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Use of Satellite Image Referencing Algorithms to Characterize Asphaltic Concrete Mixtures

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Abstract— A natural way to test the structural integrity of a pavement is to send signals with different frequencies through the pavement and compare the results with the signals passing through an ideal pavement. For this comparison, we must determine how, for the corresponding mixture, the elasticity E depends on the frequency f in the range from 0.1 to 10^5 Hz. It is very expensive to perform measurements in high frequency area (above 20 Hz). To avoid these measurements, we can use the fact that for most of these mixtures, when we change a temperature, the new dependence changes simply by scaling. Thus, instead of performing expensive measurements for different frequencies, we can measure the dependence of E on moderate frequencies f for different temperatures, and then combine the resulting curves into a single “master” curve. In this paper, we show how fuzzy techniques can help to automate this “combination”.

I. INTRODUCTION

A. Practical Problem

Many materials are *viscoelastic*, i.e., they possess both the elastic property of a solid and the viscous behavior of the liquid. Examples of such materials range from asphaltic concrete mixtures used in road pavement to elastomers used in aerospace industry.

In many applications of these materials, it is desirable to use non-destructive techniques to test the structural integrity and the properties of the corresponding structures. In these techniques, we send signals to the structure, we measure the signals that passed through the structure, and we analyze this detected signal to see if the structure has any faults. For this analysis, we need to know how, for the corresponding material, the elasticity E depends on the frequency f . To detect different types of faults, we must know this dependence for the frequency f in the range from 0.1 to 10^5 Hz. It is very expensive to perform measurements in high frequency area above 20 Hz.

To avoid these measurements, we can use the fact that many practically used viscoelastic materials – including most asphaltic concrete mixes and many elastomers used in the aerospace industry – are *thermally simple*, meaning

that when we change a temperature, the new dependence changes simply by scaling, i.e.,

$$E_{\text{new}}(f) = E(\lambda \cdot f). \quad (1)$$

Thus, instead of performing expensive measurements for different frequencies, we can measure the dependence of E on moderate frequencies f for different temperatures, and then try to apply appropriate scaling to combine the resulting curves into a single “master” curve.

To describe frequencies ranging from 0.1 to 10^5 Hz, researchers usually use logarithmic scale, i.e., they use the logarithm $F = \ln(f)$ instead of f . In the logarithmic scale, scaling is described as a shift, so the equation (1) takes an even simpler form:

$$E_{\text{new}}(F) = E(F + c), \quad (2)$$

where $c = \ln(\lambda)$ is the corresponding shift. In the logarithmic scale, the above idea can be reformulated as follows: we measure the dependence of E on F for different temperatures, and then shift the resulting curves in horizontal direction together so that they form a single curve; see, e.g., [1, 6, 7, 8, 22].

This approach has been successfully used; the problem is that at present, the shifts are done manually by an expert. It is desirable to automate this process.

B. Fuzzy Methods May Be Helpful

The main reason why there are no automatic shift methods is that we do not know the exact dependence of the shift c on the temperature. To determine the corresponding shifts, we rely on the experts – who are unable to represent their expertise in precise terms. To automate this problem, it is therefore desirable to use techniques specifically designed to formalize such expert knowledge – namely, the techniques of fuzzy logic; see, e.g., [11, 17].

C. We Can Also Use Experience of Referencing Satellite Images

The problem of determining a shift c between two known functions $E(F)$ and $E_{\text{new}}(F)$ occurs not only in the analysis of asphaltic concrete mixtures; a similar problem occurs in the analysis of satellite images. Two satellite images of the same area often differ by an unknown shift. There exist automatic methods of determining the corresponding shift, i.e., of “referencing” the corresponding

images. In this paper, we will show how these methods can be used to combine the curves corresponding to different temperatures.

II. FUZZY-BASED ALGORITHM

A. Fuzzy Analysis

Ideally, it would be great if we could find the value of the shift c for which, for every value F , $E_{\text{new}}(F)$ is exactly equal to $E(F + c)$. However, due to inevitable noise and measurement inaccuracies, the two curves are not exactly the same. All we can hope for is that for every F , the values $E_{\text{new}}(F)$ and $E(F + c)$ are *close* to each other, i.e., that for every F , the difference $e \stackrel{\text{def}}{=} E_{\text{new}}(F) - E(F + c)$ is small.

We have finitely many measurement results corresponding to different frequencies. Let us denote the total number of measurement results by K , and the corresponding differences by e_1, \dots, e_K . In these terms, the requirement for choosing the shift c is that all the differences e_i are small, i.e., that e_1 is small, e_2 is small, \dots , and e_K is small. A natural way to formalize this requirement is to use fuzzy logic. Let $\mu(x)$ be a membership function that describes the natural-language term “small”. Then, our degree of belief that e_1 is small is equal to $\mu(e_1)$, our degree of belief that e_2 is small is equal to $\mu(e_2)$, etc. To get the degree of belief d that all K conditions are satisfied, we must use a t-norm (a fuzzy analogue of “and”), i.e., use a formula $d = \mu(e_1) \& \dots \& \mu(e_K)$, where $\&$ is this t-norm.

In [10, 16], it is shown that within an arbitrary accuracy, an arbitrary t-norm can be approximated by a strictly Archimedean t-norm. Therefore, for all practical purposes, we can assume that the t-norm that describes the experts’ reasoning, is strictly Archimedean and therefore, has the form $a \& b = \varphi^{-1}(\varphi(a) + \varphi(b))$ for some strictly decreasing function φ [11, 17]. Thus, $d = \varphi^{-1}(\varphi(\mu(e_1)) + \dots + \varphi(\mu(e_K)))$. We want to find the values of the parameters for which our degree of belief d (that the model is good) is the largest possible. Since the function φ is strictly decreasing, d attains its maximum if and only if the auxiliary characteristic $D = \varphi(d)$ attains its minimum. From the formula that describe d , we can conclude that $D = \varphi(\mu(e_1)) + \dots + \varphi(\mu(e_K))$. Thus, the condition $D \rightarrow \min$ takes the form

$$\psi(e_1) + \dots + \psi(e_K) \rightarrow \min, \quad (3)$$

with $\psi(x) = \varphi(\mu(x))$.

This formula is actively used – as a heuristic formula – in statistics [9, 20]. This formula – known as an *M-method* – is one of the formulas of *robust* statistics, i.e., formulas designed for the case when we do not know the exact probability distribution.

The above text shows that this heuristic formula can be justified within fuzzy logic; see, e.g., [12, 14, 15]. This justification enables us to answer the natural question: what function $\psi(x)$ should we choose. We should base

this choice on the opinion of the experts. From these experts, we extract the membership function $\mu(x)$ that corresponds to “small”, and the function $\varphi(x)$ that best describes the experts’ “and”.

In particular, if we require that the M-method be invariant relative to rescaling $e \rightarrow \lambda \cdot e$, then the only possible M-methods are methods corresponding to $\psi(x) = |x|^p$ for some parameter p ; see [13] for the exact formulation and proof. For such function $\psi(x)$, the criterion (3) takes the form

$$|e_1|^p + \dots + |e_K|^p \rightarrow \max. \quad (4)$$

What value p should we choose? Since we are trying to formalize fuzzy expert knowledge, the value of p is not well defined: in addition to the original value of p , we can use nearby values as well. We can use this freedom to select p for which the resulting computations are the simplest possible.

Out of all methods (4), the method corresponding to $p = 2$ is the simplest, because for $p = 2$ (i.e., for the least squares method) the condition that the derivative of the objective function is equal to 0 becomes a linear equation. Thus, from the fuzzy logic viewpoint, it is reasonable to use the least squares method.

B. Towards an Algorithm

Let us transform the above ideas into an algorithm for finding the shift between the two curves $E^{(1)}(F)$ and $E^{(2)}(F)$. Each measurement corresponds to a narrow range of frequencies. Usually, within a narrow range, the dependence of E on F is monotonic. Thus, for every E , there exists at most one value F_1 for which $E^{(1)}(F_1) = E$. Similarly, there exists at most one value F_2 for which $E^{(2)}(F_2) = E$. In the idealized no-noise case, for every F , the value $E^{(2)}(F)$ is exactly equal to $E^{(1)}(F + c)$. In particular, for $F = F_2$, we have $E = E^{(2)}(F_2) = E^{(1)}(F_2 + c)$. Since F_1 is the only value for which $E^{(1)}(F_1) = E$, we thus conclude that $F_1 = F_2 + c$. So, in this idealized case, we can determine c as $F_2 - F_1$.

In reality, the values $E^{(2)}(F)$ and $E^{(1)}(F + c)$ are only approximately equal. Thus, the shift c is only approximately equal to the difference $F_2 - F_1$. We can repeat the same argument for the values $F_1(E)$ and $F_2(E)$ corresponding to different values E , and get difference estimates $F_2(E) - F_1(E)$ for the desired shift c .

Let E_1, \dots, E_K be the values for which we perform these computations. We want to find a single estimate for the shift which is close to all these estimates, i.e., for which all K values $e_i \stackrel{\text{def}}{=} c - (F_2(E_i) - F_1(E_i))$ are close to 0. The above argument shows that we must use the least squares method to determine this value c , i.e., that we must find c for which

$$(c - (F_2(E_1) - F_1(E_1)))^2 + \dots + (c - (F_2(E_K) - F_1(E_K)))^2 \rightarrow \min_c.$$

Differentiating by c and equating the derivative to 0, we conclude that

$$c = \frac{(F_2(E_1) - F_1(E_1)) + \dots + (F_2(E_K) - F_1(E_K))}{K},$$

i.e., that the shift can be estimate as the average horizontal shift between the two curves. Thus, we arrive at the following algorithm for finding the shift c between the two curves $E^{(1)}(F)$ and $E^{(2)}(F)$:

- We pick several values E_1, \dots, E_K .
- For each of these values E_i , we find the values $F_1(E_i)$ and $F_2(E_i)$ for which $E^{(1)}(F_1(E_i)) = E^{(2)}(F_2(E_i)) = E_i$, and compute the difference $c_i = F_2(E_i) - F_1(E_i)$.
- Finally, as an estimate c for the shift, we take the arithmetic average of K values c_1, \dots, c_K .

This methods indeed leads to a reasonable combination of curves corresponding to different temperatures.

III. THE FFT-BASED REFERENCING ALGORITHM

Many existing referencing methods use Fast Fourier Transform (FFT). The best of known FFT-based referencing algorithms is presented in [18]; see also [2, 3, 4, 5, 18, 19, 21]. The main ideas behind FFT-based referencing in general and this algorithm in particular are as follows.

A. The Simplest Case: Shift Detection in the Absence of Noise

Let us first consider the case when two images differ only by shift. It is known that if two images $I(\vec{x})$ and $I'(\vec{x})$ differ only by shift, i.e., if $I'(\vec{x}) = I(\vec{x} + \vec{a})$ for some (unknown) shift \vec{a} , then their Fourier transforms

$$\mathcal{F}(\vec{\omega}) = \frac{1}{2\pi} \cdot \int \int I(\vec{x}) \cdot e^{-2\pi \cdot i \cdot (\vec{x} \cdot \vec{\omega})} dx dy,$$

$$\mathcal{F}'(\vec{\omega}) = \frac{1}{2\pi} \cdot \int \int I'(\vec{x}) \cdot e^{-2\pi \cdot i \cdot (\vec{x} \cdot \vec{\omega})} dx dy,$$

are related by the following formula:

$$\mathcal{F}'(\vec{\omega}) = e^{2\pi \cdot i \cdot (\vec{\omega} \cdot \vec{a})} \cdot \mathcal{F}(\vec{\omega}). \quad (5)$$

Therefore, if the images are indeed obtained from each other by shift, then we have

$$M'(\vec{\omega}) = M(\vec{\omega}), \quad (6)$$

where we denoted

$$M(\vec{\omega}) = |\mathcal{F}(\vec{\omega})|, \quad M'(\vec{\omega}) = |\mathcal{F}'(\vec{\omega})|. \quad (7)$$

The actual value of the shift \vec{a} can be obtained if we use the formula (5) to compute the value of the following ratio:

$$R(\vec{\omega}) = \frac{\mathcal{F}'(\vec{\omega})}{\mathcal{F}(\vec{\omega})}. \quad (8)$$

Substituting (5) into (8), we get

$$R(\vec{\omega}) = e^{2\pi \cdot i \cdot (\vec{\omega} \cdot \vec{a})}.$$

Therefore, the inverse Fourier transform $P(\vec{x})$ of this ratio is equal to the delta-function $\delta(\vec{x} - \vec{a})$.

In other words, in the ideal no-noise situation, this inverse Fourier transform $P(\vec{x})$ is equal to 0 everywhere except for the point $\vec{x} = \vec{a}$; so, from $P(\vec{x})$, we can easily determine the desired shift by using the following *algorithm*:

- first, we apply FFT to the original images $I(\vec{x})$ and $I'(\vec{x})$ and compute their Fourier transforms $\mathcal{F}(\omega)$ and $\mathcal{F}'(\omega)$;
- on the second step, we compute the ratio (8);
- on the third step, we apply the inverse FFT to the ratio $R(\vec{\omega})$ and compute its inverse Fourier transform $P(\vec{x})$;
- finally, on the fourth step, we determine the desired shift \vec{a} as the only value \vec{a} for which $P(\vec{a}) \neq 0$.

B. Shift Detection in the Presence of Noise

In the ideal case, the absolute value of the ratio (8) is equal to 1. In real life, the measured intensity values have some noise in them. For example, the conditions may slightly change from one overflight to another, which can be represented as the fact that a “noise” was added to the actual image.

In the presence of noise, the observed values of the intensities may differ from the actual values; as a result, their Fourier transforms also differ from the values and hence, the absolute value of the ratio (8) may be different from 1.

We can somewhat improve the accuracy of this method if, instead of simply processing the measurement results, we take into consideration the additional knowledge that the absolute value of the actual ratio (8) is exactly equal to 1. Let us see how this can be done.

Let us denote the actual (unknown) value of the value $e^{2\pi \cdot i \cdot (\vec{\omega} \cdot \vec{a})}$ by r . Then, in the absence of noise, the equation (5) takes the form

$$\mathcal{F}'(\vec{\omega}) = r \cdot \mathcal{F}(\vec{\omega}). \quad (9)$$

In the presence of noise, the computed values $\mathcal{F}(\vec{\omega})$ and $\mathcal{F}'(\vec{\omega})$ of the Fourier transforms can be slightly different from the actual values, and therefore, the equality (9) is only approximately true:

$$\mathcal{F}'(\vec{\omega}) \approx r \cdot \mathcal{F}(\vec{\omega}). \quad (10)$$

In addition to the equation (10), we know that the absolute value of r is equal to 1, i.e., that

$$|r|^2 = r \cdot r^* = 1, \quad (11)$$

where r^* denotes a complex conjugate to r .

As a result, we know two things about the unknown value r :

- that r satisfies the approximate equation (10), and
- that r satisfies the additional constraint (11).

We would like to get the best estimate for r among all estimates which satisfy the condition (11). We have already argued that to get the optimal estimate, we can use the *Least Squares Method* (LSM). According to this method, for each estimate r , we define the error

$$E = \mathcal{F}'(\vec{\omega}) - r \cdot \mathcal{F}(\vec{\omega}) \quad (12)$$

with which the condition (10) is satisfied. Then, we find among all estimates which satisfy the additional condition (11), a value r for which the square $|E|^2 = E \cdot E^*$ of this error is the smallest possible.

The square $|E|^2$ of the error E can be reformulated as follows:

$$\begin{aligned} E \cdot E^* &= \\ &(\mathcal{F}'(\vec{\omega}) - r \cdot \mathcal{F}(\vec{\omega})) \cdot \\ &(\mathcal{F}'^*(\vec{\omega}) - r^* \cdot \mathcal{F}^*(\vec{\omega})) = \\ &\mathcal{F}'(\vec{\omega}) \cdot \mathcal{F}'^*(\vec{\omega}) - r^* \cdot \mathcal{F}^*(\vec{\omega}) \cdot \mathcal{F}'(\vec{\omega}) - \\ &r \cdot \mathcal{F}(\vec{\omega}) \cdot \mathcal{F}'^*(\vec{\omega}) + r \cdot r^* \cdot \mathcal{F}(\vec{\omega}) \cdot \mathcal{F}^*(\vec{\omega}). \end{aligned} \quad (13)$$

We need to minimize this expression under the condition (11).

For conditional minimization, there is a known technique of Lagrange multipliers, according to which the minimum of a function $f(x)$ under the condition $g(x) = 0$ is attained when for some real number λ , the auxiliary function $f(x) + \lambda \cdot g(x)$ attains its unconditional minimum; this value λ is called a *Lagrange multiplier*.

For our problem, the Lagrange multiplier technique leads to the following unconditional minimization problem:

$$\begin{aligned} &\mathcal{F}'(\vec{\omega}) \cdot \mathcal{F}'^*(\vec{\omega}) - r^* \cdot \mathcal{F}^*(\vec{\omega}) \cdot \mathcal{F}'(\vec{\omega}) - \\ &r \cdot \mathcal{F}(\vec{\omega}) \cdot \mathcal{F}'^*(\vec{\omega}) + r \cdot r^* \cdot \mathcal{F}(\vec{\omega}) \cdot \mathcal{F}^*(\vec{\omega}) + \\ &\lambda \cdot (r \cdot r^* - 1) \rightarrow \min. \end{aligned} \quad (14)$$

We want to find the value of the complex variable r for which this expression takes the smallest possible value. A complex variable is, in effect, a pair of two real variables, so the minimum can be found as a point at which the partial derivatives with respect to each of these variables are both equal to 0. Alternatively, we can represent this equality by computing the partial derivative of the expression (14) relative to r and r^* . If we differentiate (14) relative to r^* , we get the following linear equation:

$$\begin{aligned} &-\mathcal{F}^*(\vec{\omega}) \cdot \mathcal{F}'(\vec{\omega}) + r \cdot \mathcal{F}(\vec{\omega}) \cdot \mathcal{F}^*(\vec{\omega}) + \\ &\lambda \cdot r = 0. \end{aligned} \quad (15)$$

From this equation, we conclude that

$$r = \frac{\mathcal{F}^*(\vec{\omega}) \cdot \mathcal{F}'(\vec{\omega})}{\mathcal{F}(\vec{\omega}) \cdot \mathcal{F}^*(\vec{\omega}) + \lambda}. \quad (16)$$

The coefficient λ can be now determined from the condition that the resulting value r should satisfy the equation (11). The denominator $\mathcal{F}(\vec{\omega}) \cdot \mathcal{F}^*(\vec{\omega}) + \lambda$ of the equation (16) is a real number, so instead of finding λ , it is sufficient to find a value of this denominator for which $|r|^2 = 1$. One can easily see that to achieve this goal, we should take, as this denominator, the absolute value of the numerator, i.e., the value

$$|\mathcal{F}^*(\vec{\omega}) \cdot \mathcal{F}'(\vec{\omega})| = |\mathcal{F}^*(\vec{\omega})| \cdot |\mathcal{F}'(\vec{\omega})|. \quad (17)$$

For this choice of a denominator, the formula (15) takes the following final form:

$$r = \frac{\mathcal{F}^*(\vec{\omega}) \cdot \mathcal{F}'(\vec{\omega})}{|\mathcal{F}^*(\vec{\omega})| \cdot |\mathcal{F}'(\vec{\omega})|}. \quad (18)$$

So, in the presence of noise, instead of using the exact ratio (8), we should compute, for every $\vec{\omega}$, the optimal approximation

$$R(\vec{\omega}) = \frac{\mathcal{F}^*(\vec{\omega}) \cdot \mathcal{F}'(\vec{\omega})}{|\mathcal{F}^*(\vec{\omega})| \cdot |\mathcal{F}'(\vec{\omega})|}. \quad (19)$$

In the ideal non-noise case, the inverse Fourier transform $P(\vec{x})$ of this ratio is equal to the delta-function $\delta(\vec{x} - \vec{a})$, i.e., equal to 0 everywhere except for the point $\vec{x} = \vec{a}$. In the presence of noise, we expect the values of $P(\vec{x})$ to be slightly different from the delta-function, but still, the value $|P(\vec{a})|$ should be much larger than all the other values of this function. So, we arrive at the following *algorithm* for determining the shift \vec{a} :

- first, we apply FFT to the original images $I(\vec{x})$ and $I'(\vec{x})$ and compute their Fourier transforms $\mathcal{F}(\omega)$ and $\mathcal{F}'(\omega)$;
- on the second step, we compute the ratio (19);
- on the third step, we apply the inverse FFT to the ratio $R(\vec{\omega})$ and compute its inverse Fourier transform $P(\vec{x})$;
- finally, on the fourth step, we determine the desired shift \vec{a} as the point for which $|P(\vec{x})|$ takes the largest possible value.

C. Application to Pavement Analysis

We have applied the above referencing algorithm to pavement analysis, and it indeed leads to a automatic generation of high-quality combined master curve, a master curve which is comparable with the good manual combination by a skilled expert.

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