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# DETECTING CRACKS IN THIN PLATES BY USING LAMB WAVE SCANNING: GEOMETRIC APPROACH

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**Abstract.** *A crack in a thin plate reflects ultrasonic waves; therefore, it is reasonable to determine the location of the crack by measuring the reflected waves. The problem of locating the crack can be reformulated in purely geometric terms. Previously, time-consuming iterative numerical methods were used to solve the resulting geometric problem. In this paper, we show that explicit (and fast to compute) formulas can be used instead.*

**Formulation of the engineering problem.** One of the most common problems in aging aircraft structures is the presence of cracks. These cracks are often not visible because they are hidden inside the structure or covered with paint. It is therefore necessary to use techniques of non-destructive testing (NDT) such as ultrasonic Lamb waves.

Lamb waves in thin plates are very convenient in detecting cracks in large-scale structures because these waves can propagate long distances and thus, can help us explore large portions of the plate; see, e.g., (Viktorov 1967).

In a faultless plate, a Lamb wave can travel long distances without dispersion or reflection. Defects reflect and scatter these waves; as a result, the very presence of a reflected wave indicates a defect. It is reasonable to determine the location of the crack by measuring the reflected waves.

**Reduction to a geometric problem.** To locate the crack, we generate a wave pulse that is sent, via a transmitter T, to the plate. This pulse propagates through the plate and reaches a sensor S.

In a faultless plate, the only signal we receive at S is a signal that goes directly from T to S; this signal is received at a time  $t_1 = t_0 + d_0/v$ , where  $t_0$  is the moment of time when the original signal was sent,  $d_0$  is the distance between T and S, and  $v$  is the (known) velocity with which the Lamb waves propagate.

In a plate with defects, in addition to this direct signal, we also observe the signal reflected from a defect; this reflected signal arrives at S at a moment  $t_2 = t_0 + d/v$ , where  $d$  is the length of the path  $TFS = TF + FS$  from T to S via a reflecting point F on the fault. Since we measure  $t_2$  and we know the values  $t_0$  and  $v$ , we can therefore determine the distance  $d$  as  $v \cdot (t_2 - t_0)$ .

If we move the sensor a little bit, to a new location  $S'$  at a small distance  $s$  from the old one, then the reflection point shifts a little bit to a new point  $F'$ , and the path length changes from  $d$  to a new value  $d'$ .

On a large scale, a crack is usually reasonably smooth. Therefore, between the two close points F and  $F'$ , the shape of a crack can be approximated by a straight line segment. Thus, we arrive at the following geometric problem (see Fig. 1):

- We know the location of three points T, S, and  $S'$  on the plane.
- We know that there is a segment  $FF'$  of a straight line  $\ell$  on the same plane.
- We know the length  $d$  of the two-line-segment path that starts at T, gets reflected by  $\ell$  at a point  $F \in \ell$ , and ends at S.
- We also know the length  $d'$  of the two-line-segment path that starts at T, gets reflected by  $\ell$  at a point  $F' \in \ell$ , and ends at  $S'$ .
- Our objective is to locate the points F and  $F'$ .

**How this problem was solved before.** For the (unknown) reflection point F, we know the sum  $TF + FS$  of the distances from two known points: T and S. It is a known geometrical fact that for any given two points T and S, the set of all points F with a given sum  $TF + FS$  is an ellipse. Due to Snell's law describing wave reflection, the

angle between the incoming wave and the crack must be the same as between the crack and the outgoing wave.

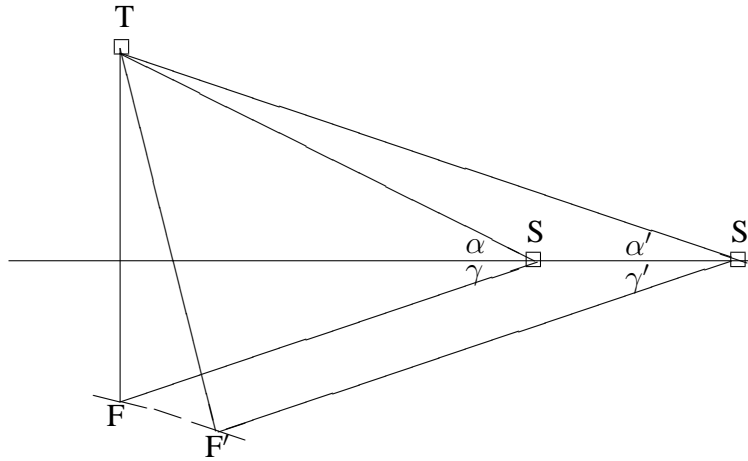


Fig. 1

Due to the properties of an ellipse, we can conclude that the crack is tangent to this ellipse at the reflection point F. Similarly, the crack is tangent to an ellipse of all the points  $F'$  for which  $TF' + F'S = d'$ . Thus, the crack can be determined as a common tangent to two known ellipses. In (De Villa et al., 2001), this idea was used to determine the crack location: explicit equation for tangents were written down, and the resulting system of equations was solved by a numerical technique.

**What is main deficiency of the known solution.** In (De Villa et al., 2001), a (time-consuming) iterative numerical methods were used to solve the resulting geometric problem. It is desirable to use (if possible) an explicit, faster-to-compute method instead. Such a method is presented in this paper.

**Main ideas.** The first idea is to take into consideration that the path  $d = TF + FS$  is equal to the distance between the sensor S and the reflection R of the transmitter T in the straight line  $\ell$  that extends the fault segment  $FF'$  (see Fig. 2):

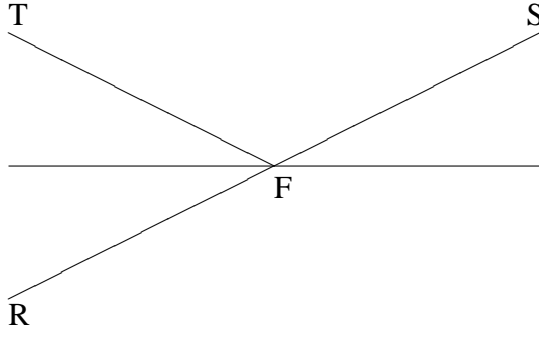


Fig. 2

Indeed,  $TF = RF$ , and due to Snell's law,  $RF$  is a continuation of  $FS$ , so  $RS = RF + FS = TF + FS = d$ . Similarly,  $RS' = d'$ . In the triangle  $\triangle RSS'$ , we thus know all three sides and hence, we can use the Law of Cosines to determine the angle  $\angle RSS' = \pi - \gamma$ :

$$(d')^2 = d^2 + s^2 - 2d \cdot s \cdot \cos(\pi - \gamma),$$

hence, since  $\cos(\pi - \gamma) = -\cos(\gamma)$ , we conclude that

$$\cos(\gamma) = \frac{(d')^2 - d^2 - s^2}{2d \cdot s}. \quad (1)$$

We now know the direction from the sensor  $S$  to the fault point  $F$ ; to determine the distance  $r$  from  $S$  to  $F$ , we can apply the Law of Cosines to the triangle  $\triangle TFS$ . In this triangle, we know the angle  $\angle TSF = \alpha + \gamma$  and we know that  $TS = d_0$ ,  $SF = r$ , and  $TF = d - r$  (see Fig. 3):

Therefore,

$$(d - r)^2 = r^2 + d_0^2 - 2r \cdot d_0 \cdot \cos(\alpha + \gamma).$$

Opening parentheses and canceling the terms  $r^2$  in both sides, we get a linear equation for  $r$ , hence

$$r = \frac{d^2 - d_0^2}{2(d - d_0 \cdot \cos(\alpha + \gamma))}. \quad (2)$$

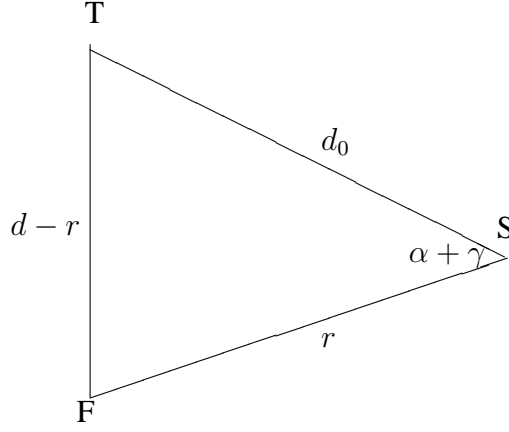


Fig. 3

**New algorithm and results.** The resulting new algorithm is as follows: We know the propagation speed  $v$  of the Lamb waves. Based on the known location of the points T, S, and S', we compute the distance  $d_0 = TS$ , the distance  $s = SS'$ , and the angle  $\alpha$ . We send a pulse signal at time  $t_0$ , we measure the time  $t_2$  when the second pulse arrives at the sensor S, and we compute the distance  $d = v \cdot (t_2 - t_0)$ . We move S to a new location S' at a known distance  $s$  from S, repeat the experiment and compute the new distance  $d'$ . Then, we use the formula (1) to compute the angle  $\gamma$  between the known line SS' and the direction to the fault, and we compute the distance  $r = SF$  by using the formula (2). Once we know the angle and the distance, we can find the location of the fault point F.

Similarly, we can find the location of F'. As we move the sensor along the line SS', we can find several points on the fault and thus, the location and shape of the fault.

We have successfully used this algorithm to find cracks, in particular, to find cracks near rivet holes where other methods have difficulty finding these cracks; see, e.g., (Osegueda et al. 2002) and (Osegueda et al. 2003).

**Alternative geometric set-up.** In the previous set-up, we fix the location of the transmitter T, and moved the sensor S. As we move

the sensor further away from T, the signal fades, and the sensitivity of this method decreases. An alternative idea is therefore to fix the connection between T and S and to move both T and S at the same time (in the direction  $SS'$  which is orthogonal to  $TS$ ), so that  $TS = T'S' = d_0$ . How can we now find the fault location?

**Geometric analysis of the new set-up.** In the new set-up, the path  $d = TFS$  measured by the first sensor is equal to the distance  $SR$  between S and the reflection R of the point T in the fault  $FF'$ . Similarly, the path  $d' = T'F'S'$  measured by the second sensor is equal to the distance  $S'R'$  between  $S'$  and the reflection  $R'$  of the point  $T'$  in the fault  $FF'$ .

Let  $\beta$  denote the angle between the fault  $FF'$  and the direction  $SS'$  in which the sensor moves. We know that  $TS \perp SS'$ , and, due to the properties of reflection,  $RT \perp FF'$ ; therefore, the angle  $\angle RTS$  between  $RT$  and  $TS$  is also equal to  $\beta$ . Similarly,  $\angle R'T'S' = \beta$ .

Due to the properties of reflection, the distance  $RR'$  is equal to  $TT' = s$ . Since  $TT' \parallel SS'$ , the line  $TT'$  is at angle  $\beta$  to the reflecting line  $FF'$ , hence  $RR'$  is also at angle  $\beta$  from the reflecting line; see Fig. 4. If from R and T we draw the lines  $RR''$  and  $TT''$  that are parallel to  $FF'$  (and which are hence orthogonal to  $RT$  and  $R'T'$ ), then we conclude that  $R'R'' = T'T'' = s \cdot \sin(\beta)$  hence

$$R'T' - RT = R'R'' + T'T'' = 2s \cdot \sin(\beta).$$

If we denote an average of  $RT$  and  $R'T'$  by  $M$ , we can thus conclude that  $RT = M - d \cdot \sin(\beta)$  and  $R'T' = M + d \cdot \sin(\beta)$ .

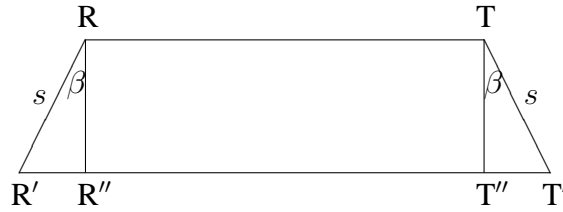


Fig. 4

In the triangle  $\triangle RTS$ ,  $RT = M - s \cdot \sin(\beta)$ ,  $\angle RTS = \beta$ ,  $TS = d_0$ , and  $RS = d$ ; therefore, the Law of Cosines leads to:

$$d^2 = (M - s \cdot \sin(\beta))^2 + d_0^2 - 2(M - s \cdot \sin(\beta)) \cdot d_0 \cdot \cos(\beta). \quad (3)$$

Similarly,

$$(d')^2 = (M + s \cdot \sin(\beta))^2 + d_0^2 - 2(M + s \cdot \sin(\beta)) \cdot d_0 \cdot \cos(\beta). \quad (4)$$

Subtracting (3) from (4), we conclude that:

$$(d')^2 - d^2 = 4M \cdot s \cdot \sin(\beta) - 4s \cdot d_0 \cdot \sin(\beta) \cdot \cos(\beta),$$

hence for

$$z_1 \stackrel{\text{def}}{=} \frac{(d')^2 - d^2}{4s} \quad (5)$$

we get the formula

$$z_1 = \sin(\beta) \cdot (M - d_0 \cdot \cos(\beta)). \quad (6)$$

Averaging (3) and (4), we conclude that for

$$z_2 \stackrel{\text{def}}{=} \frac{(d')^2 + d^2}{2}, \quad (7)$$

we get the formula

$$z_2 = M^2 + s^2 \cdot \sin^2(\beta) + d_0^2 - 2M \cdot d_0 \cdot \cos(\beta). \quad (8)$$

From (6), we conclude that

$$z_1^2 = \sin^2(\beta) \cdot (M^2 - 2M \cdot d_0 \cdot \cos(\beta) + d_0^2 \cdot \cos^2(\beta)), \quad (9)$$

and from (8), that

$$z_2 \cdot \sin^2(\beta) = \sin^2(\beta) \cdot (M^2 + s^2 \cdot \sin^2(\beta) + d_0^2 - 2M \cdot d_0 \cdot \cos(\beta)). \quad (10)$$

Subtracting (9) from (10), we conclude that

$$z_2 \cdot \sin^2(\beta) - z_1^2 = s^2 \cdot \sin^4(\beta) + d_0^2 \cdot \sin^4(\beta),$$

i.e., that

$$(s^2 + d_0^2) \cdot \sin^4(\beta) - z_2 \cdot \sin^2(\beta) + z_1^2 = 0.$$



This is a quadratic equation in terms of the unknown  $\sin^2(\beta)$ , so

$$\sin^2(\beta) = \frac{z_2 - \sqrt{z_2^2 - 4z_1^2 \cdot (s^2 + d_0^2)}}{2(s^2 + d_0^2)}. \quad (11)$$

Once we know the angle  $\beta$ , we can use the formula (6) to determine  $M = z_1 / \sin(\beta) + d_0 \cdot \cos(\beta)$  and hence,  $RT = M - s \cdot \sin(\beta)$  as

$$RT = \frac{z_1}{\sin(\beta)} + d_0 \cdot \cos(\beta) - s \cdot \sin(\beta). \quad (12)$$

Let us select the coordinate system in which the x-axis is parallel to  $SS'$ , and the y-axis is parallel to  $ST$ . We know the coordinates  $x_T$  and  $y_T$  of the point T, we know the angle  $\beta$  between TS (i.e., the y-axis) and the direction TR, and we know the distance RT; thus, we can find the coordinates  $(x_R, y_R)$  of the point R as

$$x_R = x_T - RT \cdot \sin(\beta); \quad y_R = y_T - RT \cdot \cos(\beta). \quad (13)$$

The midpoint  $m$  between the point T and its reflection R in the line that extends  $FF'$  is a point on this extended line  $\ell$ ; its coordinates are

$$x_m = \frac{x_T + x_R}{2}; \quad y_m = \frac{y_T + y_R}{2}. \quad (14)$$

By definition of the angle  $\beta$ , the fault segment  $FF'$  forms an angle  $\beta$  with the line  $SS'$  (i.e., with the x-axis). Therefore, the line  $\ell$  goes through this point  $m$  at the angle  $\beta$  with the x-axis, hence the line  $\ell$  is described by the equation:

$$y = y_m - \tan(\beta) \cdot (x - x_m). \quad (15)$$

The fault point F is the intersection between the line  $\ell$  and the line SR. The point S has coordinates

$$x_S = x_T; \quad y_S = y_T - d_0; \quad (16)$$

therefore, the equations of the line SR can be described as:

$$y = y_R + \frac{y_S - y_R}{x_S - x_R} \cdot (x - x_R). \quad (17)$$

We can therefore find the coordinates  $x$  and  $y$  of the fault point  $F$  as the solution to the system of two linear equations (15) and (17) with two unknowns – a solution that can be obtained explicitly in terms of the coefficients.

**Resulting algorithm for the alternative set-up.** We know the propagation speed  $v$  of the Lamb waves, we know the distance  $d_0$  between the transmitter  $T$  and the sensor  $S$ . We send a pulse signal at time  $t_0$ , we measure the times  $t_2$  when the second pulse arrives at the sensor  $S$ , and we compute the distance  $d = v \cdot (t_2 - t_0)$ . Then, we move the combination of  $T$  and  $S$  to a new location  $T'S'$  at a distance  $s$  from  $TS$ , repeat the experiment and compute the new distance  $d'$ .

We compute  $z_1$  and  $z_2$  by using the formulas (5) and (7), then the angle  $\beta$  by using the formula (11), then  $RT$  from the formula (12), and the coordinates of the points  $R$ ,  $m$ , and  $S$  from formulas (13), (14), and (16). After that, we solve the system of two linear equations (15) and (17) with two unknowns  $x$  and  $y$  and find the coordinates of the point  $F$  on the fault.

Similarly, we can find the location of  $F'$ . As we move the transmitter and the sensor, we can find several points on the fault and thus, the location and shape of the fault.

**Open problem.** In the above algorithms, we approximated (locally) a smooth-shaped crack as a straight line segment. The closer the sensor locations  $S$  and  $S'$  are to each other, the better this approximation. However, if we make these locations too close, then the difference between the signals received at these locations will get below the noise level and thus, we will be unable to locate the fault. So, to increase the approximation accuracy – and thus, to increase the accuracy of fault location – it is desirable to use a more accurate approximation to the fault shape.

A natural idea is to take second order (curvature) terms into consideration and represent a crack as a circular arc. For this representation, can we still get explicit formulas for reconstructing fault location?

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## References

F. de Villa, E. Roldan, C. Tirado, R. Mares, S. Nazarian, and R. Osegueda, “Defect detection in thin plates using  $S_0$  Lamb wave scanning”, In: *NDE for Health Monitoring and Diagnostics*, Proceedings of the SPIE/International Society for Optical Engineering, Vol. 4335, San Diego, CA, March 2001.

R. Osegueda and V. Kreinovich, “Detecting and locating curved cracks in thin plates by Lamb wave reflection: validated geometric approach”, *Extended Abstracts of 2002 SIAM Workshop on Validated Computing*, Toronto, Canada, May 23–25, 2002, pp. 149–151.

R. Osegueda, V. Kreinovich, S. Nazarian and E. Roldan, “Detection of cracks at rivet holes in thin plates using Lamb-wave scanning”, In: *Smart NDE and Health Monitoring of Structural and Biological Systems*, Proceedings of the SPIE/International Society for Optical Engineering, Vol. 5047, San Diego, CA, March 2–6, 2003 (to appear).

I. A. Viktorov, *Rayleigh and Lamb waves: physical theory and applications*, Plenum Press, New York, 1967.