12-1998

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Comments:
Technical Report: UTEP-CS-98-32
Published in: *Proceedings of the Second International Workshop on Intelligent Virtual Environments*, Xalapa, Veracruz, Mexico, September 11-12, 1998, pp. 25-41.

Interval Computations, Soft Computing, and Aerospace Applications

(Research Report)

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1 Introduction: Data Processing and Interval Computations

Data processing. In many real-life problems, we are interested in the value y of a physical quantity which is *difficult* or *impossible* to measure directly.

> *For example, we cannot directly measure the distance to a star, or the amount of oil in a given area.*

To measure this quantity y, we:

- measure some other quantities $x_1, \ldots, x_n$ which are related to y by a known dependence $y = f(x_1, \ldots, x_n)$, and then
- compute the estimate $\widetilde{y}$ for the desired quantity y by applying the algorithm f to the results $\widetilde{x}_i$ of measuring the quantities x_i: $\widetilde{y} = f(\widetilde{x}_1, \ldots, \widetilde{x}_n)$.

This two-stage process is called *indirect measurement*, and computing f is called *data processing*.

> *For example, to estimate the amount of oil in a given area, we may use geophysical data plus satellite images of this area.*

Error estimation of the results of data processing: mathematical statistics and interval computations. Values $\widetilde{x}_i$ come from measurements, and measurements are never 100% accurate; therefore, $\widetilde{x}_i \neq x_i$. Due to the inaccuracies $\Delta x_i = \widetilde{x}_i - x_i$ of direct measurements, the result $\widetilde{y} = f(\widetilde{x}_1, \ldots, \widetilde{x}_n)$ is, in general, different from the desired value $y = f(x_1, \ldots, x_n)$: $\Delta y = \widetilde{y} - y \neq 0$. In practical applications, it is extremely important to know what are the possible values of the difference Δy.

> *For example, if our estimate for amount of oil in a given area is ≈100 mln. ton, then whether we start exploiting this oil or not depends on the accuracy of this estimate:*
>
> – *If the measurement error Δy does not exceed 10 mln. ton, then the actual value can be anywhere from 90 to 100, and we should recommend exploitation.*
>
> – *On the other hand, if the measurement error Δy can be as large as 100 mln. ton, then this means that the actual value y can actually be equal to 0 (meaning that there may be no oil at all). In this case, further, more accurate measurements are needed because we can make a decision.*

To estimate Δy, we must have some information about the errors Δx_i of direct measurements. What type of information can we have?

- The manufacturer of the measuring instrument gives us a *guaranteed* error Δ_i, i.e., a value for which $|\Delta x_i| \leq \Delta_i$.

  Without such a guarantee, a measurement result does not restrict possible values of x_i and thus, it is not a measurement.

- In some cases, in addition to the upper bounds Δ_i, we know *probabilities* of different values of Δx_i.

If we know probabilities, then we have a typical problem of *mathematical statistics*: given probability distributions for $\Delta x_i = \widetilde{x}_i - \Delta x_i$, find the probability distribution for $y = f(x_1, \ldots, x_n)$. To get the probabilities of Δx_i, we *calibrate* the measuring instrument, i.e., we compare its results with the results of a better (standard) measuring instrument.

> An application of statistical methods to environmentally-oriented multi-spectral satellite image processing is given in [29].

However, there are two important situations when we do not know these probabilities:

- In *fundamental physics*, we perform measurements on the *cutting edge*, so no better instrument is possible at all.

- In *manufacturing*, calibration of all sensors is potentially possible, but, in practice, too expensive.

When we do not know the probabilities, we only know that $|\widetilde{x}_i - x_i| \le \Delta_i$, i.e., the only information about x_i is that x_i belongs to the *interval* $[\widetilde{x}_i - \Delta_i, \widetilde{x}_i + \Delta_i]$.

> *For example, if the measured value of the current is* $\widetilde{x} = 1$ *A, and the manufacturer guarantees the measurement error to be within* ± 0.1 *A, then the actual value of* x *can be any number from the interval* $[0.9, 1.1]$.

In this case, the problem of estimating the error of indirect measurement can be reformulated as follows:

- we know n intervals $\mathbf{x}_i = [\widetilde{x}_i - \Delta_i, \widetilde{x}_i + \Delta_i]$,
- we know an algorithm f which transforms n real numbers $x_1, \ldots, x_n$ into a real number y, and
- we want to compute the interval

$$\mathbf{y} = f(\mathbf{x}_1, \ldots, \mathbf{x}_n) = \{f(x_1, \ldots, x_n) \mid x_i \in \mathbf{x}_i\}.$$

This problem is called the basic problem of *interval computations.*

Linearization is not always possible. If a function f is smooth, and the errors Δx_i are small, then we can neglect quadratic terms in f, and get explicit formulas for $\mathbf{y}$. Due to our approximation, we get *approximate* endpoints of the interval $\mathbf{y}$: the actual values y can be, therefore, slightly outside this approximate interval.

In many applications, it is OK, but in some real-life situations, the consequences of a possible error are so serious that we need to *guarantee* that y is contained in the resulting interval $\mathbf{y}$. An example of this problem is planning a mission to the Moon. To get guaranteed estimates for this problem, Ramon E. Moore, then Stanford's Ph.D. student working on 1959 NASA-oriented project, designed new techniques called *interval computations.*

2 Interval Computations in Aerospace Applications: Why

Let us enumerate the reasons why methods of interval computations are needed in aerospace applications:

- First, we want to *guarantee* a mission, we want to *guarantee* that a spaceship hits the Moon (or another planet), and interval computations provide us with the *guaranteed* computation results.

- Second, according to the new NASA paradigm, we need all the missions to be *faster, better, cheaper*. This means, in particular, that we should preferably use off-shelf components, with no time to individually calibrate all of them (and thus, no time to find all the probabilities).

- Third, many NASA missions are missions into the unknown. We simply do not know the exact values of the parameters characterizing the distant planet's surface, or the corresponding probabilities; the only thing we may know for planning a mission are *intervals* of possible values of these parameters.

- Finally, one of the main goals of NASA missions is to produce *solid scientific results*, and "solid" means *guaranteed.*

3 Aerospace Applications of Interval Computations: Examples

Robot navigation. A mobile robot has to navigate in an unknown environment by using imprecise sensors. Traditionally, statistical approach was used to describe the sensor's uncertainty, but this approach has two main drawbacks: it is very costly to calibrate, and it cannot be applied in an unknown environment, when we have no time to calibrate first. To avoid these problems, we used *interval uncertainty* in a UTEP robot. This robot won 1st place in the international competition at AAAI'97: it was more efficient, less error-prone, and at the same time rather simple to program. This technique can be used in future planetary missions.

Telemanipulation [42]**.** The idea of telemanipulation, when a robotic arm repeats the movements of the operator's arm, works perfectly well in the movies, but not so perfectly well in the real space exploration. The reasons for this imperfection are simple: both sensors (which measure the operator's movements) and the actuators (which copy them) are inaccurate. The more complicated the robotic arm, the more actuators it uses, and the more inaccuracy accumulates. It turns out that if we take interval inaccuracy into consideration, we can greatly improve the performance of the telemanipulator – namely, of the state-of-the-art MIT/Utah robotic arm.

Multi-spectral satellite imaging [30.58]**.** The existing Earth-imaging satellites of Landsat series, whose ability is restricted to 7 channels only, already send Gigabytes of difficult-to-process information. For some imaging problems, 7 channels are not sufficient, so new satellites will be able to scan 500 channels. With 100 times more data, we need at least 100 times more time to process it; even now, processing all the satellite data is a problem, and with the expected two orders of magnitude increase, this processing seems to be getting close to impossible. Solution: take interval uncertainty into consideration. It turns out

that with this uncertainty in mind, we can use *linear* models where previously only complex models were used; computations become *faster* and thus, quite feasible.

Non-destructive testing of aerospace constructions [22,63]**.** Failure of an aerospace apparatus can be disastrous, and therefore, all mechanical parts must be thoroughly tested. Exhaustive testing, however, is extremely expensive. Here also intervals help. It turns out that:

- when the tested surface is smooth (no faults, no cracks, etc.), the dependence of the measured signal on the test ultrasound signal is also smooth, and since the test signals are small, we can approximate it by a linear dependence;
- on the other hand, if there are non-smoothnesses (faults, cracks, etc.), then non-linear terms are no longer negligible.

Checking whether the actual data is consistent with the linear dependence (within interval uncertainty), we can thus test whether there is a non-smoothness. Experiments confirmed that this is a viable and expense-saving testing method.

We also analyzed the problem of choosing the best sensor locations for aerospace testing [26,55,56].

Geophysical tomography [4]**.** Interval computations help in reconstructing the geophysical structure from observations.

Energy from space: a possible future application of interval computations. Solar energy is a very prospective renewable energy resource, but on-Earth Solar stations are not perfect: they occupy large pieces of land, they do not work in bad weather, etc. An ideal solution would be to use *orbital* solar power stations, which would generate electricity and then transmit it to Earth as a microwave beam. The problem with this solution is that a high-energy microwave beam may damage whatever it accidentally hits. So, the better solution is to have several orbital stations and several receivers, so that the resulting beams do not reach the dangerous level. Again, interval methods provide a solution to this problem.

4 Related Research: Feasible Algorithms and Impossibility Results

First specific problem: space is unreliable [2]**.** When designing algorithms for space applications, we face a specific problem: space is unreliable; a computer may stop before finishing computations. It is therefore desirable to have algorithms which produce some (approximate) results when interrupted. It turns out that for guaranteed (interval) algorithms, it is theoretically possible

to transform each algorithm into an interruptible one without greatly increasing its computation time. This is still a rather theoretical result, with few practical examples.

Second specific problem: reusing software [36]**.** A huge portion of a space mission's cost consists in designing software. A natural way of saving this costs is to *reuse* the software which was already produced for other missions (or for similar computational problems). Therefore, it seems natural to design new software in such a way that this software be used not only for this particular mission, but for similar future missions as well. The necessity to take the future use into consideration adds cost to writing new software. Hence, if we promote reuse:

- on one hand, we *save costs* on reusing software components, but
- on the other hand, we *add costs* to make new software components reusable.

It is, therefore, not clear whether a reuse policy will actually save costs or not. In [36], we show how the use of interval uncertainty can help in answering this question.

General research in interval computations. Due to the importance of interval computations in aerospace applications, we have researched the possibility of designing feasible algorithms for solving various interval computation problems.

- The general *analysis* is given in [1] (for linear systems).
- Feasible algorithms are produced:
  - in [23,49] for error estimation for linearized indirect measurements;
  - in [7,8] for function approximation;
  - in [13,37] for optimization.

  In most of these cases, we produced the *optimal* algorithms based on the general group-theoretic approach (borrowed from physics [62]).
- In some cases, we showed that the corresponding interval problem cannot, in the general case, be feasibly solved; these results cover, in particular:
  - solving systems of interval linear equations [6];
  - optimal function approximation [7,8], and
  - signal processing [9].

In some cases, it is clear whether an algorithm is feasible or not, but in some borderline cases, checking feasibility requires a complicated theoretical analysis [18,20].

All major results have been summarized in our monograph [10]; aerospace applications are surveyed in [47].

Comment. Some of these results also have non-aerospace applications, e.g., to medicine [22,31,60,63].

5 From Interval Computations to Soft Computing

Why soft computing. As we have mentioned, some interval computation problems are not feasible; this means that if we do not have any additional information, we cannot, in general, solve these problems efficiently. We can rephrase this negative result in a positive form: to solve these problems, we must add some *expert knowledge*. The methodologies which use expert knowledge to solve problems are known as *soft computing*; so, we can reformulate our conclusion as saying that many aerospace problems require soft computing.

We have shown that the use of soft computing methods can indeed make these problems feasibly solvable [34].

Two main problems of satellite data processing. One of the main objectives of PACES is processing satellite data with the purpose of extracting useful geophysical, environmental, and other earth-related information. For this data processing to be successful, we need to solve two major problems:

- First, satellite imaging provides us with an unusually *enormous* amount of data; traditional methods of data processing, which work well for smaller amounts of data, often require too long a time when applied to satellite images; thus, new methods are needed.

- Second, many traditional data and image processing techniques depend on *experts* to *do* many *routine subtasks* such as mosaicking images, identifying different vegetation or cloud patterns, etc. With a huge amount of data coming from the satellites, it is no longer possible to use experts to process all this data, these subtasks need to be automated.

In solving both problems, soft computing techniques such as fuzzy, neural, etc., naturally emerge.

Soft computing helps in solving the first problem of satellite data processing.

- Traditional methods of data processing are based on thorough statistical analysis of the problems.

- Due to the continuing progress in satellite imaging techniques and to the continuing discovery of new applications, there is no time to follow a (rather slow) traditional statistical analysis approach. Therefore, new heuristic methods are needed, methods which use, in addition to statistics, also informal expert ideas.

Fuzzy, neural, and other soft computing techniques allow us:

- to *formalize* these expert ideas, and
- which is very important, to formalize these ideas in a scientifically justified consistent fashion, thus *increasing* the *reliability* of the results of data processing.

Examples of such formalizations are given in [16,28,32,39,40]. An important heuristic idea is the idea of choosing the *simplest* explanation. In computer science, there are natural measures of complexity and simplicity, such as the length and the time of the program, but with respect to all these formal measures, finding the simplest explanation becomes a computationally un-feasible task; soft computing enables us to explain the existing feasible modifications of this idea and to come up with alternative feasible modifications [11,21,24,33,44].

These explanations help not only in heuristic image processing and data processing, but also:

- in education [43],
- in decision making [61],
- in humanities [25], etc.

Soft computing helps in solving the first problem of satellite data processing.

- Experts have trouble describing how exactly they mosaic or how exactly they identify features.
- Experts can, at best, formulate their rules in terms of words of natural language (like "a little bit"). To us these informal rules, we must use a special techniques for transforming such rules into automated control: fuzzy logic.
- If even rules are not available, then the only way to automate is to observe the experts' behavior in several cases and extrapolate. One of the best extrapolation techniques, which is the most appropriate for our purposes because it simulates the way humans do extrapolation, is neural networks.

Applications of soft computing methodology include *image processing* (including processing satellite images and clustering) [27,45,46], as well as related problems such as:

- optimization [15];
- control [14,51,53]; and
- modeling [12].

A general survey of soft computing methodology is given in [52].

In many real-life situations, the existing soft computing techniques are still too computationally intensive [50]; in the attempts to solve this problem, the following direction were pursued:

- thorough analysis of the modifications of soft computing methodologies which have already been proposed but which have not yet been practically used, with the hope that some of these modifications will help to make our problems computationally feasible [17];
- designing new (e.g., multi-D or hierarchical) modifications of soft computing methodologies [19,41,48,52,57,59], with the hope that these new methodologies will lead to feasible solutions to the problems;
- combining soft computing methods with alternative computationally feasible techniques for processing uncertainty, such as *logic programming* [16,54];
- analyzing the possibility of using new physical and engineering ideas in computer design [38].

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