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Optimizing Trajectories for Unmanned Aerial Vehicles (UAVs) Patrolling the Border

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Optimizing Trajectories for Unmanned Aerial Vehicles (UAVs) Patrolling the Border

Chris Kiekintveld, Vladik Kreinovich, and Octavio Lerma

***Abstract*— At first glance, most aspects of border protection activity look like classical examples of zero-sum games, in which the interests of the two sides are exactly opposite. This is how such situations are planned now: this is how border patrol agents are assigned to different segments of the border, this is how routes of coast guard ships are planned, etc. However, there is a big difference between such situations and the traditional zero-sum games: in the traditional zero-sum games, it is assumed that we know the exact objective function of each participant; in contrast, in border protection planning (e.g., in counter-terrorism planning), the adversary's objective function is rarely known in precise terms; at best, we have the description of this objective function in terms of words from natural language. In this paper, on an example of an UAV patrolling the border, we show how fuzzy techniques can help in planning border protection strategies under such uncertainty.**

I. Patrolling the Border: a Practical Problem

Remote areas of international borders can be (and are) used by the adversaries: to smuggle drugs, to bring in weapons. It is therefore desirable to patrol the border, to minimize such actions.

Even with the current increase in the number of border patrol agents, it is not possible to effectively man every single segment of the border. It is therefore necessary to rely on other types of surveillance.

Unmanned Aerial Vehicles (UAVs) are an efficient way of patrolling the border:

- from every location along the border, they provide an overview of a large area, and
- if needed at a different location, they can move reasonably fast to the new location, without being slowed down by clogged roads or rough terrain.

However, while the area covered by the UAV is large, it is still limited. Due to resource limitations, we cannot have all the points on the border under a constant UAV surveillance. Thus, within a portion of the border that is covered by a UAV, it is necessary to keep the UAV moving.

II. How to Describe Possible UAV Patrolling Strategies

For simplicity, let us assume that the UAV can fly reasonably fast along the border, so that for each point, the interval between two consequent overflies does not exceed the time $2T$ needed to successfully cross the border area back-and-forth.

In the ideal case, this would means that the UAV is capable of detecting all adversaries – and thus, preventing all border violations. In reality, however, a fast flying UAV can miss the adversary. It is therefore desirable to select a trajectory that would minimize the effect of this miss.

The faster the UAV goes pass a certain location, the less time it spends in the vicinity of this location, the more probable it is that the UAV will miss the adversary. From this viewpoint, an important characteristic of the trajectory is the velocity $v(x)$ with which the UAV passes through the location x. So, by a patrolling strategy, we will mean a function $v(x)$ that describes how fast the UAV flies at different locations.

This strategy must be selected in such a way that a total time for a UAV to go from one end of the area to another one is equal to the given value T. The time during which a UAV passes from the location x to the location $x + \Delta x$ is equal to

$$\Delta t = \frac{\Delta x}{v(x)}. \tag{1}$$

Thus, the overall flight time is equal to the sum of these times, i.e., to

$$T = \int \frac{dx}{v(x)}, \tag{2}$$

where the integral is taken over the whole length of the border segment.

From the mathematical viewpoint, an arbitrary non-negative function $v(x)$ can describe the velocity at different locations. In practice, not every function $v(x)$ can be implemented, since the UAV has the largest possible velocity V, so we must have $v(x) \le V$ for all x.

From the computational viewpoint, it is convenient, instead of the velocity $v(x)$, to use its reciprocal

$$s(x) \stackrel{\text{def}}{=} \frac{1}{v(x)}. \tag{3}$$

In the geosciences, this reciprocal is called *slowness*; see, e.g., [1] and references therein; we will use this term in this paper as well.

In terms of slowness, the requirement that the overall time be equal to T has a simpler form

$$T = \int s(x)\, dx. \tag{4}$$

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In terms of slowness $s(x)$, the velocity limitation

$$v(x) \le V \tag{5}$$

takes the form $s(x) \ge S$, where $S \stackrel{\text{def}}{=} \dfrac{1}{V}$. Since $s(x) \ge S$, the value $s(x)$ can be represented as $S + \Delta s(x)$, where $\Delta s(x) \stackrel{\text{def}}{=} s(x) - S$ satisfy the simpler constraint $\Delta s(x) \ge 0$.

In terms of $\Delta s(x)$, the requirement that the overall time be equal to T has a simpler form

$$T = S \cdot L + \int \Delta s(x)\,dx, \tag{6}$$

where L is the total length of the piece of the border that we are defending, or, equivalently,

$$T_0 = \int \Delta s(x)\,dx, \tag{7}$$

where $T_0 \stackrel{\text{def}}{=} T - S \cdot L$.

III. Probability of Detection

In order to select a reasonable patrolling strategy, we must find out, for each strategy, what is the probability that under this strategy, the adversary can still cross the border.

Let h denote a distance at which the UAV can still see. This means that when the adversary is trying to cross at location x, a UAV can, in principle, observe this adversary when it is located in the zone between $x - h$ and $x + h$. The width of this zone is equal to

$$(x + h) - (x - h) = 2h. \tag{8}$$

We have denoted the UAV's velocity at location x by $v(x)$. So, the time that it takes for a UAV to cross the zone of width $2h$ is equal to

$$t_{\text{obs}} = \frac{2h}{v(x)}. \tag{9}$$

In terms of slowness, this expression takes a simpler form

$$t_{\text{obs}} = 2h \cdot s(x). \tag{10}$$

Let Δt denote the time during which a UAV takes one snapshot of the underlying area. In these terms, during the crossing time t_{obs}, the UAV can take

$$n(x) = \frac{t_{\text{obs}}}{\Delta t} = \frac{2h}{\Delta t} \cdot s(x) \tag{11}$$

snapshots.

Let p_1 be the probability that an adversary can avoid detection based on a single snapshot. Then, to avoid detection during several snapshots means to avoid detection during the first snapshot, during the second snapshot, etc. It is reasonable to assume that the misses corresponding to different snapshots are statistically independent. Under this assumption, the probability $p(x)$ to be missed under $n(x)$ snapshots is equal to the product of $n(x)$ probabilities of a miss corresponding to different snapshots, i.e., equal to

$$p(x) = p_1^{n(x)}. \tag{12}$$

Substituting the above expression for $n(x)$ in terms of $s(x)$, we conclude that

$$p(x) = p_1^{(2h/\Delta t)\cdot s(x)}, \tag{13}$$

i.e., that

$$p(x) = \exp(-k \cdot s(x)), \tag{14}$$

where we denoted

$$k \stackrel{\text{def}}{=} \frac{2h}{\Delta t} \cdot |\ln(p_1)|. \tag{15}$$

IV. Relative Importance of Different Locations

We also need to take into account that different locations along the border have different importance.

For example, if smugglers succeed in bringing drugs to the vicinity of the city of El Paso, they can store in a safe place and distribute it without exposure. On the other hand, if they bring the same shipment in the remote desert area, they still need to bring it close to a town or a city, and risk being detected while they are transporting this shipment.

In the case of smugglers, this importance can be described in monetary terms: a shipment available in city can be sold for a much larger amount than a shipment available at some remote location from which it still has to be transported to a city. The corresponding price $w(x)$ of the shipment successfully transported across the border at a point with coordinate x can be used as a measure of potential benefit, for the adversary, of penetrating the border at this particular location.

For other types of border penetration, we can also similarly estimate the potential benefit to the adversary.

We will start our analysis with a simplified case when we know the exact value of $w(x)$ for all x. After that, we will explain how to deal with a more realistic case, when we only know $w(x)$ with uncertainty.

V. Decision Making: Reminder

We assume that the adversary has observed the UAV, so the adversary knows the slowness function $s(x)$ and is, thus, capable of computing the probability $p(x)$ of avoiding detection. How does an adversary make decisions based on this knowledge?

A standard way to describe preferences of a decision maker is to use the notion of *utility*; see, e.g., [3], [4], [5], [9], [11]. To describe the utility of an outcome A, we need to select two extreme outcomes: a very unfavorable alternative A_- and a very favorable outcome A_+.

We assume that all outcomes A in which we are interested are better than A_- and worse than A_+. If we denote the relation "the decision maker prefers A' to A" by $A \le A'$, then we can describe this assumption as $A_- \le A \le A_+$.

Then, for each probability $p \in [0, 1]$, we can consider a *lottery* $L(p)$ in which we have A_+ with probability p and A_- with the remaining probability $1 - p$.

For $p = 1$, the lottery $L(p)$ coincides with A_+, so we have $A \le A(1)$. For $p = 0$, the lottery $L(p)$ coincides with A_-, so we have $A(0) \le A$. The larger p, i.e., the larger the

probability of a beneficial event A_+, the more beneficial is the lottery $L(p)$ for the decision maker. So, if $p < q$, then $L(p) < L(q)$.

Let p_0 be the infimum (greatest lower bound) of the set of all the values p for which $A \leq L(p)$. Then:

- When $p < p_0$, then for $\widetilde{p} = (p + p_0)/2$, we have $\widetilde{p} < p_0$ and thus, by definition of the infimum, we cannot have $A \leq L(\widetilde{p})$. Thus, we have $L(\widetilde{p}) \leq A$. Since $p < \widetilde{p}$, we have $L(p) < L(\widetilde{p}) \leq A$ and thus, $L(p) < A$.
- When $p > p_0$, then, since p_0 is the greatest lower bound, p is not a lower bound, i.e., there exists a value $\widetilde{p}$ for which $A \leq L(\widetilde{p})$ and $\widetilde{p} < p$. Since $\widetilde{p} < p$, we have $L(\widetilde{p}) < L(p)$ hence $A < L(p)$.

Thus, we have the value p_0 that has the following property:

- when $p < p_0$, the corresponding lottery is worse than the event A:
$$L(p) < A; \tag{16}$$
- when $p > p_0$, the corresponding lottery is better than the event A:
$$L(p) > A. \tag{17}$$

This threshold value p_0 is called the *utility* of the event A. The utility is usually denoted by $u(A)$.

We can simplify the above somewhat complicated relation between A and p_0 by saying that the event $L(p_0)$ is *equivalent* to A. We will denote this equivalence by $A \sim L(p_0)$.

The notion of utility depends on the choice of the outcomes A_- (for which utility is 0) and A_+ (for which utility is 1). In principle, we select different outcomes A'_- and A'_+. One can show that the new value $u'(A)$ is linearly related to the old one: $u'(A) = a \cdot u(A) + b$, where:

- $b = u'(A_-)$ is the utility of A_- in the new scale, and
- $a + b = u'(A_+)$ is the utility of A_+ in the new scale, so we can determine a as $u'(A_+) - u'(A_-)$.

In other words, utility is defined modulo an arbitrary linear transformation
$$u(A) \to u'(A) = a \cdot u(A) + b. \tag{18}$$

In practice, we can rarely predict the exact consequences of each decision. The consequences depend on the circumstances. For example, if we decide whether to take an umbrella or not, the consequences of this decision depend on whether it will rain or not. In the ideal situation, we know the probabilities $p_1, \ldots, p_n$ of different possible consequences $E_1, \ldots, E_n$. In other words, the action leads to E_1 with probability p_1, to E_2 with probability p_2, …, and to E_n with probability p_n.

By definition of the utility, the event E_1 is equivalent to a lottery $L(u(E_1))$ in which we get A_+ with probability $u(E_1)$, the event E_2 is equivalent to a lottery $L(u(E_2))$ in which we get A_+ with probability $u(E_2)$, etc. Thus, the original action is equivalent to the composite lottery, in which:

- with probability p_1, we get a lottery that results in A_+ with probability $u(E_1)$, and in A_- otherwise;
- with probability p_2, we get a lottery that results in A_+ with probability $u(E_2)$, and in A_- otherwise;
- …

In this composite lottery, we get either A_+ or A_-, and the probability of getting A_+ can be easily computed as
$$u \stackrel{\text{def}}{=} p_1 \cdot u(E_1) + p_2 \cdot u(E_2) + \ldots + p_n \cdot u(E_n). \tag{19}$$

Thus, the original action is equivalent to the lottery $L(u)$. By definition of the utility, this means that the utility of the action is equal to u.

From the mathematical viewpoint, u is the expected value of the utility of different consequences, so we can conclude that the utility of an action is the expected value of utilities of its consequences.

VI. Strategy Selected by the Adversary

We have already mentioned that utility is defined modulo an arbitrary linear transformation. For convenience, let us select the utility scale in such a way that for the adversary, the utility of not being able to cross the border is 0.

In this scale, let $w(x)$ denote the utility of the adversary succeeding in crossing the border at location x. We have assumed that we know the exact value of $w(x)$ for every location x.

According to decision theory, the adversary will select a location x at which the expected utility
$$u(x) = p(x) \cdot w(x) = \exp(-k \cdot s(x)) \cdot w(x) \tag{20}$$
is the largest possible.

Thus, for each slowness function $s(x)$, the adversary's gain $G(s)$ is equal to
$$G(s) = \max_x u(x) = \max_x \left[\exp(-k \cdot s(x)) \cdot w(x)\right]. \tag{21}$$

VII. Towards an Optimal Strategy for Patrolling the Border

Our objective is to select a strategy $s(x)$ for which the gain $G(s)$ is the smallest possible.

Let x_m be the location at which the utility $u(x) = \exp(-k \cdot s(x)) \cdot w(x)$ attains its largest possible value. If close to x_m, we have a point x_0 for which $u(x_0) < u(x_m)$ and $s(x_0) > S$, then we can slightly decrease the slowness $s(x_0)$ at the vicinity of x_0 (i.e., go faster in this vicinity) and use the resulting time to slow down (i.e., to go slower) at all locations x at which $u(x) = u(x_m)$. As a result, we slightly decrease the value $u(x_m) = \exp(-k \cdot s(x_m)) \cdot w(x_m)$.

Yes, we also slightly increase the value
$$u(x_0) = \exp(-k \cdot s(x_0)) \cdot w(x_0), \tag{22}$$
but for small changes, this value is still smaller that $u(x_m)$ and thus, does not affect the maximum $\max_x u(x)$. As a result, the gain $G(s)$ decreases (this argument is similar to the one presented in [6]).

So, when the adversary's gain is minimized, we get
$$u(x) = u_0 = \text{const} \tag{23}$$

hence

$$\exp(-k \cdot s(x)) = \frac{u_0}{w(x)}, \tag{24}$$

thence

$$s(x) = \frac{1}{k} \cdot (\ln(w(x)) - \ln(u_0)) \tag{25}$$

and

$$\Delta s(x) = \frac{1}{k} \cdot \ln(w(x)) - \Delta_0, \tag{26}$$

where

$$\Delta_0 \stackrel{\text{def}}{=} -\frac{1}{k} \cdot \ln(u_0) - S. \tag{27}$$

When this value gets to $s(x) = S$ and $\Delta s(x) = 0$, we get $\Delta s(x) = S$. Thus, we conclude that

$$\Delta s(x) = \max\left(\frac{1}{k} \cdot \ln(w(x)) - \Delta_0, 0\right). \tag{28}$$

The value Δ_0 can be determined from the condition that

$$\int \Delta s(x)\, dx =$$

$$\int \max\left(\frac{1}{k} \cdot \ln(w(x)) - \Delta_0, 0\right) dx = T_0. \tag{29}$$

Since this integral monotonically decreases with Δ_0, we can use bisection to find the appropriate value Δ_0; see, e.g., [2].

VIII. Towards Taking Fuzzy Uncertainty into Account

The above algorithm is based on the assumption that we know the exact value of the adversary's gain $w(x)$ at different locations. In reality, as we have mentioned, we only have expert estimates for $w(x)$. To formalize these estimates, we can use fuzzy techniques; see, e.g., [8], [10].

Once we have the fuzzy values $w(x)$, we can apply Zadeh's extension principle to the above crisp formulas and thus, come up with fuzzy recommendations about the slowness, such as "go somewhat slow here", "go fast", etc. It is well known (see, e.g., [8], [10]) that Zadeh's extension principle is equivalent to processing α-cuts. Specifically, if we know a relation $y = f(x_1, \ldots, x_n)$ between the inputs $x_1, \ldots, x_n$ and the desired value y, and we know the fuzzy values $X_1, \ldots, X_n$ of the inputs, then the resulting fuzzy value Y of the output can be obtained as follows: for every $\alpha \in (0, 1]$, we have

$$Y(\alpha) = f(X_1(\alpha), \ldots, X_n(\alpha)) =$$

$$\{f(x_1, \ldots, x_n) : x_1 \in X_1(\alpha), \ldots, x_n \in X_n(\alpha)\}, \tag{30}$$

where for each fuzzy value Z with a membership function $\mu_Z(z)$, its α-cut $Z(\alpha)$ is defined as

$$Z(\alpha) \stackrel{\text{def}}{=} \{z : \mu_Z(z) \geq \alpha\}. \tag{31}$$

When a fuzzy value is a fuzzy number, each α-cut is an interval $Z(\alpha) = [\underline{Z}(\alpha), \overline{Z}(\alpha)]$. When all the inputs are fuzzy numbers, the above formula takes the simplified form

$$[\underline{Y}(\alpha), \overline{Y}(\alpha)] =$$

$$\{f(x_1, \ldots, x_n) : x_i \in [\underline{X}_i(\alpha), \overline{X}_i(\alpha)]. \tag{32}$$

When the function $y = f(x_1, \ldots, x_n)$ is an increasing function of all its variables, then its largest value is attained when all its inputs attain their largest values, and its smallest value is attained when all its inputs attain their smallest values. In other words, the desired α-cut has the form $[\underline{Y}(\alpha), \overline{Y}(\alpha)]$, where

$$\underline{Y}(\alpha) = f(\underline{X}_1(\alpha), \ldots, \underline{X}_n(\alpha)); \tag{33}$$

$$\overline{Y}(\alpha) = f(\overline{X}_1(\alpha), \ldots, \overline{X}_n(\alpha)). \tag{34}$$

Similarly, when the function $y = f(x_1, \ldots, x_n)$ is an increasing function of the variables $x_1, \ldots, x_k$ and decreasing in $x_{k+1}, \ldots, x_n$), then the α-cut has the form $[\underline{Y}(\alpha), \overline{Y}(\alpha)]$, where

$$\underline{Y}(\alpha) =$$

$$f(\underline{X}_1(\alpha), \ldots, \underline{X}_k(\alpha), \overline{X}_{k+1}(\alpha), \ldots, \overline{X}_n(\alpha)); \tag{35}$$

$$\overline{Y}(\alpha) =$$

$$f(\overline{X}_1(\alpha), \ldots, \overline{X}_k(\alpha), \underline{X}_{k+1}(\alpha), \ldots, \overline{X}_n(\alpha)). \tag{36}$$

In our case, for each location x, we know the fuzzy value $W(x)$ of the corresponding gain. This means that for each degree α, we know the corresponding α-cut $W(x)(\alpha) = [\underline{W(x)}(\alpha), \overline{W(x)}(\alpha)]$.

In the crisp case, based on the gains $w(x)$, we first compute the value Δ_0 and then the corresponding changes $\Delta s(x)$ in the UAV's slowness. Thus, in the fuzzy case, we need to find the α-cuts for Δ_0 and then, α-cuts for $\Delta s(x)$.

According to the above formula for Δ_0, its value is an increasing function of all the inputs $w(x)$. Thus, we conclude that for every α, the α-cut for Δ_0 has the form $[\underline{\Delta}_0(\alpha), \overline{\Delta}_0(\alpha)]$, where $\underline{\Delta}_0(\alpha)$ can be determined from the condition that

$$\int \max\left(\frac{1}{k} \cdot \ln(\underline{W(x)}(\alpha)(x)) - \underline{\Delta}_0(\alpha), 0\right) dx = T_0, \tag{37}$$

and $\overline{\Delta}_0(\alpha)$ can be determined from the condition that

$$\int \max\left(\frac{1}{k} \cdot \ln(\overline{W(x)}(\alpha)(x)) - \overline{\Delta}_0(\alpha), 0\right) dx = T_0. \tag{38}$$

The value $\Delta s(x)$ is increasing in $w(x)$ and decreasing in Δ_0. Thus,

- the smallest value $\underline{\Delta s(x)}(\alpha)$ is attained when $w(x)$ is the smallest and $\overline{\Delta_0}$ is the largest, and
- the largest value $\overline{\Delta s(x)}(\alpha)$ is attained when $w(x)$ is the largest and Δ_0 is the smallest:

$$\underline{\Delta s(x)}(\alpha) = \max\left(\frac{1}{k} \cdot \ln(\underline{W(x)}(\alpha)) - \overline{\Delta}_0(\alpha), 0\right); \tag{39}$$

$$\overline{\Delta s(x)}(\alpha) = \max\left(\frac{1}{k} \cdot \ln(\overline{W(x)}(\alpha)) - \underline{\Delta}_0(\alpha), 0\right). \tag{40}$$

The resulting recommendations can be used either as a guidance for a human controller, or – by using fuzzy control – in the design of the automatic controller.

Comment. Fuzzy techniques can be similarly used in other problems related to security, e.g., in finding optimal placement for bio-weapon detectors [7].

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