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Edward Vidal

Luc Longpre
The University of Texas at El Paso, longpre@utep.edu

Vladik Kreinovich
The University of Texas at El Paso, vladik@utep.edu

Huang Haitao

Yeung Yam

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Asymptotically Optimal Algorithms For Weather Applications of Smart Dust

Edward Vidal
Army Research Lab.
White Sands
New Mexico
USA

Luc Longpré, Vladik Kreinovich
Huang Haitao
Dept. Computer Science
University of Texas
El Paso, TX 79968, USA
vladik@cs.utep.edu

Yeung Yam
Dept. Mechanical &
Automation Engineering
The Chinese University of
Hong Kong
yyam@mae.cunh.edu.hk

Abstract

Smart Dust is a collection of small sensor-equipped leaves which send their information to two or more receivers. When a receiver gets a signal from a sensor, it can determine the direction from which this signal came. By combining the directions from two different receivers, we can determine the 3-D locations of all the leaves, and thus, transform their sensor readings into a 3-D picture of the corresponding parameters (temperature, moisture, etc.). In this paper, we describe an asymptotically optimal algorithm for such reconstruction.

1 What Is "Smart Dust"

Smart Dust is a project developed by the University of California at Berkeley under the US DARPA funding [2, 6].

In this project, small surfaces shaped like maple leaves are equipped with temperature and moisture sensors; each particle costs about $30. A small automatic 8-in simple plane lifts a bunch of these leaves up and throws them down. The leaves slowly descend and as they descend they send signals to Earth-based receivers.

One of the potential future applications of this system is to trace wind profiles in the Bay area; it is important for the US Environmental Protection Agency.

2 Weather Application of Smart Dust: Possibilities and Problems

2.1 Possibilities

Since the leaves which form the Smart Dust are spread around the 3-D zone, they provide us with a unique opportunity to measure weather parameters (temperature, moisture, wind, etc.) in different points within this zone and thus, to create a 3-D weather map.

2.2 Problems Related to Measuring Temperature and Moisture

With respect to measuring temperature and moisture, the main difficulty of creating such a 3-D weather map is that when we receive a signal from a leaf,

- we know the *direction* from which we received this signal, but
- we do not know the *distance* to the location of this leaf.

Therefore, we do not know the exact 3-D position of a point where the measurements were made.

2.3 Problems Related to Measuring Wind Velocity and Direction

Similarly, since the leaves do not measure the wind velocity or direction directly, a natural indirect way to determine these parameters is to trace how the location of the leaves change in time. For that, we also need to know their exact 3-D locations (we can also use Doppler measurements of leaves' velocity).

3 Natural Solution to the Problem: Use Two or More Receivers

Since by using a single receiver, we can only determine the *direction* from which the leaf is sending this information but not the exact 3-D *location* of the leaf, a natural idea is to use two or more receivers.

If we know the exact direction from two different receivers, then:

- we know, for each receiver, the straight line on which this leaf is located, and
- thus, we can, in almost all cases, uniquely determine the 3-D location of the leaf as the unique point – which is the intersection of the corresponding two straight lines.

The only case when we cannot uniquely determine this location is when these two lines coincide, i.e., when the leaf is located exactly on the line which connects the two receivers. This is a rare possibility, but if we want to have a unique reconstruction for all the leaves, then we need to add the third receiver; this receiver will lead to a guaranteed uniqueness, and it will also increase the accuracy with which we measure the leaves' locations.

An alternative solution would be, for each leaf:

- to pick the signals from several different beams,
- to use this comparison to determine its exact coordinates (like in GPS), and
- to transmit these coordinates together with its readings.

Unfortunately, this would require adding a lot of sophisticated equipment to the leaf, and it is still not even clear how to place the existing equipment within the required size parameters.

4 Related Problem: Matching Signals Coming From the Same Leaf to Different Receivers

When we use two receivers, then for each receiver, we get a lot of signals from different leaves. To process this information, we must match the signals coming from the same leaf to two different receivers.

We can try to match leaves which send the exact same sensor information, but it is possible that two nearby locations have the same temperature and moisture.

For this match, it is thus beneficial to assign a unique ID to each leaf, an ID which is transmitted together with the sensor information, so that we will be able to trace individual leaves. However, as we have mentioned, at present, no additions to the leaves are possible. Therefore, we must match the leaves without such ID's.

5 In Principle, the Matching Problem Is Solvable

Let R_1 and R_2 be 3-D locations of receivers, and $L_1, \ldots, L_n$ be 3-D locations of leaves. For each leave L_i, the first receiver detects the direction to L_i; based on this information, we can conclude that this sensor is located on the straight-line ray $R_1 \to L_i$ which goes from this receiver R_1 in the observed direction.

The second receiver R_2 also detects the directions to different leaves. We can describe these directions by placing a plane ("screen") S near the receiver and describing each direction by the unique point of intersection S_j between the ray $R_2 \to L_j$ and this screen S.

On the screen plane, the projections of the points from each ray $R_1 \to L_i$ form a 1-D ray r_i which all start at the same point P – projection of R_1 on this plane S. So, we have:

- n points $S_1, \ldots, S_n$ which reflects the directions from this receiver, and
- n rays r_i starting at P.

Rays describe results of the first receiver, points describe the results of the second receiver. To find the 3-D location of each leaf, we must therefore find out which measurements correspond to which, i.e., we must put points S_i in 1-to-1 correspondence with rays r_j.

If a point S_i describes the same leaf as the ray r, then the point belongs to the ray. For almost all configurations R_1, R_2, $L_1, \ldots, L_n$, a point S_i does not belong to the ray r_j if $i \neq j$. Thus, we can match each point S_i with the unique ray to which this point belongs.

6 A Straightforward Algorithm and Its Drawbacks

If we simply check, for each of n points S_i, whether it belongs to the 1^{st}, 2^{nd}, $\ldots$, n^{th} ray, then in the worst case, we will need n^2 checks. For a large number n of leaves, this will mean a lot of computations.

7 An Asymptotically Optimal Matching Algorithm

To decrease the computation time, we can use, in S, polar coordinates with a center in P. Then, S_i belongs to the ray r_j if and only if their angles coincide. So, to match points with rays, we do the following:

- sort the rays by the angle (which takes $n \cdot \log(n)$ time; see, e.g., [1]), and
- then, for each of n points S_i, we use the binary search ($\log(n)$ steps) to find the ray with the same angle.

The total time is thus $2n \cdot \log(n) \ll n^2$.

8 Remaining Open Problem

8.1 Cause of this Problem: Measurement Inaccuracy

The above simple geometric considerations assume that we can measure the exact direction to each leaf. In this case, the more leaves we send, the more points we cover by our measurements and thus, the better the resulting 3-D weather description.

In reality, we can only measure this direction with a certain accuracy $\varepsilon > 0$. So, if the measured direction to the leaf corresponds to a point $\widetilde{S}_i$, the (unknown) point S_i which corresponds to the actual direction to this leaf may lie anywhere within a certain distance from $\widetilde{S}_i$; in other words, the only information that we have about this point is that lies within a *disk* D_i with a center in $\widetilde{S}_i$.

Similarly, on this screen S, the directions from the leaves to the first receiver are described not by rays r_i, but by *sectors* s_i bounded by two close rays which start at P. In this case, we match a disk D_i and a sector s_j when $D_i \cap s_j \neq \emptyset$.

8.2 Measurement Inaccuracy Influences Matching

The more leaves we take, the larger the area covered by the corresponding sectors. For a certain number m of leaves, these sectors will cover the whole screen S ($s_1 \cup \ldots \cup s_m = S$). Thus, if we add one more leaf, we will not be able to match it properly, because the corresponding disk D_{m+1} will intersect not only with its corresponding sector s_{m+1}, but also – since

$$D_{m+1} \subseteq S = s_1 \cup \ldots \cup s_m$$

– with a sector s_i ($1 \le i \le m$) corresponding to one of the previous m leaves.

8.3 Problem: What Is the Optimal Number of Leaves for a Given Accuracy?

Thus, if we take measurement uncertainty into consideration, adding new leaves does not necessarily make the resulting 3-D picture better: if we add too many leaves, we lose the ability to match the signals on two receivers and thus, we do not get any 3-D picture at all. So, if we start with a single leave and add one leaf at a time, then:

- at first, we get better and better pictures, but
- after a certain number of leaves, we reach an optimum after which adding further leaves would only decrease the resulting number of matched measurements.

A natural question is:

For a given measurement accuracy ε, what is the optimal number of leaves?

For example, if we assume that n leaves are uniformly distributed in a given 3-D area, for what n is the expected number of matched leaves the largest possible?

8.4 Expected Solution to This Problem

Since the relative size of each sector is $\sim 2\varepsilon$, the leaves start covering the entire plane when $n \cdot 2\varepsilon \approx 2\pi$. So, intuitively, we expect that this optimal number is $n \sim 1/\varepsilon$. It is desirable to confirm (or disprove) this intuitive estimate, and to get more accurate results.

9 Towards Control of Smart Dust Leaves

9.1 Control of Smart Dust Leaves Must Be As Simple As Possible

For the above scheme to work, we must design and control Smart Dust particles. Due to severe limitations on the leaf size, the control algorithm must be as simple as possible.

9.2 Fuzzy Control Is Desirable

It is known that in many applications, fuzzy control is much simpler than the traditional control; see, e.g., [5] and references therein. It is therefore desirable to use fuzzy control. However, fuzzy control often requires a large number of rules and is, therefore, not very simple.

9.3 Further Simplification Is Necessary

Most current applications of fuzzy control deal with reasonably simple systems, where a small number of control rules is sufficient; straightforward application of the same methodology to more complicated systems leads, sometimes, to unrealistically many rules.

This problem (and methods of handling it) has been described, e.g., in [3, 4].

9.4 Singular Value Decomposition: A Technique for Achieving the Desired Simplification

We will show how, for a reasonable class of fuzzy controllers, we can decrease the number of rules. Let's consider a system with inputs x_1, x_2 and control rules

$$\text{“if } A_{1,i}(x_1) \text{ and } A_{2,j}(x_2) \text{ then } u = u_{i,j}\text{”},$$

where $A_{\ell,j}$ are fuzzy properties, and $u_{i,j}$ are given values. We assume that for $\ell = 1, 2$, the corresponding properties $A_{\ell,i}$ form a *fuzzy partition*, i.e., for every x_ℓ, $\sum_i A_{\ell,i}(x_\ell) = 1$. If we use $a \cdot b$ as a t-norm, addition for combining rules, and center-of-gravity defuzzification, the resulting control is

$$u(x_1, x_2) = \sum_{i,j} u_{i,j} \cdot A_{1,i}(x_1) \cdot A_{2,j}(x_2).$$

The fewer non-zero coefficients $u_{i,j}$, the fewer rules we need. So, to decrease the number of rules, we represent the corresponding bilinear form

$$F = \sum_{i,j} u_{i,j} \cdot y_i \cdot z_j$$

as

$$F = \sum_{p=1}^{P} u'_p \cdot Y_p \cdot Z_p,$$

where Y_p are linear combinations of y_i, Z_p are linear combinations of z_j, i.e.,

$$Y_p = \sum_i c^{(1)}_{p,i} \cdot y_i,$$

$$Z_p = \sum_j c^{(2)}_{p,j} \cdot z_j,$$

and the number P is the smallest possible. Then, we can describe the fuzzy controller as

$$u(x_1, x_2) = \sum_{p=1}^{P} u'_p \cdot A'_{1,p}(x_1) \cdot A'_{2,p}(x_2),$$

where

$$A'_{\ell,p}(x_\ell) = \sum_i c^{(\ell)}_{p,i} \cdot A_{\ell,i}(x_\ell)$$

are the corresponding linear combinations. Thus, P rules are sufficient.

If the matrix $u_{i,j}$ is symmetric, then the desired representation of the bilinear form corresponds to eigenvalues u'_p and eigenvectors $Y_p = Z_p$; in general, the desired representation is known as a Singular Value Decomposition (SVD).

A similar reduction can be achieved if we have three or more inputs x_i. Practical examples (see, e.g., [7]) show that this reduction can indeed be drastic.

Comment. It is worth mentioning that by formally applying the SVD techniques, we will end up with *positive* values of membership functions $A'_{\ell,p}$, which sum up to 1 at all values within the input domain. However, these new membership functions may not all attain a maximum membership degree of 1 at certain point within their input domain, as normally expected of them (see [7]).

9.5 Conclusion

This paper describes the Smart Dust project and outlines the difficulties in determining the 3-D locations of the leaves. In particular, a solution of using two receivers and solving the resultant matching signal problem is described. An asymptotically optimal algorithm which minimizes drastically the computational load compared to the straightforward checking approach is given. The paper also describes the open problem of measurement inaccuracy and its effects. For control, the work proposes using fuzzy controller and applying SVD methods to simplify complexity.

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