March 18–20, 2011.

[4] M. Nagamachi, *Kansei/Affective Engineering*, CRC Press, Boca Raton, Florida, 2011.

[5] H. T. Nguyen, O. Kosheleva, V. Kreinovich, and S. Ferson, "Trade-off between sample size and accuracy: case of dynamic measurements under interval uncertainty", In: V.-N. Huynh, Y. Nakamori, H. Ono, J. Lawry, V. Kreinovich, and H. T. Nguyen (eds.), *Interval/Probabilistic Uncertainty and Non-Classical Logics*, Springer-Verlag, Berlin-Heidelberg-New York, 2008, pp. 45–56.

[6] H. T. Nguyen, O. Kosheleva, V. Kreinovich, and S. Ferson, "Trade-off between sample size and accuracy: case of measurements under interval uncertainty", *International Journal of Approximate Reasoning*, 2009, Vol. 50, No. 8, pp. 1164–1176.

[7] H. T. Nguyen and V. Kreinovich, "Trade-off between sample size and accuracy: case of static measurements under interval uncertainty", In: V.-N. Huynh, Y. Nakamori, H. Ono, J. Lawry, V. Kreinovich, and H. T. Nguyen (eds.), *Interval/Probabilistic Uncertainty and Non-Classical Logics*, Springer-Verlag, Berlin-Heidelberg-New York, 2008, pp. 32–44.

[8] M. Ryoke, Y. Nakamori, and V.-N. Huynh, "Personalized recommendation for traditional crafts using fuzzy correspondence analysis with Kansei data and OWA operator", In: V.-N. Huynh, Y. Nakamori, H. Ono, J. Lawry, V. Kreinovich, and H/ T. Nguyen (eds.), *Interval/Probabilistic Uncertainty and Non-Classical Logics*, Springer Verlag, Berlin, Heidelberg, New York, 2008, pp. 311–325.

[9] H. Yan, V.-N. Huynh, T. Murai, and Y. Nakamori, "Kansei evaluation based on prioritized multi-attribute fuzzy target-oriented decision analysis", *Information Sciences*, 2008, Vol. 178, No. 21, pp. 4080–4093.

[10] H. Yan, V.-N. Huynh, Y. Nakamori, "A group nonadditive multiattribute consumer-oriented Kansei evaluation model with an application to traditional crafts", *Annals of Operations Research*, 2012, Vol. 195, No. 1, pp. 325–354.