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Complexity of Collective Decision Making Explained by Neural Network Universal Approximation Theorem

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STRONG NEGATION: ITS RELATION TO INTERVALS AND ITS USE IN EXPERT SYSTEMS

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I. INTERVALS IN DIFFERENT LOGICS: THE ORIGIN OF NON-STANDARD NEGATIONS

Order And Intervals in Logic

One of the main objectives of logic is to study statements. From the commonsense viewpoint, statements (propositions) can be described as phrases that can be true or false.

Some phrases of this type (e.g., "2+2=4") refer to abstract mathematical constructs and are, therefore, true or false irrespective of what is happening in the real world. Some other phrases of this type refer to directly observable facts and are, therefore, true or false depending on what we observe. The most interesting case is when a statement A describes some phenomenon that is not easily directly observable. In this case, we cannot easily detect whether the corresponding statement is true or false; instead, we can try to figure out whether this statement S is true or false under different assumptions about the real world. In more precise terms, we are interested in knowing whether S *follows* from a statement (or statements) T that describe these assumptions.

In other words, this "follows from" relation is one of the basic relations in every logic. Intuitively, each statement follows from itself, and if A follows from B, and B follows from C, then A follows from C. Hence, "follows from" is a reflexive and transitive relation, i.e., in mathematical terms, a *pre-ordering*. In the following text, we will denote this relation by $\geq$.

How can we prove that A follows from B, i.e., that $A \geq B$? Due to transitivity, if we cannot prove this implication directly, we can try to do it in two steps: namely, we can try to find some intermediate statement C for which we will be able to prove that $A \geq C$ and $C \geq B$. The set $\{C \mid B \leq C \leq A\}$ of all such intermediate statements C forms what is usually called an *interval*; the interval is usually denoted by $[A, B]$.

Logical Connectives and Their Relation to Order/Interval Structure of a Logic

In the above text, we just talked about statements as a whole, without getting into the structure of an individual statement.

Some statements are structure-less (basic, "atomic"), but some statements can have a complicated structure. For example, we can combine several atomic statements into a single complicated one by using logical connectives such as "and", "or", "not", etc.

The natural question is: when we describe the logic, do we need to describe these connectives "on top" of the order and interval structures, or we can describe the connectives in terms of the order/interval structure. It turns out that the answer to this question depends on what exactly logic we use.

In Classical Logic, Connectives Are Uniquely Determined by the Order/Interval Structure of Logic [2]

- *Disjunction* $A \vee B$ (A or B) can be defined as the weakest statement C (weakest in the sense of $\le$) from which both A and B follow, i.e.,
  - $A \le C$, $B \le C$, and
  - $C \le D$ for all D for which $A \le D$ and $B \le D$.
- Similarly, *conjunction* $A\&B$ (A and B) can be defined as the strongest statement C (strongest in the sense of $\le$) that follows both from A and from B, i.e.,
  - $C \le A$, $C \le B$, and
  - $D \le C$ for all D for which $D \le A$ and $D \le B$.
- *Implication* $A \to B$ can be defined as the strongest statement C (strongest in the sense of $\le$) for which B follows from $A \vee C$, i.e.,
  - $A\&C \le B$, and
  - $D \le C$ for all D for which $A\&D \le B$.
- *Negation* $\neg A$ can now be defined as $A \to F$, where F stands for "false".

The fact that all the connectives can be uniquely described in terms of order/interval relation, means that if we have an *order automorphism* of the original logic L, i.e., a mapping $\varphi : L \to L$ from the set of all statements L into itself that preserves the order/interval structure, then all logical connectives are also preserved under this automorphism, i.e., $\varphi(A \vee B) = \varphi(A) \vee \varphi(B)$, etc.

Quantum Logic and Fuzzy Logic: the Situation is Somewhat Different

Quantum logic (see, e.g., [1, 4]) and fuzzy logic (see, e.g., [3, 5]) are two most well-known generalizations of classical logic:

- *Quantum logic* is oriented towards formalizing physics. In its basic version, statements are linear subspaces of a Hilbert space (i.e., of the space of all square integrable complex-valued functions $\psi(x)$), and $A \le B$ means $A \subseteq B$.
- *Fuzzy logic* is oriented towards formalizing commonsense reasoning. In the basic version of this logic, the "truth values" of different statements are characterized by real numbers from the interval $[0, 1]$, and $A \le B$ means that the real number A is indeed smaller than or equal to the real number B.

In both logics, we can use the above classical-logic construction to describe *disjunction* and *conjunction*:

- In *quantum logic*, conjunction $A\&B$ is an intersection of subspaces A and B, and disjunction $A \vee B$ is a linear space generated by the union $A \cup B$ of the two subspaces
- In *fuzzy logic*, $A \vee B = \max(A, B)$ and $A\&B = \min(A, B)$.

In both logics, there are natural *negation* operations:

- In *quantum logic*, it is natural to define $\neg A$ as the space of all functions ψ that are orthogonal to all functions $\chi \in A$: $\int \psi(x) \cdot \chi^*(x)\, dx = 0$ (where a^* means complex conjugate).
- In *fuzzy logic*, it is reasonable to define $\neg A = 1 - A$.

In both logics, however, this negation operation cannot be uniquely defined based on an order/interval structure, because there exist transformations that preserve the order/interval structure, but that do not preserve this negation operation:

- In *quantum logic*, we can take an arbitrary 1-1 linear transformation $T : L \to L$. This transformation preserves the order/interval structure, but, unless it is an orthogonal transformation, it does not preserve orthogonality and hence, does not preserve negation.
- In *fuzzy logic*, we can take an arbitrary continuous strictly increasing function $T : [0, 1] \to [0, 1]$ for which $T(0) = 0$ and $T(1) = 1$, e.g., $T(x) = x^2$. Such transformations preserve order/interval structure, but not the negation operation.

So, in both logic, negation is not uniquely determined by the order/interval structure. Since both quantum logic and fuzzy logic have direct applied meaning, it is desirable to interpret this non-uniqueness in terms of the corresponding application areas.

Physical Meaning of Non-Uniqueness in Quantum Logic

Intuitively, the non-uniqueness of quantum logic ties in well with the two major ideas underlying modern physics:

- First, it ties in very well with the idea of *non-determinism* (i.e., *non-uniqueness of predictions*), the idea that underlies quantum physics in general as opposed to deterministic pre-quantum (Newtonian) physics.

- Second, the fast that this non-uniqueness is expressed in terms of symmetries ties in very well with the fundamental role of symmetries and symmetry groups in modern physics.

Commonsense Meaning of Non-Uniqueness in Fuzzy Logic

The mathematical non-uniqueness of negation in a formalism of fuzzy logic, a formalism that is aimed at describing commonsense reasoning, seems to imply that, in contrast to two-valued logic, in commonsense reasoning, there exist *different* negation operations.

Such operations have indeed been considered, e.g., *strong negation* (see, e.g., [7] and references therein).

II. STRONG NEGATION AND ITS USE IN AN EXPERT SYSTEM FOR TRADITIONAL ORIENTAL MEDICINE

Introduction

Traditional oriental medicine incorporates hundreds (maybe even thousands) of years of experience. Some parts of it have already been described in precise terms and used in the West. However, there are still methods and ideas in Oriental medicine that seem to work well for various diseases but that are not yet formalized and not yet widely used. It is, therefore, desirable to formalize these methods. In this talk, we describe how these methods can be formalized.

We Need an Expert System

One of the biggest *problems* in incorporating the methods of traditional Oriental medicine is that these methods are not described in precise terms – they rely largely on the experience of the experts. To help other people use these methods, it is, therefore, desirable to incorporate the expert knowledge into a computer-based expert system. To design such a system, we use the experience of designing medical expert systems that formalize Western medicine.

Previous Attempts of Formalizing Methods of Oriental Medicine

Starting from the 1980s, several computer-oriented models of Oriental medicine have been published. These methods followed two approaches that have been successfully used in general expert system designs:

- *Bayesian* approach, in which uncertainty is described by *probabilities.*

- *fuzzy* approach, in which uncertainty is described by *degrees of certainty*, i.e., by numbers from the interval $[0, 1]$ that characterize the expert's degree of certainty.

Alas, none of these models lead to a practical expert system. The main problem with probabilistic models is that they require the knowledge of probabilities.

Finding the *probabilities* requires that we have a large statistical sample of different patients and different treatments. For example, to get the probability with accuracy 10%, we need at least 100 similar cases; to get the probability with accuracy 1%, we need 10,000 similar cases. In most situations, we do not have this statistics.

Previous *fuzzy* models for Oriental medicine followed the pattern that was designed and used in another successful application of fuzzy logic: *fuzzy control.* In fuzzy control, the main knowledge to formalize is *positive*, like: "if the car is going fast, and the road conditions are not good, so the ride is getting rough, then slow down." In Oriental medicine, *negative* knowledge (what you should not do) is also very important. It is, therefore, desirable to incorporate *negative* knowledge into the fuzzy-based expert system. In fuzzy logic, the fact that we have a certain degree of belief in a standard negation $\neg A$ of a statement A may mean two different things:

- That we simply do not have enough information to believe in A 100% (*weak* negation).

- That we know that in some sense or in some cases, A is indeed false (*strong* negation).

On the other hand, negative knowledge, e.g., that some acupuncture procedure is harmful with degree of belief 0.4, is only *strong* negation. So, we must formalize this strong negation.

What We Have Done

We used the resulting formalism with strong negation to design a prototype expert system that describes diagnostics methods of Oriental medicine. This is still a system in progress. So far, we have not formalized all the rules, only the most widely used ones (about 20 positive and about 10 negative rules).

This system is written in C++ on a Pentium-based PC. The results appear practically on-line.

It is a hierarchical system that consists of three subsystems (submodules):

The main module, called *Eight Rule diagnosis*, classifies a patient according to eight dichotomies, which, in Oriental medicine, are called Yin–Yang, cold–heat, deficiency–excess, etc.

After that, the second module called *organ diagnosis* tries to find out which organs are affected.

The third module combines the results (*syndromes*) of the first two subsystems into a single diagnosis.

In addition to the system for *diagnostics*, we have designed several companion systems that transform the diagnosis into the actual *treatment*. Different companion systems describe different types of treatment in Oriental medicine, such as acupuncture, moxibustion, massage, acupression, etc. These systems are written in Turbo Prolog 2.0, with Turbo Pascal as a display interface for images.

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