2008-01-01

Allocating Emergency Response Vehicles To Cover Critical Infrastructures

Hao Lei
University of Texas at El Paso, hlei@miners.utep.edu

ALLOCATING EMERGENCY RESPONSE VEHICLES TO COVER CRITICAL INFRASTRUCTURES

HAO LEI

Department of Civil Engineering

APPROVED:

Ruey (Kelvin) Cheu, Ph.D., Chair

Carlos M. Chang Albitres, Ph.D.

Raed Aldouri, Ph.D.

Patricia Witherspoon, Ph.D.
Dean of the Graduate School

ALLOCATING EMERGENCY RESPONSE VEHICLES TO COVER CRITICAL INFRASTRUCTURES

by

HAO LEI, B.S., M.S.

THESIS

Presented to the Faculty of the Graduate School of

The University of Texas at El Paso

in Partial Fulfillment

of the Requirements

for the Degree of

MASTER OF SCIENCE

Department of Civil Engineering

THE UNIVERSITY OF TEXAS AT EL PASO

December 2008

ACKNOWLEDGEMENT

I would like to thank my graduate advisor, Dr. Ruey Long Cheu for his invaluable and patient guidance, continuous and whole-hearted support in carrying out my research work as well as my personal matters. Without his help and encouragement, this thesis would not have been possible. I would like to thank Dr. Carlos Chang and Dr. Raed Aldouri for funding my previous research. The experience of working with them helps me scale my academic heights.

I would also like to express my gratitude to Lt. Michael W. Fogelman and Mr. Luis Uriel Flores from El Paso Fire Department for providing invaluable data needed for this thesis.

I would also like to thank the friends in Border Intermodal Gateway Lab who have in one way or another supported me.

Finally, this thesis is dedicated to my parents and my wife for their love, encouragement, and support.

ABSTRACT

Optimal deployment of limited emergency response service units in a metropolitan area is of interests to public agencies. The limited emergency units not only have to respond to the demand for service by residents and businesses, but also cover Critical Infrastructures (CIs). This thesis formulates an improved optimization model to allocate different types of Emergency Response Service (ERS) units among their candidate base stations. The allocation of units must ensure maximum coverage to CIs, subject to the capacities of the base stations, service standard in terms of time to reach the CIs, and the availability of the ERS units (when there are other competing demands for services). Unlike past models, the new modeling approach accounts for the fluctuation of travel time and demand frequency at different time periods of a typical day. The applicability of the model has been demonstrated in a case study in the city of El Paso, Texas which allocates 45 fire fighting units and 23 ambulances among 34 active fire stations to cover 138 CIs, including Critical Transportation Infrastructures (CTIs), hospitals and schools.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

1. INTRODUCTION

1. 1. Background and Motivation

Emergency Response Services (ERSs) such as fire, law enforcement and medical services are important lifelines in most communities. ERSs provide vital services to the public in order to minimize the losses caused by incidents (such as accidents, fire, medical emergency, natural and man-made events). For this to take place, ERS units must cover the maximum possible area within the city and yet arrive at the incident scene at the shortest possible time. In the day-to-day operation of an ERS department, its units must respond to the calls requesting services from the residents and businesses. In addition, the ERS units must be able to respond quickly to any incident that occurs at Critical Transportation Infrastructures (CTIs), schools, hospitals, and other important facilities in a city.

The CTIs are important facilities such as interchanges, tunnels, bridges, transit terminals, etc within the city's transportation network. Their continuing operation at the maximum possible capacity is essential to the daily economic and social functions of the city. The ERS units must be able to reach the CTIs quickly so that the capacity of the affected transportation node is restored at the earliest possible time to minimize the delay to the users. Schools are densely populated buildings with children and young adults who are vulnerable to any disruptive or life-threatening event. Hospitals are the central providers of medical services, and have high concentration of medically impaired persons. Moreover, hospitals must continue to function round the clock even in the event of a large-scale disaster. In this thesis, the CTIs, schools and hospitals are termed collectively as Critical Infrastructures (CIs).

Locating ERS units at their potential base stations to serve CIs may be viewed as a facility location problem (FLP), which has been studied by researchers in operations research for more than three decades. A comprehensive review on the various simplified problems and solution algorithms on this topic can be found in (Owen and Daskin, 1998). The problem may be modeled to minimize the number of base stations with the requirement that every CI in the network should be covered (i.e., could be reached) within a certain service time (Toregas et al. 1971, Toregas and ReVelle, 1973). When the quantity of stations is limited, the objective can be set as to maximize the coverage of the total number of CIs with the limited number of stations (Church and ReVelle, 1974, Schilling, et al., 1979). Extension of the models from one type of ERS units to simultaneously allocate different types of ERS units among their respective base stations has also been proposed (Church and ReVelle, 1974). Recognizing that an ERS unit may be busy when a new demand for service arises, the concept of backup coverage was introduced to give additional coverage to certain CIs (Hogan and ReVelle, 1986). These earlier models, categorized as deterministic covering models, do not model the probability that ERS unit being unavailable when it is busy serving other demand. Daskin (1983) and ReVelle and Marianov (1991) formulated the co-called probabilistic covering models to account for the competing demand in the allocation of one type of ERS units. Cheu et al. (2008) expanded the earlier formulation of the probabilistic covering model from one type of ERS to three types of ERSs and incorporated the concept of backup coverage. The above models computed travel times from the base stations to the demand nodes using either straight arc distance or distance along a road network. As an improvement from the abovementioned integer programming model, Huang et al. (2007) provided the mixed integer programming formulation of the model originally presented

by Cheu et al. 2008 and demonstrated the different applications of the model, such as sensitivity and budget analyses. They have used Singapore as the example, and computed the times to reach the scenes that took into account the driving speed and other traffic regulations in the actual road network, which was not found in earlier literature.

To date, all the models assume that the travel time from the base stations to the scenes are the same throughout the day. Furthermore, they assume that the frequency of competing demand is the same throughout the city and remains the same throughout the day. In reality, the travel times from the base stations to the scenes depends on the actual traffic of the road network, which fluctuates from hour to hour. Moreover, the demand frequency varies not only by hour at one station, but also from station to station in the same hour.

1. 2. Objective and Goal

This research is to formulate and solve an improved version of the probabilistic covering mixed integer programming model. The model allocates multiple types of ERS units among the set of candidate stations, with station capacity and service time constraints. Unlike past models, this model takes into account (1) the spatial and temporal distribution of competing demands; and (2) the temporal distribution of travel time due to congestion during peak hours.

To demonstrate the application of this model, the case of El Paso Fire Department (EPFD) allocating its fire fighting units and ambulances among its fire stations have been used. The CIs defined in the case study are CTIs, schools and hospitals.

Further extensions and applications of this model are provided after the cast study.

1. 3. Outline

The remaining parts of this thesis are organized as follow.

Chapter 2 reviews three classes facility location problems: covering problems, centre problems and p-median problems. Emphasis is then placed on the research works that used these models in locating ERSs.

Chapter 3 introduces the formulation of the improved probabilistic mixed integer programming covering model in allocating emergency service vehicles to serve the CTIs.

Chapter 4 gives details on how the standard ERS data should be pre-processed prior to the application of the formulated model. All the procedures, including data extraction, demanding frequency and travel time calculations, are described in this chapter.

Chapter 5 applies the improved model with a case study. The results under different levels of service reliability are compared using the fire stations and CIs in El Paso as an example.

Chapter 6 provides some possible extension and further application for this model.

Chapter 7 concludes this research and suggests a few possible research directions.

2. LITERATURE REVIEW

The model developed in this research is based on the FLPs. Three general classes of FLPs namely, covering problems, centre problems and P-median problems, are first presented with typical taxonomies. In the next section, the ERS siting problems are discussed. The mixed integer programming probabilistic covering models (Huang et al., 2007) that directly motivated this research are reviewed.

2.1 Background

Facility Location Problem has been extensively studied in the past decades. The study in location decisions arises from a variety of public and private sector problems mentioned by Daskin (1995). The FLP is designed to make decisions on locations in a quantifiable way and identify algorithms for finding optimal or near optimal facility locations. Daskin (1995) noted that mathematical location models were designed to address a number of questions including:

1) How many facilities should be sited?
2) Where should each facility be located?
3) How large should each facility be and the number of service units to be accommodated in the facility?
4) How should demand for service be allocated to the facilities?

He categorized the FLPs into three general types of facility location problems based on their objectives. Covering problems locate facilities relative to the demand points according to some pre-specified performance standard. The performance standard may be measured by

distance or travel time. In a traditional covering model, a demand node is served only if it is within a pre-specified distance of a facility. Centre problems locate facilities so as to minimize the maximum travel cost that any customer will travel to a facility. Median problems attempt to locate facilities so as to minimize the total weighted travel cost between demand locations and a facility (Miller et al., 2001). Detailed descriptions of these problems are given in the following subsections.

2.2 Three General Classes of Facility Location Problems

2.2.1 Covering Problems

In the context of facility location problems, service to customers depends on the distance between the customer and the facility to which the customer is assigned to. The term distance may refer to Euclidean distance or driving distance. Often, the service is deemed adequate if the customer is within a given distance of the facility and is deemed inadequate if the distance exceeds a critical value. This leads to the notion of coverage. The Location Set Covering Problem (LSCP) (Toregas et al., 1971 and 1973) is perhaps the simplest facility location models. The LSCP sought to position the minimum number of facilities in such a way that every demand node on the network had at least one server initially positioned within some distance or time standard *S*. This may be formulated mathematically using the following notation:

Minimize $$Z = \sum_{j \in J} x_j \tag{2.1}$$

Subject to $$\sum_{j \in N_i} x_j \geq 1 \qquad \forall i \in I \tag{2.2}$$

$$x_j \in \{0,1\} \qquad \forall j \in J \tag{2.3}$$

where

J =set of potential facility locations indexed by j;

I =set of demand nodes indexed by i;

x_j =1 if a server is stationed at j and 0 otherwise;

$N_i = \left\{ j \middle| t_{ji} \leq S \right\}$; the set of nodes j located within the distance or time standard for node i with

t_{ji} = shortest distance or time from potential facility location j to demand node i;

S = the time or distance standard for service coverage;

If a call for service originating at node i is answered by available servers stationed inside a neighborhood defined by S, it will be answered within the time or distance standard.

The objective function (2.1) minimizes the total number of facilities required. Constraint (2.2) states that the demand at each node i must be covered by at least one server located within the time or distance standard S. This problem can be easily solved by linear programming relaxation in which the integer variables are required simply to be non-negative.

Daskin (1995) also mentioned that this set covering model has been applied to a broad range of problems, for example, the application in airline crew scheduling and tool selection in flexible manufacturing system.

One potential shortcoming associated with the set covering model is that the number of facilities that are needed to cover all demand nodes is likely to exceed the number that can actually be built. Furthermore, the set covering model treats all demand nodes of equal importance. The LSCP was thus soon superseded by more sophisticated models which, in turn, have served to generate more realistic problem formulations.

The Maximum Covering Location Problem (MCLP) was introduced by Church and ReVelle (1974). In essence, in the MCLP model, the requirement that all demand nodes be covered is relaxed. This model considers fixing the limited number of facilities to be located and maximizing the number of covered demands:

Maximize $$\sum_{i \in I} h_i Z_i \tag{2.4}$$

Subject to $$Z_i \le \sum_{j \in J} a_{ij} x_j \qquad \forall i \in I \tag{2.5}$$

$$\sum_{j \in J} x_j \le P \tag{2.6}$$

$$x_j = 0,1 \qquad \forall j \in J \tag{2.7}$$

$$Z_i = 0,1 \qquad \forall i \in I \tag{2.8}$$

where

h_i =demand at node i;

P =number of facilities to locate;

a_{ij} =1 if candidate site j can cover demands at node i within S and 0 otherwise;

x_j =1 if a facility is located at j and 0 otherwise;

Z_i =1 if demand node i is *covered* and 0 otherwise.

The objective function (2.4) maximizes the number of covered demands. Constraint (2.5) states that the demand at node i cannot be covered unless at least one of the facility sites j within S of node i is selected. Constraint (2.6) gives the total number of facilities that can be sited. Finally, constraints (2.7) and (2.8) are the binary integer constraints on the decision variables. The frequency of demand h_i, at the node i, is considered. The MCLP gives more weights to the demand nodes with higher demand frequencies.

Relaxed linear programming supplemented by occasional use of branch and bound method was utilized by Church and ReVelle (1974) to provide solutions to this problem. Heuristic algorithms such as Greedy Adding Algorithm and Lagrangian Relaxation are required to solve the MCLP, when the size of the problem becomes very large. One of the assumptions of MCLP is that whenever a demand node i requests a service from a facility j, there is always a server in that facility available to provide the service to that demand node. In reality, all the servers in facilities $j \epsilon J$ may be busy when there are so many simultaneous requests for service from the competent demand nodes that all servers, which can serve that demand node, are not available to respond this request. In this scenario, the demand at node i is not fulfilled.

An extension of MCLP is made to account for the possibility of server congestion or being busy. This consideration leads to development of the Maximum Expected Covering Location Problem (MEXCLP) in which a fixed number of facilities were located to maximize the coverage of expected number of demands (Daskin 1983). The MEXCLP will be explicitly reviewed in Section 2.3.

2.2.2 Center Problems

With the objective of improving the shortcomings of the set covering model, another strategy, namely P-center problem or a minimax problem was developed. In particular, this problem has the same objective as the LSCP which requires all demand nodes covered. However, instead of using an exogenously specified coverage distances and asking the model to minimize the number of facilities needed to cover all the demand nodes, the P-center problem minimizes the maximum coverage distance or travel time such that each demand node is covered by one of the facilities. Since it aims to minimize the maximum possible coverage distance or travel time between a demand and the nearest facility, this model is therefore named as the minimax problem. The center problem is usually categorized as (1) the absolute centre problems in which facilities can be located anywhere on the network, and (2) the vertex centre problem in which the facilities can only be located on the nodes in the network.

It is noted that for a general graph, the P-center problem is NP-complete (Kariv and Hakimi, 1979; Garey and Johnson, 1979). That is why in the past decades, many researchers have tried to design different efficient algorithms to solve the problem. The vertex formulation of the P-center problem is given below:

Minimize $$W \tag{2.9}$$

Subject to $$\sum_{j \in J} y_{ij} = 1 \qquad \forall i \in I \tag{2.10}$$

$$\sum_{j \in J} x_j = P \tag{2.11}$$

$$y_{ij} \le x_j \qquad \forall i \in I \;\; \forall j \in J \tag{2.12}$$

$$W \geq \sum_{j \in J} d_{ij} y_{ij} \qquad \forall i \in I \tag{2.13}$$

$$x_j = 0,1 \qquad \forall j \in J \tag{2.14}$$

$$y_{ij} \geq 0 \qquad \forall i \in I \ \forall j \in J \tag{2.15}$$

where

h_i =demand at node i;

P =number of facilities to locate;

d_{ij} =distance from demand node i to candidate facility site j;

x_j =1 if facility j is located at j and 0 otherwise;

y_{ij} =fraction of demand at node i that is served by a facility at node j;

W =maximum distance between a demand node and the nearest facility.

The objective (2.9) minimizes the maximum distance between a demand node and the closest facility to the node. Constraint (2.10) states that all the demand at node i must be assigned to a facility at some nodes j. Constraint (2.11) stipulates that there are P facilities to be located. Constraint (2.12) states that the demands at node i cannot be assigned to a facility at node j unless a facility is located at node j. Constraint (2.13) states that the maximum distance between a demand node and the nearest facility to the node (W) must be greater than the distance between any demand node i and the facility j to which it is assigned. Constraints (2.14) and (2.15) are the integer and nonnegative constraints, respectively.

A number of authors such as Minieka (1977) and Martinich (1988) have considered extensions to the centre problems. Minieka (1977) extended the notion of coverage to the need to

cover all points on all links of the graph. Thus, the problem becomes one of locating a fixed number of facilities to minimize the maximum distance between all nodes as well as the infinite number of intermediate points on each of the arcs and the nearest facilities. Martinich (1988) proposed a vertex closing heuristic for solving the vertex centre problem. A vertex closing algorithm may be thought of as a "greedy subtraction" algorithm in the sense that such heuristics begin with facilities located at all candidate locations and proceed to close facilities in an intelligent manner.

2.2.3 Median Problems

All variants of covering and center problems discussed above assume that a demand node receives full service from facility *j* if the facility *j* is within the distance or travel time standard of node *i* and no service if facility *j* is outside the distance or travel time standard of node *i*. In many cases, however, the level of service associated with a demand node-facility pair decreases gradually with the distance. Median problems were introduced to account for the distance or travel time between a facility *j* and a demand node *i* and the related cost associated with the facility-demand node pairs.

The P-median problem (Hakimi, 1964) is to find the locations of *P* facilities in a network so that the total service cost is minimized. The cost of serving demand at node *i* is given by the product of the demand at node *i* and the distance or travel time between node *i* and the nearest facility *j*. It was formulated below:

Maximize $$\sum_{i \in I} \sum_{j \in J} h_i d_{ij} y_{ij} \qquad (2.16)$$

Subject to $\sum_{j \in J} y_{ij} = 1 \quad \forall i \in I$ (2.17)

$$\sum_{j \in J} x_j = P \tag{2.18}$$

$$y_{ij} - x_j \le 0 \quad \forall i \in I \ \forall j \in J \tag{2.19}$$

$$x_j = 0,1 \quad \forall j \in J \tag{2.20}$$

$$y_{ij} = 0,1 \quad \forall i \in I \ \forall j \in J \tag{2.21}$$

where

P =number of facilities to locate;

d_{ij} =distance from demand node i to candidate facility site j;

x_j =1 if facility j is located at j and 0 otherwise;

y_{ij} =fraction of demand at node i that is served by a facility at node j;

The objective (2.16) is to minimize the total demand-weighted distance between each demand node and the nearest facility. Constraint (2.17) requires each demand node i to be assigned to exactly one facility j. Constraint (2.18) states that exactly P facilities are to be located. Constraint (2.19) stipulates that demands at node i cannot be assigned to a facility at node j unless a facility is located at node j. Constraints (2.20) and (2.21) are binary integer constraints on decision variables.

The P-median problem is also NP-complete. Effective heuristic algorithm is necessary if one wants to solve problems of realistic size in a reasonable amount of time.

2.3 Emergency Response Service Siting Problems

The ERS siting problem is among the most common applications of the FLP. Significant amount of research has been done, and continues to be focused, on the development of models for coverage availability in ERS systems (Schilling et al., 1993).

2.3.1 Deterministic Models

Early literature on the siting of ERSs was based on the traditional set covering problem. Given a set of constraints and requirements, mathematical programming models were used to optimally deploy ERS units geographically. The LSCP, reviewed in subsection 2.1.2, is the first emergency service covering model which seeks to position the minimum number of ambulance base stations such that every demand point on the network has at least one server (ambulance station) positioned within some distance standard (Toregas et al., 1971 and 1973).

The LSCP model has been used by a number of researchers. Berlin and Liebman (1971) used the model to identify Emergency Medical Service (EMS) station locations. The model was also used to determine EMS stations in rural areas (Jarvis et al., 1975). Walker (1974) used the LSCP model to locate fire stations.

The greatest weakness of LSCP is that it gives no consideration to demand frequency. The requirement of having all the demand points covered all the time may lead to excessive resources and budget. Recognizing these weaknesses, the MCLP model was then introduced to maximize the total number of demand points that could be served within a maximum service

distance, given a fixed number of server facilities (Church et al., 1974). The formulation of MCLP has been presented in Subsection 2.1.2.

Eaton et al. (1979, 1980, and 1985) utilized the MCLP model to determine EMS vehicle locations in Austin, Texas. Bennett et al. (1982) used it to locate rural health care workers in developing countries. The MCLP model and its different variants have been used for the location of health clinics (Eaton et al., 1981) and hierarchical health services (Moore and ReVelle, 1982).

Repede and Bernardo (1994) created TIMEXCLP, a model derived from MEXCLP. The model sought time varying ambulance positions that maximized coverage throughout a day at one-hour intervals with differing call frequencies. It allowed the number of vehicles at a site to vary at different hours of a day as demand evolved. However, in this model, only one type of vehicle is considered.

Both the LSCP and MCLP models consider only one type of server. Two relatively advanced models were then developed from the maximal covering principle to deal with the case of more than one type of server (Schilling et al., 1979). The Tandem Equipment Allocation Model (TEAM) locates two types of vehicles: primary and secondary equipments. Secondary equipments are positioned only at points where primary equipments have already been located. A more general formulation called Facility-Location Equipment-Emplacement Technique (FLEET) locates two types of vehicles, fire engines and fire trucks among the fire stations in an optimal way that maximizes the population or calls having at least one fire engine and one fire truck positioned within the distance or time standard. Coverage in this model is considered sufficient

only when both types of servers are both within their respective distance or time standards. The FLEET model is formulated as follows:

Maximize $$Z = \sum_{i \in I} h_i y_i \tag{2.22}$$

Subject to $$y_i \le \sum_{j \in NE_i} x_j^E \qquad \forall i \in I \tag{2.23}$$

$$y_i \le \sum_{j \in NT_i} x_j^T \qquad \forall i \in I \tag{2.24}$$

$$\sum_{j \in J} x_j^E = P^E \tag{2.25}$$

$$\sum_{j \in J} x_j^T = P^T \tag{2.26}$$

$$\sum_{j \in J} z_j = P^z \tag{2.27}$$

$$x_j^E \le z_j \qquad \forall j \in J \tag{2.28}$$

$$x_j^T \le z_j \qquad \forall j \in J \tag{2.29}$$

$$x_j^E, x_j^T, z_j = 0,1 \qquad \forall j \in J \tag{2.30}$$

$$y_i = 0,1 \qquad \forall i \in I \tag{2.31}$$

Where

y_i =1 if node *i* is covered by a fire engine and a fire truck, 0 otherwise;

x_j^E =1 if a fire engine is located at node *j*, 0 otherwise;

x_j^T =1 if a fire truck is located at node *j*, 0 otherwise;

$NE_i = \left\{ j \middle| t_{ji} \le S^E \right\}$, the set of fire stations with fire engines located within the distance standard of demand node *i*.

$NT_i = \left\{ j \middle| t_{ji} \le S^T \right\}$, the set of fire stations with fire trucks located within the distance standard of demand node i.

P^E, P^T, P^z = the numbers of fire engines, fire trucks and fire stations.

The objective (2.22) maximizes the number of calls covered by both fire engines and fire trucks simultaneously. Constraints (2.23) and (2.24) ensure that a demand point i is covered only if it is covered by both engines and trucks, stationed within their respective standards. Constraints (2.25), (2.26) and (2.27) set the number of fire engines, fire trucks, and fire stations to be sited. Constraints (2.28) and (2.29) force fire engines and fire trucks to be positioned only at nodes where fire stations have been selected. The final two constraints, (2.30) and (2.31) are binary integer constraints for the decision variables.

The FLEET model requires only one unit of each type of service vehicle within their respective distance standards. When a call requires the service of more than one server of a particular type, the siting of more than one server at a site could be a good decision. Therefore, co-location of servers has to be considered. The hybrid FLEET (Bianchi and Church, 1988) has explicitly dealt with the possibility of co-location and further considered the station capacity. Marianov and ReVelle (1991 and 1992) proposed and solved a capacitated model that sought the placement of a fixed number of engines and trucks so that the population or calls for service with a full standard response could be maximized.

2.3.2 Probabilistic Models

A limitation of the FLEET, hybrid FLEET model and other earlier models is the assumption that servers, once placed, are always available to respond to calls for services. The idea of server congestion and as a result, vehicle and equipment availability has not been addressed. Recognizing this limitation, newer generation of coverage models were developed to incorporate randomness in server availability. In these models, the probability of at least one server being available to serve each demand node must be greater than or equal to a predefined constant α. To estimate this probability, an estimate of the probability of a server being busy (q) must first be found. q is called the busy fraction and was assumed to be independent of other servers' q. Daskin (1983) utilized notion of a server busy fraction to formulate the Maximum Expected Covering Location Problem (MEXCLP). He utilized a single system-wide value of busy fraction q in his model and assumed a binomial probability of k servers being busy. ReVelle and Hogan (1988) introduced a local estimate of the busy fraction, q_i, in the coverage area around node i. This local busy fraction is estimated from historical data, using the total actual service time in the region, divided by the total available service time in the region, that is:

$$q_i = \frac{\bar{t} \sum_{k \in M_i} f_k}{24 \sum_{j \in N_i} x_j} = \frac{\rho_i}{\sum_{j \in N_i} x_j} \tag{2.32}$$

where

$\bar{t}$ =average duration of a single call, in hours;

f_k =frequency of calls for service at demand nod k, in calls per day;

M_i =set of demand nodes located within S of node i;

$N_i = \left\{ j \middle| t_{ji} \le S \right\}$; that is N_i is the set of nodes j located within the time or distance standard S of demand node i;

ρ_i =utilization ratio.

The probability that at least one server is available within time or distance standard S when node i requests for service is 1 minus the probability that all servers within S of node i are busy:

$$1\text{-}P\ [\text{all servers within } S \text{ of node } i \text{ are busy}] \tag{2.33}$$

Using the notation of local busy fraction, the probability of at least one server being available to be greater than or equal to α is

$$1-\left[\frac{\rho_i}{\sum_{j\in N_i} x_j}\right]^{\sum_{j\in N_i} x_j} \ge \alpha \tag{2.34}$$

The probability constraint (2.34) does not have an analytical linear deterministic equivalent. ReVelle and Hogan found the numerical deterministic equivalent to be:

$$\sum_{j\in N_i} x_j \ge b_i \tag{2.35}$$

where b_i is the smallest integer which satisfies:

$$1-\left(\frac{\rho_i}{b_i}\right)^{b_i} \ge \alpha \tag{2.36}$$

here b_i is the smallest numbers of servers that must be located within S from node i.

2.3.3 Backup Coverage

Parallel to the probabilistic methods of introducing reliability and busy fraction of servers, Hogan and ReVelle (1986) introduced a concept of backup coverage with the purpose of ensuring a time invariant coverage (the coverage does not vary by time) and a minimum of at least one server to serve a particular neighborhood at all times. Deterministic models named Backup Coverage Problem 1 (BACOP1) and Backup Coverage Problem 2 (BACOP2) were proposed to address the problem of server congestion (Hogan and ReVelle, 1986). The formulation of BACOP1 model is as follows:

Minimize $$Z_1 = \sum_{j \in J} x_j \tag{2.37}$$

Maximize $$Z_2 = \sum_{i \in I} a_i u_i \tag{2.38}$$

Subject to $$\sum_{j \in N_i} x_j - u_i \geq 1 \qquad \forall i \in I \tag{2.39}$$

$$\sum_{j \in M_i} x_j \geq 1 \qquad \forall i \in I \tag{2.40}$$

where

u_i =1 if demand node *i* is covered twice, 0 otherwise;

x_j =integer number of facilities located at potential site *j*;

a_i =population at demand node *i*;

$N_i = \{ j \mid d_{ij} \leq S \}$, that is N_i is set of node *j* located within the standard distance *S*;

$M_i = \{ j \mid d_{ij} \leq T \}$, that M_i is set of node *j* located within the standard distance *T*;

S, *T*= distance standards such that *S*>*T*; and

I, J = the sets of demand nodes and potential facility sites, respectively.

The BACOP1 model seeks to maximize the demand nodes covered by a second server, in addition to the coverage provided by the first servers. It focuses on the first redundant server of a node.

To relax the primary or first coverage requirement, Hogan and ReVelle (1986) formulated the BACOP2 model, which trades off first (primary) coverage against backup (secondary) coverage. The formulation is:

Maximize $$Z_1 = \sum_{i \in I} a_i y_i \quad (2.41)$$

$$Z_2 = \sum_{i \in I} a_i u_i \quad (2.42)$$

Subject to: $$\sum_{j \in N_i} x_j - y_i - u_i \geq 0 \qquad \forall i \in I \quad (2.43)$$

$$u_i - y_i \leq 0 \qquad \forall i \in I \quad (2.44)$$

$$\sum_{j \in J} x_j = p \quad (2.45)$$

$$\sum_{j \in P_i} x_j \geq 1 \qquad \forall i \in I \quad (2.46)$$

where

y_i =1 if demand node i is covered once, 0 otherwise;

u_i =1 if demand node i is covered twice, 0 otherwise;

x_j =integer number of facilities located at potential site j;

a_i =population at demand node i;

$N_i = \{ j | d_{ij} \le S \}$, that is, N_i is set of node j located within the standard distance S;

$P_i = \{ j | d_{ij} \le R \}$, that is, P_i is set of node j located within the standard distance R;

S, R= distance standards such that $S<R$;

P = total number of facilities;

I, J = the sets of demand nodes and potential facility sites, respectively.

Objectives (2.41) and (2.42) maximize primary coverage and secondary coverage, respectively. Constraint (2.43) states that coverage by the first and second servers is bounded by the number of servers stationed in the neighborhood defined by service standard S of demand node i. Thus, if only one server is located in the neighborhood, only the primary or first coverage is achieved. If there are two or more servers, first (primary) and backup (secondary) coverage are achieved. Constraint (2.44) indicates that secondary coverage cannot be achieved without prior achievement of the first or primary coverage. Constraint (2.45) limits the number of servers to be deployed. The last constraint is a mandatory constraint that at least one facility should be located at node j.

A hybrid model, Multiple Cover, One-unit Facility Location, Equipment Emplacement Technique (MOFLEET) which incorporates the concepts of MEXCLP and FLEET was later developed to site stations and allocate ambulances (Bianchi and Church, 1988). This model has been applied to locate ERS vehicles in Fayetteville, North Carolina (Tavakoli and Lightner, 2004). The FLEET model was then further extended to the probabilistic FLEET model through the introduction of randomness into the availability of servers (ReVelle and Marianov, 1991).

This model maximizes the expected value of population coverage by both fire engines and fire trucks with minimum probability of α within the time standard, given a fixed number of facilities to be located on the network. Both of the models described above aim to maximize the expected coverage. In contrast, ReVelle and Hogan (1988 and 1989) examined the problem in a different way that they introduced a model named Probabilistic Location Setting Covering Problem (PLSCP) to minimize the total number of utilized servers, while they constrained the level of server availability to be greater than or equal to a preset value of α .

2.3.4 Simultaneous Allocation of Multiple Vehicle Types

Although some progress has been made in the siting of ERSs, the simultaneous siting of fire and ambulance services (2 types of vehicles) has rarely been explored. ReVelle and Snyder (1995) attempts to address this by an integrated model called Fire and Ambulance Service Technique (FAST) which extends and blends the MCLP for ambulance siting and the FLEET for fire service placement to simultaneously site both types of services, but allowing ambulances to be positioned at locations other than the fire stations. The objective of the FAST model is to maximize and trade-off the coverage provided by ambulances and fire engines subject to constraints on the total number of vehicles and stations. However, being a deterministic model, FAST does not consider server availability.

Cheu et al. (2007) expanded the earlier formulation of the probabilistic covering model from one type of ERS to three types of ERSs to cover CTIs. In their model, sufficient coverage of a CTI is achieved only when all types of ERSs are each individually capable of reaching the

CTI within the specified distance standard and reliability level. A variant of the model that incorporates the concept of backup coverage to give more emphasis to the highly critical transportation infrastructures relative to the less critical transportation infrastrucutre is also proposed.

2.4 Discussion

For all of the models reviewed in the previous sections, some are deterministic models with the assumptions that all the servers are available whenever there is a new request. The probabilistic models, taking into account that a server may be busy serving other requests, usually assume that all the servers have the same busy fraction. Although newer probabilistic models use the concept of the local busy fractions, the local busy fractions are the same throughout the network and throughout the day. None of them considers the variations of demands during a day and across the city.

2.5 Summary

In this chapter, the background and previous research work in FLPs were reviewed according to three general classifications: covering problems, centre problems and P-median problems. Comparisons in model formulations and applications of the three types of problems have also been provided to better understand the advantages and shortcomings of each of them.

A complete review has been made on deterministic models and probabilistic models applied to the field of siting ERSs. New extension of probabilistic covering models from one

type of ERS to multiple types of ERSs, and the incorporation of the conception of backup coverage are also reviewed.

Based on the review conducted, the potential improvements in the formulation of an improved model for ERS vehicle allocation problem to optimally serve CIs have been identified. The new model will be presented in the next chapter.

3. FORMULATION OF IMPROVED PROBABILISTIC COVERING MIXED INTEGER PROGRAMMING MODEL

3.1 Introduction

In this chapter, an improved probabilistic covering mixed integer programming model, which addresses the spatial and temporal distribution of competing demands and the temporal distribution of travel time due to congestion during peak hours, is formulated.

3.2 Problem Statement

The ERSs of a city usually faces the problem how to provide prompt responses to the vast amount of emergency events during daily operation. Of all the emergency events, those that happened in those CIs, such as schools, hospitals and critical transportation infrastructures are of special interests to the emergency response departments. These CIs are vital to the socio-economic life of a city and are worthy of additional attentions.

Due to budget limitation, emergency response departments cannot equip as many resources, hire as many crews and build as many stations as possible. The locations of the ERS base stations are fixed after they were built as needs arisen with the development of the city. In the short term, the capacity of a station to accommodate emergency vehicles is also fixed as the land around the station is hard or expensive to expand the facility.

In addition, there is no way to accurately forecast where the emergency events will happen within the city. It is also impossible to forecast the types and frequencies of the emergency events in certain hours of a day. Certain emergency events such as traffic accidents have a higher probability of occurring during the morning and evening peak hours. Other emergencies events such as medical emergencies and fires have higher frequencies in the residential areas and in the evening than in the commercial areas. These variations of the temporal and spatial distribution must be considered.

To allocate the limited number of each type of ERS units in a set of base stations in order to maximize the temporal and spatial coverage of CIs within the respective standard service times, a sophisticated allocation model is needed to achieve this goal. As mentioned in Chapter 2, none of the previous models considers both the spatial and temporal distribution of competing demands and temporal distribution of travel time. Thus, a new model is developed to address these shortcomings.

3.3 Model Formulation

In order to address the spatial and temporal distribution of competing demands, we assume that a typical 24-hour day is divided into T discrete time periods $\tau = 1, \ldots, T$. The time period may be as short as one hour to as long as one crew-shift. In addition, we assume that, in the historical data set, for every emergency call the location of the call, the vehicle dispatch time and service time are also known. The locations of the calls reflect the spatial distribution of the

demands. The vehicle dispatch times reflect the temporal distribution of demand. Based on this data, the location and time specific busy fraction may be computed by the following equation

$$q_{i,\tau}^{v} = \frac{\sum_{l \in ME_{i,\tau}} t_l^v}{d_\tau \sum_{j \in NE_{i,\tau}} x_j^v} = \frac{\rho_{i,\tau}^v}{\sum_{j \in NE_{i,\tau}} x_j^v} \tag{3.1}$$

where $q_{i,\tau}^{v}$ = the local busy fraction of a ERS unit of type v at CI node i in time period τ;

t_l^v = the service time of a ERS unit of type v in response to emergency call l;

$ME_{i,\tau}^{v}$ = the set of all the nodes with historical demands within S^v of CI node i that generated emergency calls in time period τ;

$NE_{i,\tau}^{v}$ = the set of fire stations within S^v of CI node i in time period τ;

d_τ = total available service time during the time period in the historical data;

$\rho_{i,\tau}^v$ = the utilization ratio for a unit of type v at CI node i in time period τ.

x_j^v = number of units of type v allocated to station j;

S^v = the service standard (maximum travel time) of ERS units of type v from its station to the call location

Most ERS departments recognize that it is impossible for their units to reach the scenes for all the emergency calls. Therefore, they normally set the target that S^v must be fulfilled at 100 α % of the time ($0 < \alpha < 1$). Typical values of α are 0.90, 0.95 and 0.99. In another words, when CI node i is requesting for an emergency service, the probability of having at least one

ERS unit of type v available within S^v of node i must be larger than or equal to α. The above condition is equivalent to the expression

$$1-\left[\frac{\rho_{i,\tau}^{v}}{\sum_{j\in NE_i} x_j^v}\right]^{\sum_{j\in NE_i} x_{j,\tau}^v} \geq \alpha \,. \tag{3.2}$$

The above equation can be replaced by

$$1-\left[\frac{\rho_{i,\tau}^{v}}{e_{i,\tau}^{v}}\right]^{e_{i,\tau}^{v}} \geq \alpha\,, \tag{3.3}$$

where $e_{i,\tau}^{v}$ = the smallest integers which satisfy the inequality in (3.3).

Typically $\rho_{i,\tau}^{v}$ is estimated from historical data. Equation (3.3) is used to obtain the $e_{i,\tau}^{v}$ values which will be entered as constants in the linear programming problem.

The new mixed integer linear programming problem is then

Maximize $$\sum_{\tau=1}^{T}\sum_{i\in I} y_{i,\tau} w_{i,\tau} \tag{3.4}$$

subject to $$\sum_{j\in NE_i,\tau} x_j^v \geq e_{i,\tau}^{v} y_{i,\tau} \quad \forall i \in I\,,\ \forall v \in V\,,\ \tau=1,\ldots,T \tag{3.5}$$

$$\sum_{j\in J} x_j^v \leq P^v \qquad \forall v \in V \tag{3.6}$$

$$0 \leq \sum_{v\in V} x_j^v \leq B_j \qquad \forall j \in J\,,\ \forall v \in V \tag{3.7}$$

$$y_{i,\tau} = 0,1 \qquad \forall i \in I\,,\ \tau=1,\ldots,T \tag{3.8}$$

$$x_j^v \geq 0 \qquad \forall i \in I\,, \forall v \in V \tag{3.9}$$

where $y_{i,\tau}$ = the binary variable that defines the coverage of CI node i in time period. $y_{i,\tau}$ =1 if node i is covered by $e^{v}_{i,\tau}$ units within S^{v}, $\forall v \in V$ simultaneously, otherwise, $y_{i,\tau}$ =0;

$w_{i,\tau}$ = the weight for the coverage of a particular CI node i in time period τ. For example, the modeler may want to use a relatively smaller weight for the school nodes after school hour, or a relatively higher weight for a CTI because there is no alternate link that connects both sides of a city;

B_j = the capacity of station j. The station capacity constraint in (3.7) has been modified such that all the ERS units (of different types) that are allocated to station j cannot exceed B_j.

The objective function (3.4) maximizes the weighted sum of covered demands during different period of a day. Different demand nodes are assigned a different weight based on the importance. Constraint (3.5) states that the demand node i cannot be covered during period τ unless at least $e^{v}_{i,\tau}$ number of vehicle of type v are available in time period τ. Constraint (3.6) restricts the total number of vehicle of type v that can be assigned to the stations. Constraint (3.7) states that the total number of the vehicles assigned to station j cannot exceed the capacity of that station. Constraint (3.8) is the binary integer constraint of the decision variables.

Compared to the earlier models, the number of constraints in this model is approximately multiplied by the factor T. Another point worth noting is that, in this formulation, different CI

nodes may have different reliability requirement in different time periods, resulting in different $e_{i,\tau}^{v}$ values in Equation (3.5).

4. IMPLEMENTATION METHODOLOGY

4.1 Introduction

As mentioned in the previous chapter, the improvements of this new model include more accurate calculation of (1) local utilization ratio $\rho_{i,\tau}^{v}$ and (2) travel times between base stations and demand nodes at the different time periods of the day. In this chapter, the improved methods used to calculate $\rho_{i,\tau}^{v}$ and travel times are presented. The values of $\rho_{i,\tau}^{v}$ for different types of ERS vehicle located in each station are calculated using the historical data provided by the El Paso Fire Department (EPFD).

Travel time estimation is another important part of the new model. In constraint (3.5), $NE_{i,\tau}^{v}$, all potential stations within the service standard travel time S^{v} of demand node i in time period τ must be known. To find out these potential stations, the travel time in time period τ between all available stations and all demand nodes must be estimated and compared with the standard service time S^{v}. The actual travel times between stations and demand nodes are impacted by the traffic load on the transportation network in that time period. The traffic volume and hence travel times vary during the different period of the day. The simple method of dividing the distance by speed limit, as used in past research, is not sufficiently accurate for this new model. A more sophisticated method that takes this variation into account is adopted in this modeling effort.

4.2 Calculation of Local Utilization Ratios

The demand frequency for emergency service varies not only by station, but also by hour of a day. Moreover, the types of requested services which resulted in different ERS vehicles being dispatched also show this kind of temporal and spatial variation by stations.

To calculate the demand frequency for each station at different time periods of a day, historical data from EPFD is used as an example. The formats of these data follow the national standard for recording emergency response events. Thus, this calculation method can be used by other cities following the same national standard.

The raw data, however, cannot directly be used to calculate local utilization ratio $\rho_{i,\tau}^{v}$. The monthly raw data provided by EPFD come in two separate text files. The first data file, as shown in Figure 1, records the incident no. [INCI NO], ID of ERS vehicle dispatched [UNIT ID], creating time of the event record [EVT CREATE TIME], starting status of the event [STATUS], starting time of the event in that status [UNT TIME FMT], ending time of the event in that status [END TIME FMT], and case number [CASE NUMBER]. The second data file, as shown in Figure 2, records incident no. [CAT#], ID of the dispatched ERS vehicle [INC#], dispatching time [DISPATCH], time when the dispatched vehicle became available for the next call [AVAILABLE] and event end time [CLOSED]. To distinguish these two data files, the first data file is called the dispatch file, and the second data file is called the event file, based on the stored information.

INCI NO	UNIT ID	EVT CREATE TIME	STATUS	UNT TIME FMT	END TIME FMT	NEXT STATUS	CASE NUMBER
1151	LR21	061201005125	DM	12/01/06 00:52:34	12/01/06 00:53:05	E	06065062
1151	P21	061201005125	DM	12/01/06 00:52:34	12/01/06 00:53:07	E	06065062
1151	LR21	061201005125	E	12/01/06 00:53:05	12/01/06 00:55:15	S	06065062
1151	P21	061201005125	E	12/01/06 00:53:07	12/01/06 00:55:14	S	06065062
1151	P21	061201005125	S	12/01/06 00:55:14	12/01/06 01:10:57	TR	06065062
1151	LR21	061201005125	S	12/01/06 00:55:15	12/01/06 01:10:41	TR	06065062
1165	L11	061201010048	DM	12/01/06 01:02:01	12/01/06 01:02:59	E	06065063
1165	R11	061201010048	DM	12/01/06 01:02:01	12/01/06 01:03:01	E	06065063
1168	R12	061201010137	DM	12/01/06 01:02:30	12/01/06 01:03:22	E	06065064
1168	P12	061201010137	DM	12/01/06 01:02:30	12/01/06 01:03:23	E	06065064
1165	L11	061201010048	E	12/01/06 01:02:59	12/01/06 01:05:00	R	06065063
1165	R11	061201010048	E	12/01/06 01:03:01	12/01/06 01:04:29	S	06065063
1168	R12	061201010137	E	12/01/06 01:03:22	12/01/06 01:06:22	S	06065064
1168	P12	061201010137	E	12/01/06 01:03:23	12/01/06 01:06:45	S	06065064
1165	R11	061201010048	S	12/01/06 01:04:29	12/01/06 01:09:52	TR	06065063
1165	L11	061201010048	R	12/01/06 01:05:00	12/01/06 01:05:09	I	06065063
	L11	061201010048	I	12/01/06 01:05:09	12/02/06 03:38:07	DM	06065063
1168	R12	061201010137	S	12/01/06 01:06:22	12/01/06 01:33:09	TR	06065064
1168	P12	061201010137	S	12/01/06 01:06:45	12/01/06 01:39:00	R	06065064
1165	R11	061201010048	TR	12/01/06 01:09:52	12/01/06 01:19:08	H	06065063
1151	LR21	061201005125	TR	12/01/06 01:10:41	12/01/06 01:31:57	H	06065062
1151	P21	061201005125	TR	12/01/06 01:10:57	12/01/06 01:11:03	TR	06065062
1151	P21	061201005125	TR	12/01/06 01:11:03	12/01/06 01:26:57	H	06065062
1168	LR9	061201010137	DM	12/01/06 01:13:43	12/01/06 01:15:02	E	06065064
1168	LR9	061201010137	E	12/01/06 01:15:02	12/01/06 01:15:02	E	06065064
1168	LR9	061201010137	E	12/01/06 01:15:02	12/01/06 01:26:28	S	06065064

Figure 1: Data format of the dispatch file

CAD#	INC#	DISPATCH	AVAILABLE	CLOSED
1151	06065062	12/01/06 00:52:34	12/01/06 01:50:17	12/01/06 01:50:45
1165	06065063	12/01/06 01:02:01	12/01/06 01:28:19	12/01/06 01:34:06
1168	06065064	12/01/06 01:02:30	12/01/06 02:08:46	12/01/06 02:19:12
1185	06065065	12/01/06 01:15:49	12/01/06 01:39:19	12/01/06 01:40:00
1195	06065066	12/01/06 01:18:21	12/01/06 01:27:42	12/01/06 01:27:50
1214	06065067	12/01/06 01:33:45	12/01/06 01:50:15	12/01/06 01:50:38
1223	06065068	12/01/06 01:40:22	12/01/06 02:22:12	12/01/06 02:22:38
1235	06065069	12/01/06 01:50:10	12/01/06 02:37:59	12/01/06 02:38:41
1247	06065070	12/01/06 02:07:32	12/01/06 02:30:56	12/01/06 02:31:08
1250	06065071	12/01/06 02:08:31	12/01/06 02:22:59	12/01/06 02:28:06
1265	06065072	12/01/06 02:22:25	12/01/06 02:58:44	12/01/06 03:01:40
1277	06065073	12/01/06 02:27:58	12/01/06 02:48:16	12/01/06 02:48:31
1288	06065074	12/01/06 02:32:56	12/01/06 03:43:56	12/01/06 03:44:07
1291	06065075	12/01/06 02:34:34	12/01/06 02:58:34	12/01/06 03:01:33
1296	06065076	12/01/06 02:40:38	12/01/06 03:52:33	12/01/06 03:57:09
1306	06065080	12/01/06 02:53:32	12/01/06 03:18:10	12/01/06 03:18:55
1316	06065077	12/01/06 03:06:00	12/01/06 04:20:18	12/01/06 04:20:46
1324	06065078	12/01/06 03:10:31	12/01/06 03:56:25	12/01/06 03:57:16
1330	06065079	12/01/06 03:18:51	12/01/06 04:41:52	12/01/06 04:42:08
1333	06065081	12/01/06 03:19:39	12/01/06 03:39:18	12/01/06 03:39:51
1344	06065082	12/01/06 03:32:06	12/01/06 03:46:05	12/01/06 03:46:17
1358	06065083	12/01/06 03:42:07	12/01/06 04:16:51	12/01/06 04:20:53
1377	06065084	12/01/06 04:05:50	12/01/06 04:56:59	12/01/06 05:01:11
1384	06065085	12/01/06 04:11:51	12/01/06 05:15:26	12/01/06 05:15:33
1388	06065086	12/01/06 04:14:33	12/01/06 05:12:41	12/01/06 05:12:56

Figure 2: Data format of the event file

In order to calculate the duration of service time of an emergency response vehicle from a station, these two files must be merged to form complete records with essential information such as case number, event starting time, duration of the event, and dispatched vehicles. There are incomplete data records for some events. In addition, some other events did not involve ERSs. These incomplete and unrelated data records were removed.

The following flow chart shows the detailed procedure of calculating $\rho^{v}_{i,\tau}$ from the two historical event data files.

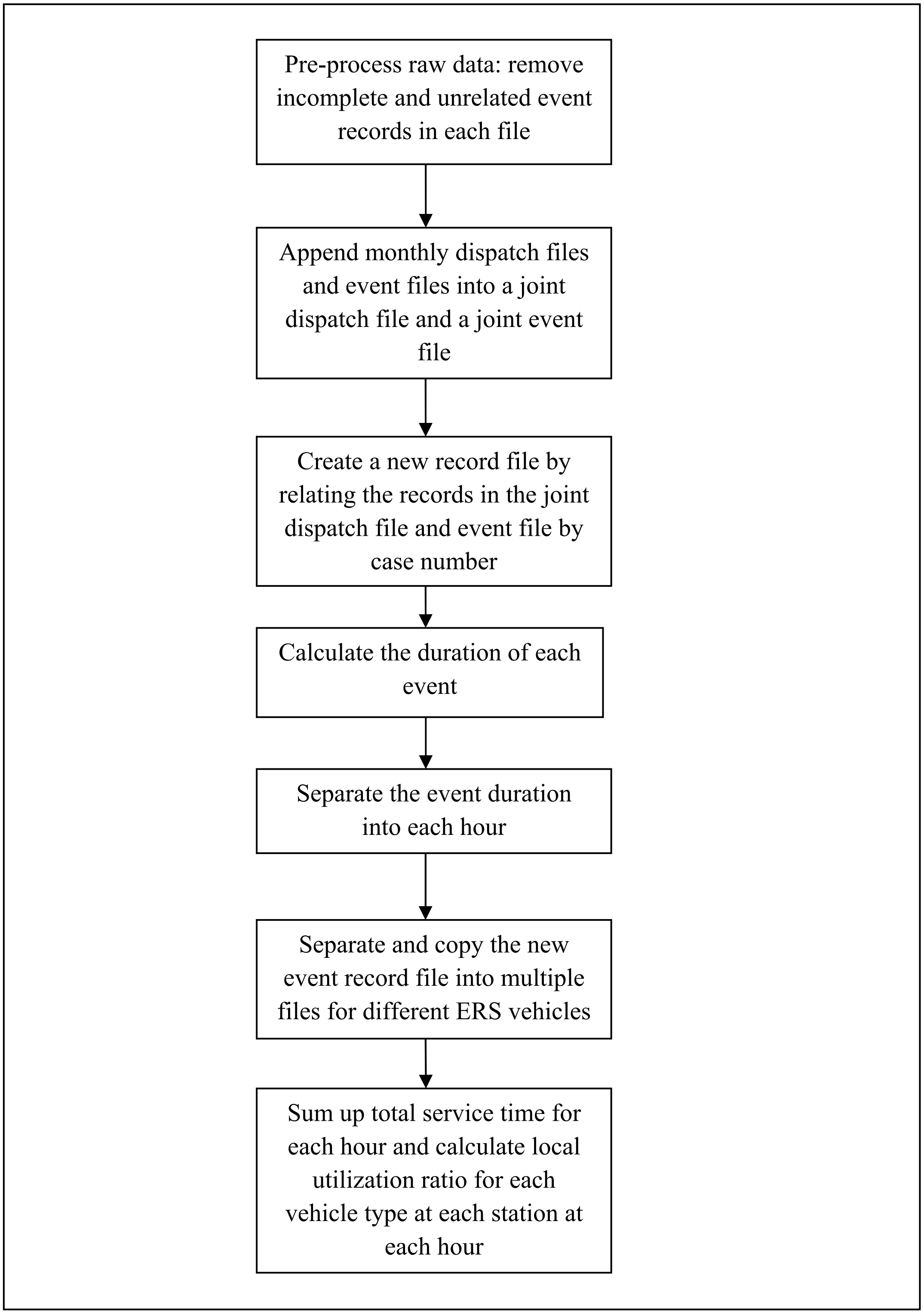

Figure 3: Flow chart for calculating local utilization ratios

First, those incomplete and unrelated records are removed from the monthly files. Then, the monthly dispatch files and event files are merged into a joint dispatch file and a joint event file. In order to get a complete record file with dispatched vehicle, duration of service time etc, these two files are merged by making use of the common field, i.e., case number. This operation cannot be done simply by typical spread sheet software such as Microsoft Excel. The joint dispatch file and joint event file are imported into database software such as Microsoft Access. The imported records are related by the case number to generate the complete record file and the necessary columns are added in the next step.

	A	B	E	F	I	J	K
1	Unit	Dispatched	INC#	HOUR	AVAILABLE	YR	MO
2	L1	2006/9/4 12:07:57	6048688	12	2006/9/4 13:02:55	2006	9
3	L1	2006/9/4 15:17:01	6048723	15	2006/9/4 16:12:41	2006	9
4	L1	2006/9/4 22:24:36	6048780	22	2006/9/4 23:11:50	2006	9
5	L1	2006/9/4 22:55:23	6048785	22	2006/9/4 23:09:21	2006	9
6	L1	2006/9/5 3:01:56	6048814	3	2006/9/5 4:05:57	2006	9
7	L1	2006/9/5 10:41:03	6048858	10	2006/9/5 11:26:14	2006	9
8	L1	2006/9/5 11:57:41	6048867	11	2006/9/5 12:04:48	2006	9
9	L1	2006/9/5 17:18:08	6048923	17	2006/9/5 17:36:33	2006	9
10	L1	2006/9/6 0:13:34	6048989	0	2006/9/6 1:02:21	2006	9
11	L1	2006/9/6 10:43:57	6049038	10	2006/9/6 11:46:19	2006	9
12	L1	2006/9/6 18:00:24	6049106	18	2006/9/6 19:06:10	2006	9
13	L1	2006/9/7 13:12:11	6049223	13	2006/9/7 13:49:14	2006	9
14	L1	2006/9/8 14:06:07	6049406	14	2006/9/8 14:57:02	2006	9
15	L1	2006/9/9 13:06:10	6049586	13	2006/9/9 13:50:57	2006	9
16	L1	2006/9/10 3:04:49	6049732	3	2006/9/10 3:57:59	2006	9
17	L1	2006/9/10 6:04:41	6049747	6	2006/9/10 7:04:46	2006	9
18	L1	2006/9/11 7:43:07	6049940	7	2006/9/11 8:37:06	2006	9
19	L1	2006/9/11 11:24:16	6049975	11	2006/9/11 12:11:08	2006	9

Figure 4: Complete record file

The duration of a vehicle being busy is calculated by subtracting the dispatching time (the [Dispatched] column) from ending time (the [AVAILABLE] column) in Figure 4. After that, the

duration a vehicle being busy in a particular hour must be considered. The reason to find out the service time of a particular hour is that the partitions of the analysis periods are usually at the start of an hour. Care must be taken in the calculation because the starting time and the ending time of an event are usually in the middle of an hour. For instance, a fire fighting vehicle may response to an emergency at 9:30 p.m. and be free at 11:15 p.m.. The actual busy time for this fire fighting vehicle should be 30 minutes in the 9:00 – 10:00 p.m. interval, 60 minutes in the 10:00 – 11:00 p.m. interval and 15 minutes in the 11:00 – 12:00 p.m. interval. This hourly service time can be done by in Excel using a simple customized function.

The next step is to separate the records into different ERS vehicle types. The ERS vehicle's names start with the first letter of the vehicle type (*L* (ladder), P (pumper), *Q* (quint) and *R* (rescue)) and follow by the number assigned by EPFD. The column [UNIT] in the complete record file is used to distinguish the type of the ERS vehicles. In this research, the vehicles are classified into two categories, fire fighting vehicles (ladder, pumper and quint) and rescue vehicles (rescue). The Filter function in Excel is used to filter these two categories of vehicles and create separate files to store the results.

Usually, each vehicle has a unique name or code in the system so that to which fire station it belongs is known. It is easy in software like Excel to select all the records for one fire station from the others. The total service times for the interested time intervals for that fire station were then summed. The local utilization ratios $\rho_{i,\tau}^{v}$ could be calculated by Equation (3.1).

4.3 Estimation of Travel Time Distribution

The purpose of estimation of travel times between demand nodes and fire stations is to find out $NE_{i,\tau}^{v}$, the potential ERS stations within the service standards of each demand node *i*. This step is the key point to construct the constraint equations. In this model, the travel time is not estimated from the free-flow travel time, which is simply the result of dividing the link length by speed limit. Instead, factors like traffic volume and congestion have significant impact on the travel time. The traffic volume varies by hour of day and by day of week. This variation is a reflection of the traffic load on the road network, and often shows as the obvious morning peak and evening peak. To accommodate this fluctuation so as to get a more accurate estimation of hourly travel time, common software in transportation planning is used to fulfill this task.

Some software, such as TransCAD (Caliper, 2005) and DYNASMART-P (Federal Highway Administration, 2008) are commonly used in transportation planning applications. Both of them have their strengths and weaknesses, and are used in different application environment to meet different project requirements. For this thesis, TransCAD is chosen as an example because TransCAD data, being in GIS format, are always available from a city's metropolitan planning organization (MPO) or department of transportation (DOT). This task is to estimate travel times during several periods of a day. This could be very conveniently estimated in TransCAD. DYNASMART-P has its strength in the estimation of travel time in dynamic (e.g., peak to off-peak) environment. However, it requires more effort in network coding. TransCAD provides powerful GISDK—a programming language to manipulate GIS maps. In this case, the

CIs and fire stations need to be added into the map. Using the GISDK, these features could be added to the map in a relatively short time.

4.3.1 Manipulation of GIS Map

The road network from a transportation planning database includes only major roads and intersections. None of the fire stations, hospitals, schools are included in that map. The transportation planning map provided by El Paso MPO is shown in Figure 5. Thus, an important task is to add those points into the map. Moreover, to make these new points accessible to each other, new links are also needed to connect these new points to the existing road network. The process also involves setting the properties for each of the new links.

Figure 5: Transportation planning map before compilation

The latest shapefiles of fire stations, schools and hospitals can be downloaded from public sources. Since TransCAD uses its own format for storing and accessing maps but is compatible with shapefiles. These shapefiles are opened in TransCAD and exported into map files in the TransCAD format. The CIs and fire stations are represented in standard coordinate (degree: minute: second) while TransCAD uses million seconds for coordinates. The coordinates of the CIs and fire stations are transferred into million seconds, loaded into TransCAD and saved as a TransCAD map file.

Though all the CIs and fire station nodes are transferred into TransCAD map files, they are still in separate node layers. To estimate the travel times between the fire stations and the CIs, they must be in the same transportation planning layer. There are two ways to do so. One way is manual editing—overlaying these node layers with the existing transportation map, zooming in to every single demand node and fire station, and connecting this node to the adjacent link. It is usually convenient to manually add a small number of nodes into an existing map. But for a big network, there will be hundreds of CIs and fire stations. This process need a lot of time, efforts and care, and will still be error-prone. Another way is to use the GIS programming—reading the coordinate of each new node from the node layers, adding respective node into the transportation planning map and generating a new link with starting/ending coordinates of the new node and the nearest existing node. By this way, efficiency is greatly improved and manual error of map compilation is avoided. In this thesis, a program written in the TransCAD GISDK (Caliper, 2005) script was implemented, and the transportation map after compilation is shown in Figure 6.

Figure 6: Transportation planning map after compilation

After those new nodes are incorporated into the existing map and, link information such as link capacity, speed limit and etc must be added. The new link may be treated as driveways. A 30 mph speed limit and 500 vehicles per hour capacity are considered reasonable and assigned to all the new links.

4.3.2 Traffic Assignment

Traffic assignment concerns with the selection of routes (alternatively called paths) between origins and destinations in transportation networks. It is the fourth step in the

conventional travel demand model, following trip generation, trip distribution, and mode choice. In trip generation, the geographical patterns of trip making are summarized by dividing the region into small travel analysis zones (TAZs). Trip origins and destinations are associated with these zones. The number of trips produced and attracted by each zone is estimated by socio-economic and demographic factors of each zone. The trip distribution step of the travel demand model is concerned with the estimation of the number of trips between all the TAZs. In a typical travel decision, a trip maker can select between several travel modes. These may include driving alone, share ride, bus, walking and so forth. The third step of the travel demand model, known as mode choice, is concerned with the trip maker's behavior regarding the selection of travel mode. The last step, known as traffic assignment, is concerned with the trip-maker's choice of path between his origin and destination by the selected travel model. The result of traffic assignment is vehicular flows in the multimodal transportation network. This step may be viewed as the equilibration model between the demand for travel estimated and the supply of transportation in terms of the physical facilities. Before the traffic assignment step, the trip production and attraction table from the earlier steps are assembled to form an origin-destination matrix (O-D matrix) with directional flow of the trips.

In order to estimate the travel times at the different time periods of the day (a period may be longer than one hour), the actual or expected traffic loading for each interval must be used. However, the O-D matrix for traffic assignment in the four-step transportation planning process is the 24-hour O-D matrix. This 24-hour O-D matrix contains the 24-hour directional flow of the trips between all the TAZs. Thus, O-D matrices for the hourly interval or periods must be generated by making use of the so-called "K-factor". The K-factors are the ratio of hourly traffic

volume to the 24-hour traffic volume. The city or state's DOT usually maintains a K-factor table for a city. If the time interval is longer than one hour, the average K-factor for that time period may be used. The new O-D matrix for a time period is obtained by multiplying the 24-hour O-D matrix by the average K-factor of the period. Then, one traffic assignment is performed for each of these matrices to produce the link volumes and travel times for each time period. The outcomes of traffic assignment is used in TransCAD to estimate the travel times between the demand nodes and the fire stations and identify which fire stations are within the service standard in terms of travel time from a CI node.

4.4 Summary

In this chapter, two key tasks in the preparation of data for the improved mixed integer programming model are presented. Because all the data used in this research are from the local MPO and fire department, the techniques used in this research can be adapted to the other cities.

5. CASE STUDY

This chapter presents a case study that applies the improved model presented in Chapter 3 to the City of El Paso to allocate all the ERS units operated by the El Paso Fire Department (EPED). Two reliability levels (90% and 95%) under the service standard of 4.5 minutes are investigated in this case study.

5.1 Background

5.1.1 El Paso Fire Department

El Paso is located at the western end of Texas, at the border between Texas, New Mexico and Mexico. It has a population of more than 600,000. The EPFD is the primary agency in the City of El Paso responsible for responding to fire, medical emergencies, hazardous materials incidents, building explosions and search and rescue events. The EPFD currently has 35 fire stations located in the City of El Paso (including 1 under construction). Figure 7 shows a map of the fire districts (by battalions) where each fire district consists of several fire stations.

The EPFD operates the following ERS vehicles: 4 ladders, 33 pumpers, 8 quints and 23 rescues. Ladders are fire trucks equipped with ladders to reach multi-story buildings. Pumpers are fire trucks with high pressure water pumps. Quints have the combined functions of ladders and pumpers. Rescues are ambulances. The ladders, pumpers and quints are collectively known as fire fighting units in this research. The existing allocations of the EPFD units among the fire stations, as provided by EPFD, are listed in Table 1. The EPFD handled 70,472 calls in the

calendar year of 2006, of which 24,992 required the dispatch of both fire fighting units and rescues. The remaining 45,480 cases resulted in the dispatch of only rescues. As for the service reliability and service standard, the EPFD requires its units to reach the scenes within 4.5 minutes for at least 90% of the cases.

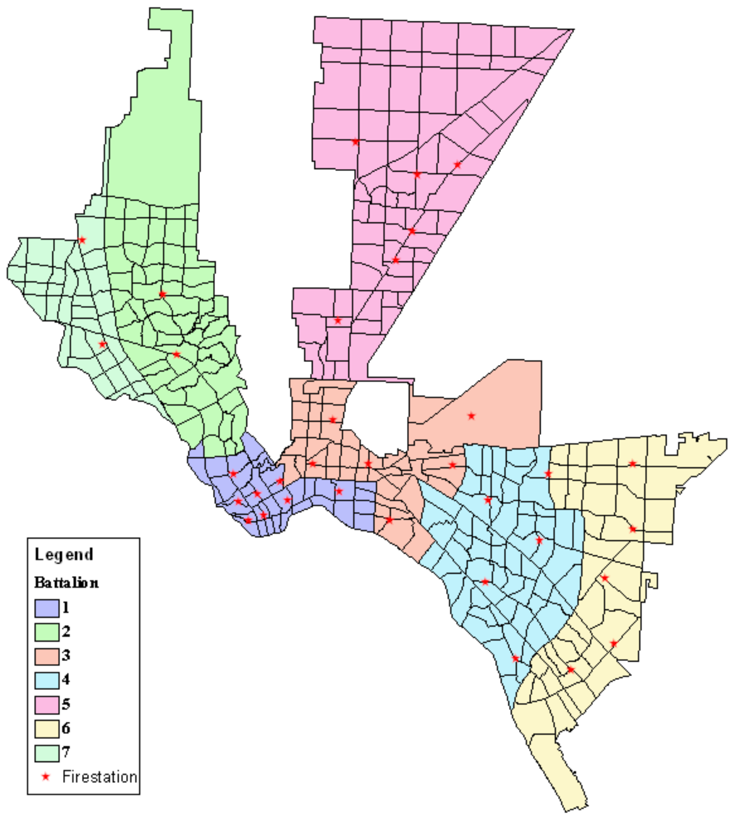

Figure 7: Fire districts and locations of fire stations

Table 1: Station capacities and unit allocations

Fire station	Station capacity	Existing number of units				After optimization with α=0.99			
		Ladder	Pumper	Quint	Rescue	Ladder	Pumper	Quint	Rescue
1	10	1	1		1		1		1
2	4		1						
3	4	1	1		1		1		1
4	1		1				2		1
5	5	1	1		1		1		1
6	3			1	1				
7	3		1	1					1
8	1		1						1
9	2		1		1			2	1
10	1		1				1		1
11	5	1	1		1			1	1
12	2		1		1			2	2
13	1		1					1	2
14	2		1		1		1		
15	3		2		1				
16	3		1	1	1				
17	3		1	1	1	2			2
18	4		1	1	1				
19	2		1		1				2
20	3		1	1	1		2		
21	2		1		1				
22	4		1	1	1				1
23	3		1		1				
24	5		1	1					
25	3		1		2	2			2
26	3		1		1		2		2
27	3		1		1			2	1
28	4		1						
29	3		1		1				
30	3		1						
31	uc	-	-	-	-	-	-	-	-
32	2				1		3		
33	3		1						
34	3		1						
35	3		1						
Total	106	4	33	8	23	4	14	8	23

uc=under construction

5.1.2 Critical Infrastructures

In this case study, the CIs are CTIs, schools and hospitals within its fire districts. With the assistance of Texas Department of Transportation (TxDOT), 20 CTIs were identified. They include the bridges, tunnels, freeway interchanges and major intersections. A total of 105 public elementary, middle and high schools were found in the web sites of the school districts. 13 public and private hospitals were identified from public online sources. The locations of the CTIs, schools and hospitals are shown in Figure 8. The total number of CIs is 138.

Figure 8: Locations of CTIs, schools and hospital in El Paso

5.1.3 Other Data

The transportation network in El Paso was obtained from TxDOT and El Paso MPO. The network is based on the model year 2005, provided in TransCAD files. It has 681 traffic analysis zones, 4836 links and 3060 nodes. The network is also shown in Figure 8. The 24-hour origin-destination (O-D) matrix consists of the trips made in a typical weekday in 2007.

The ERS activity log between September 2006 and February 2007 (6 months) was provided by EPFD. There were 32,116 emergency calls from all sources within the six month period.

5.2 Model Set-Up

For practical considerations, it was decided to divide the 24-hour day into three eight-hour periods (T=3, τ={1, 2, 3}). Although theoretically it is possible to use 24 one-hourly periods (T=24). The assumption of $T=3$ is to correspond to the eight-hour shift by the fire fighting and rescue crews. After examining the hourly fluctuation of traffic flow in the city and the hourly distribution of emergency calls, the three periods were identified as: 3:00 a.m. to 11:00 a.m. (τ=1, morning shift), 11:00 a.m. to 7:00 p.m. (τ=2, afternoon shift), and 7:00 p.m. to 3:00 a.m. (τ=3, night shift). The period from 11:00 a.m. to 7:00 p.m. (τ=2) covers the hours with the highest call frequencies.

Since EPFD operates four types of units, V={ladder, pumper, quint, rescue}. According to EPFD, a CI node was considered covered if one rescue and one fire fighting unit (ladder, pumper or quint) could reach the CI node within their respectively service standards. An easier way to handle this requirement is to regard ladder, pumper and quint as one type of unit, and rescue as another type of unit. However, after discussion with EPFD, it was decided to keep the four types of units in the model. Therefore, constraint (3.5) was modified to

$$\sum_{j \in NE_i} (x_j^{ladder} + x_j^{pumper} + x_j^{quint}) \geq e_{i,\tau}^{fire} y_{i,\tau} \quad \forall i \in I \text{ , } \tau=1,...,T \tag{5.1}$$

$$\sum_{j \in NE_i} x_j^{rescue} \geq e_{i,\tau}^{rescue} y_{i,\tau} \quad \forall i \in I \text{ , } \tau=1,...,T \tag{5.2}$$

where $e_{i,\tau}^{fire}$ denotes the minimum total number of fire fighting units (sum of ladders, pumpers and quints) that must be allocated to stations within the service standard of CI node i so as to provide sufficient service reliability.

The service standards for the fire fighting units and rescues were the same. The maximum time to reach the scene is S^v=4.5 minutes. Besides the current reliability requirement of α=0.90 set by EPFD, this case study also used a more stringent target of α=0.99.

5.3 Computation of Busy Fractions

To estimate $e_{i,\tau}^{fire}$ and $e_{i,\tau}^{rescue}$ to be used in the above Equation (5.1) and Equation (5.2), the busy fractions $q_{i,\tau}^{fire}$, $q_{i,\tau}^{rescue}$ must first be computed. Since it was relatively more complex to

obtain $ME^{v}_{i,\tau}$, and $NE^{v}_{i,\tau}$, $\forall i$, we therefore assumed that the $q^{fire}_{i,\tau}$ and $q^{rescue}_{i,\tau}$ values were the same for all the CI nodes i within each EPFD district. Furthermore, some of the CTI nodes (especially those along Interstate Freeway 10) are at the boundary of two fire districts. Therefore some of the districts that bound these CI nodes were combined. Hence the calculations of $q^{fire}_{i,\tau}$, $q^{rescue}_{i,\tau}$ were performed by districts or combined districts. The ranges of busy fractions found were $0.1150 \le q^{fire}_{i,\tau} \le 0.3260$, $0.0871 \le q^{rescue}_{i,\tau} \le 0.2470$. The computed $e^{fire}_{i,\tau}$ and $e^{rescue}_{i,\tau}$ for the different time periods of a day are listed in the Table 2.

Table 2: Minimum number of units within the service standard of a CI

(a) α=0.99

Fire District	$e^{fire}_{i,\tau}$			$e^{rescue}_{i,\tau}$		
	τ=1	τ=2	τ=3	τ=1	τ=2	τ=3
1 & 3	2	2	2	2	2	2
2 & 7	2	3	2	2	2	2
4	2	3	3	2	3	2
5	2	2	2	2	2	2
6	2	2	2	2	2	2

(b) α=0.90

Fire District	$e^{fire}_{i,\tau}$			$e^{rescue}_{i,\tau}$		
	τ=1	τ=2	τ=3	τ=1	τ=2	τ=3
1 & 3	2	2	2	2	2	2
2 & 7	2	2	2	2	2	2
4	2	2	2	2	2	2
5	2	2	2	2	2	2
6	2	2	2	2	2	2

5.4 Computation of Travel Times

Constraint (3.5) requires $NE_{i,\tau}^{v}$, the set of fire stations within the S^{v}=4.5 minutes of travel time from CI node *i*. The travel times between all the fire stations $j \in J$ and CI nodes $i \in I$ during different shifts (τ=1, 2, 3) must therefore be computed. The steps of computations were as follows. These are similar to the detail procedure presented in section

i. The CI nodes were added to the existing transportation network in the TransCAD geographic file. Links were added to connect these CI nodes and the nearest nodes in the network;

ii. The 24-hour O-D matrix was converted into three eight-hour O-D matrices with the unit expressed in vehicles per hour;

iii. The 24-hour link capacities in the network were also factored to the equivalent hourly link capacities;

iv. For each eight-hour shift (τ):

   a) Static traffic assignment (using the Frank-Wolfe algorithm) was performed in TransCAD (Caliper, 2005);

   b) The shortest paths between all the stations and CI nodes were computed, and the corresponding travel times determined;

   c) For each CI node *i*, the fire stations within 4.5 minutes of travel time were identified to form $NE_{i,\tau}^{v}$.

5.5 Mixed Integer Programming

The mixed integer programming model in Equations (3.4) and (3.5)-(3.8) was constructed with all the pre-processed data. The decision variables were x_j^{ladder}, x_j^{pumper}, x_j^{quint}, $\forall j$, and $y_{i,\tau}$, $\forall\, i,\tau$. For the objective function, it was assumed that all the CIs were of equal importance but different weights were assigned for different time periods. The historical demands in the respective periods were used to calculate the relative weights. Therefore $w_{i,1}$=0.85, $w_{i,2}$=1.40, $w_{i,3}$=1.00, $\forall i$ (the time period τ=3 was selected as the base). The model has 836 constraints. They consist of 828 service reliability constraints, 4 fleet size constraints and 34 capacity constraints. Note that fire station 31 was not considered in the model as it was under construction. The optimization model was solved by ILOG CPLEX 11.1 Interactive Optimizer (ILOG, 2007) in a standard desktop personal computer.

5.6 Results and Discussions

Table 3 presents the optimized coverage of all the 138 CIs for the three shifts (τ=1, 2, 3). Each entry in the table is the binary $y_{i,\tau}$ value. A value of $y_{i,\tau}$=1 indicates that CI node i is covered while a value of 0 means otherwise.

The optimized total coverage for the respective time periods are: 64 nodes for τ=1, 3 and 52 nodes for τ=2 (see Table 4). The time periods τ=1 and 3 have the same set of CIs covered. This is because both time periods have almost the same $e_{i,\tau}^{fire}$ and $e_{i,\tau}^{rescue}$ (see Table 2). The

afternoon shift (τ=2) has relatively lower total coverage because of the higher requirements of $e_{i,\tau}^{fire}$ in fire districts 2 & 7 and 4, and $e_{i,\tau}^{rescue}$ in fire district 4 caused by the higher frequency of competing demand during this period.

Of the CIs that are not covered within 4.5 minutes of any fire station in any of the time periods are CIs 25-28, 73, 85, 109, 111, 122-124, 132 and 137. That is, no fire station can meet the response time requirement. For these CIs to be covered (with any α), more fire stations need to be built within the 4.5 minutes travel time. As for the other CIs that are not covered in a time period, there are insufficient number of fire fighting units or rescues to meet the reliability requirement ($e_{i,\tau}^{fire}$ and $e_{i,\tau}^{rescue}$, as shown in Table 2) because of limited station capacities. For example, the only fire station within 4.5 minutes of travel time from CIs 14, 15 and 16 is fire station 4. This fire station can accommodate only one ERS unit (capacity B_4=1). Since CI nodes 14, 15 and 16 belong to fire district 4, each of them needs at least 2 fire fighting units and 2 ambulances to meet the reliability requirement. One way to cover CIs 14, 15 and 16 is to increase the capacity of fire station 4 to at least 4 units.

The optimal distribution of fire fighting units and rescues are listed in Table 1. It is observed that although EPED has 33 pumpers, only 18 are assigned to the recommended stations. This is because of the less than or equal sign in Equation (3.6). It also means that deploying more than 18 pumpers in the system will not increase the total coverage. For the coverage of CIs and because of Equation (5.1), the EPFD may swap ladders, pumpers and quints between the

assigned fire stations. However, there are other operational considerations (such as trained crews, high rise buildings etc) which are not within the scope of this thesis.

For comparison purpose, the coverage given by the current allocation of ERS units is also computed. The calculations were perform by replacing x_j^{ladder}, x_j^{pumper}, $x_j^{qu\,int}$, $\forall\, j$ in the constraints by their actual values and solve for $y_{i,\tau}$, $\forall\, i, \tau$ without optimization. The existing total coverage was found to be 35, 25 and 28 for τ=1, 2 and 3 respectively. As expected, the existing coverage in the different time periods is inversely proportional to $e_{i,\tau}^{fire}$ values in Table 2. Comparing the statistics of coverage listed in Table 4, our optimization model could improve the total coverage by 83%, 108% and 129% for the respective time periods.

Table 3: Optimal coverage at the different shifts with α=0.99

CI	τ		
i	1	2	3
1	1	0	1
2	1	1	1
3	1	0	1
4	1	0	1
5	1	1	1
6	1	1	1
7	1	1	1
8	1	0	1
9	1	0	1
10	1	0	1
11	1	0	1
12	1	0	1
13	1	0	1
14	0	0	0
15	0	0	0
16	0	0	0
17	0	0	0
18	0	0	0
19	0	0	0
20	0	0	0
21	0	0	0
22	0	0	0
23	0	0	0
24	0	0	0
25	0	0	0
26	0	0	0
27	0	0	0
28	0	0	0
29	0	0	0
30	0	0	0
31	1	1	1
32	0	0	0
33	1	1	1
34	0	0	0
35	1	1	1
36	1	1	1
37	0	0	0
38	0	0	0
39	1	1	1
40	1	1	1
41	1	1	1
42	1	1	1
43	0	0	0
44	1	1	1
45	1	1	1
46	1	1	1
47	1	1	1
48	1	1	1
49	1	1	1
50	1	1	1
51	1	1	1
52	1	1	1
53	1	1	1
54	0	0	0
55	0	0	0
56	1	1	1
57	1	1	1
58	0	0	0
59	0	1	0
60	1	1	1
61	1	1	1
62	1	1	1
63	1	1	1
64	1	0	1
65	1	0	1
66	1	1	1
67	1	1	1
68	1	1	1
69	1	1	1
70	1	1	1
71	0	0	0
72	0	0	0
73	0	0	0
74	0	0	0
75	0	0	0
76	0	0	0
77	0	0	0
78	0	0	0
79	1	1	1
80	0	0	0
81	0	0	0
82	1	1	1
83	0	0	0
84	0	0	0
85	0	0	0
86	0	0	0
87	0	0	0
88	0	0	0
89	0	0	0
90	0	0	0
91	0	0	0
92	0	0	0
93	0	0	0
94	0	0	0
95	0	0	0
96	0	0	0
97	0	0	0
98	0	0	0
99	0	0	0
100	0	0	0
101	0	0	0
102	0	0	0
103	0	0	0
104	0	0	0
105	0	0	0
106	1	1	1
107	1	1	1
108	0	0	0
109	0	0	0
110	0	0	0
111	0	0	0
112	1	1	1
113	1	1	1
114	1	1	1
115	1	1	1
116	1	1	1
117	0	0	0
118	1	0	1
119	1	1	1
120	1	1	1
121	1	1	1
122	0	0	0
123	0	0	0
124	0	0	0
125	0	0	0
126	1	1	1
127	1	1	1
128	1	1	1
129	1	1	1
130	1	1	1
131	0	0	0
132	0	0	0
133	1	1	1
134	1	0	1
135	0	0	0
136	0	0	0
137	0	0	0
138	0	0	0

The above scenarios assume a service reliability of 99%, i.e., α=0.99. The actual service reliability requirement for the EPFD is α=0.90. The $e_{i,\tau}^{fire}$ and $e_{i,\tau}^{rescue}$ values (shown in Table 2) were recalculated for α=0.90, with the same data set of service demands. It was found that the $e_{i,\tau}^{fire}$ and $e_{i,\tau}^{rescue}$ values were all equal to 2 (see Table 1). This implies that there is no distinction between the service reliability requirements of the three time periods τ and between the fire districts. However, the travel times in time periods differ to cause different $NE_{i,\tau}^{v}$ for the same CI i. For example, for CI node 59 (i=59), the travel time from fire station 27 (j=27) is 4.49 minutes for τ=2, but 4.52 minutes for τ=1. This means that fire station 27 is included in $NE_{59,2}$ but not in $NE_{59,1}^{v}$. The optimization model was run with α=0.90 for the three time periods. The existing and optimized coverage is compared in Table 4. The existing coverage with α=0.90 is smaller for τ=1 than for τ=2, 3 because of the longer travel time between the stations and the CI nodes. The maximum coverage obtained with α=0.90 is 64. As discussed earlier, this is because of the capacity constraints at the fire stations.

Table 4: Coverage of CI nodes

Scenarios	τ=1	τ=2	τ=3	Total node-shifts
α=0.99, existing coverage	35	25	28	88
α=0.99,optimized coverage	64	52	64	180
α=0.90, existing coverage	35	36	36	107
α=0.90,optimized coverage	64	64	64	192

In Table 4, the existing coverage for different time intervals under the same reliability level is different. For example, the existing coverage under reliability level α=0.90 is 35 for τ=1 and 36 for τ=2. This difference is caused by the difference travel time t_{ij} between CI and the fire station. The travel time t_{ij} for τ=1 are larger than the other two intervals because this time interval contains morning peak hour, which has greater impact on travel time than evening peak hour. Larger travel time means the fire station set $NE_{i,\tau}^{v}$ within service standard S^{v} are fewer to meet $e_{i,\tau}^{v}$. The optimized coverage for the same time intervals under different reliability level is also different. For example, the optimized coverage at the same time interval is 36 at the reliability level α=0.90 and 25 at the reliability level α=0.99.With reliability level increasing, the number of competing demands increases correspondingly. This finally poses a larger $e_{i,\tau}^{v}$.

5.7 Summary

In this chapter, using the historical emergency data provided by EPFD and transportation planning data provided by TxDOT and EP MPO, the improved mixed integer programming probabilistic covering model in terms of CIs covered has been implemented in a case study. Two scenarios with different reliability levels (0.90 and 0.95) were run. Their optimization results were compared with that of the EPFD allocations of the ERS units. For both scenarios, the coverage was significantly better than the coverage without optimization, which proves the effectiveness of this improved probabilistic covering model.

Though the case study shows a promising result for this model, improvements and refinements of this model are still possible. In the case study, all the vehicles are assumed to

occupy the same space. In reality, due to its large size, a quint may occupy more space than a pumper or a ladder. Another assumption made in the case study is that all the CIs are needed to be covered 24 hours. However, schools are not necessarily covered after school time since there will be students, so as during the summer and winter vocations. Further refinement to this model can be done to better improve this covering model.

6. FURTHER APPLICATIONS

In Chapter 5, an application of this improved probabilistic covering mixed integer programming model is introduced and the optimized results are compared with the existing coverage. In this chapter, further applications of this improved covering model are presented. The applications still use the EPFD as an illustrative example. Several applications are presented to show how this new model can be modified to cater for different scenarios. Since the service reliability adapted by the EPFD is 90%, all the applications in this chapter adapt the same service reliability of 90% (α=0.90).

6.1 Scenario 1: New Fire Station

Consider the scenario that the construction of the fire station 31 (j=31 as mentioned in Table 1) is finished and this fire station is put into service. This scenario is to find out how this new fire station affects the optimized coverage. It is a simple adjustment to the model. With this new fire station, some CIs around this station will have at least one or one more fire stations within the range of the service standard. So these CIs, which are not covered previously, may now be covered after this new fire station is put into service. Therefore the coverage is expected to improve. The basic changes to the improved optimization model are the following constraints, with the updated NE_i. For some CI nodes, the NE_i now includes the new fire stations.

$$\sum_{j \in NE_i} \left(x_j^{ladder} + x_j^{pumper} + x_j^{quint} \right) \geq e_{i,\tau}^{fire} y_{i,\tau}, \ \forall i \in I, \ \tau=1,\ldots,\mathrm{T} \tag{6.1}$$

$$\sum_{j \in NE_i} x_j^{rescue} \leq e_{i,\tau}^{rescue} y_{i,\tau}, \ \forall i \in I, \ \tau=1,\ldots,\mathrm{T} \tag{6.2}$$

A capacity of 3 is assumed for this fire station (B_{31}=3). The same procedures mentioned in Chapter 5 were followed to add this new fire station into the TransCAD road network, perform the traffic assignment, and to update the fire stations within the service range of CIs and generate the constraints for optimization.

It was found that this new fire station (j=31) is within the range of service standard of 6 CIs. Among these 6 CIs, 2 CIs (i=74, 124) had no fire stations within the range of service standard; while the other 4 CIs (i=64, 72, 108, 125) have gained one more fire stations. The constraints are rewritten. For example, for i=64, τ=1 (morning shift), the previous constraints already included fire station 15 and 16 within the service standard.

$$x_{15}^{ladder} + x_{15}^{pumper} + x_{15}^{quint} + x_{16}^{ladder} + x_{16}^{pumper} + x_{16}^{quint} - y_{64,1} \geq 0 \tag{6.3}$$

$$x_{15}^{rescue} + x_{16}^{rescue} - y_{64,1} \geq 0 \tag{6.4}$$

With the new fire stations constraints (6.3) and (6.4) have changed to:

$$x_{15}^{ladder} + x_{15}^{pumper} + x_{15}^{quint} + x_{16}^{ladder} + x_{16}^{pumper} + x_{16}^{quint} + x_{31}^{ladder} + x_{31}^{pumper} + x_{31}^{quint} - 2y_{64,1} \leq 0 \tag{6.5}$$

$$x_{15}^{rescue} + x_{16}^{rescue} + x_{31}^{rescue} - 2y_{64,1} \geq 0 \tag{6.6}$$

For those CIs previously not within the service standard of any fire stations, the previous constraints are reformulated as below. For example, for i=74, τ=1, the previous constraints were:

$$-2y_{74,1} \geq 0 \tag{6.7}$$

$$-2y_{74,1} \geq 0 \tag{6.8}$$

These two constraints were the same since there was no fire station within the range of service standard.

With the new fire stations, constraints (6.7) and (6.8) have changed to:

$$x_{31}^{ladder} + x_{31}^{pumperr} + x_{31}^{quint} - 2y_{74,1} \geq 0 \tag{6.9}$$

$$x_{31}^{rescue} - 2y_{74,1} \geq 0 \tag{6.10}$$

The optimization results show that the coverage is improved as expected. But the improvement is just slightly improved from 192 to 201 node-shifts. The coverage detail shows that with the new fire station, one previously covered CI (*i*=51) is no longer covered, but four previously uncovered CIs (*i*=72, 102, 108 and 125) become covered. Three (*i*=72, 108 and 125) are within the service standard of the new fire station. In other words, during the optimization, vehicles are re-distributed. In this case, one pumper, one quint and one rescue are allocated to this new fire station. Compared the total vehicle allocations for both scenarios, two more vehicles from each type are allocated (from 26 to 28 for fire fighting vehicles; from 21 to 23 for rescues). In the following figure, the change in coverage of CIs with and without new fire station is shown. The yellow star in the figure is the new fire station, the blue dots are CIs becoming covered for all the three shifts and the red dot is the CI becoming uncovered for all the three shifts with the new fire station.

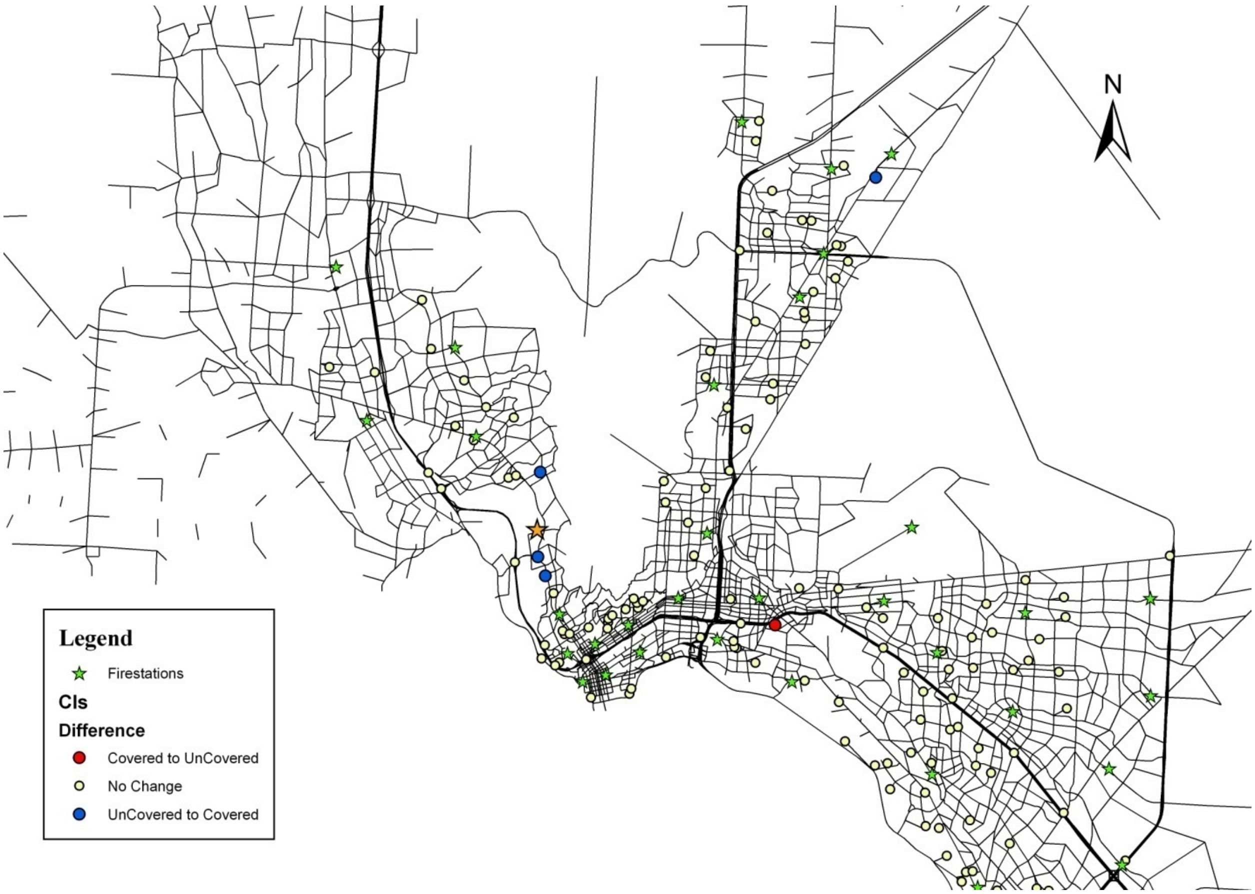

Figure 9: Coverage with and without new fire station

6.2 Scenario 2: Increase the Number of Rescues

After analyzing the optimization results and vehicle distributions in the scenarios described in Chapter 5 and Scenario 1 in this chapter, it is observed that all the rescues are assigned, but not all fire fighting vehicles. It implies that the total number of the rescues is not sufficient to match the total number of the fire fighting vehicles. Increasing the fleet size of rescue p^{rescue} will increase the optimized coverage.

In this scenario, the total number of the rescues is increased to address this mismatch. In this case, the total number of rescues p^{rescue} in the following constraint is increase once at a time.

$$\sum_{j \in NE_i} x_j^{rescue} \leq p^{rescue} \tag{6.11}$$

It is expected that the more the number of the rescues is added to the model and allocated to the fire stations, the greater the coverage improvement will be. However, it is surprising to observed, as shown in Figure 9 and 10, that after a significant improvement of the coverage from 201 to 228 node-shifts and objective value from 217.75 to 247.00 when the total number of the rescues increased from 23 to 24, there is no further improvement in both the coverage and the objective value, no matter how many more rescues are added.

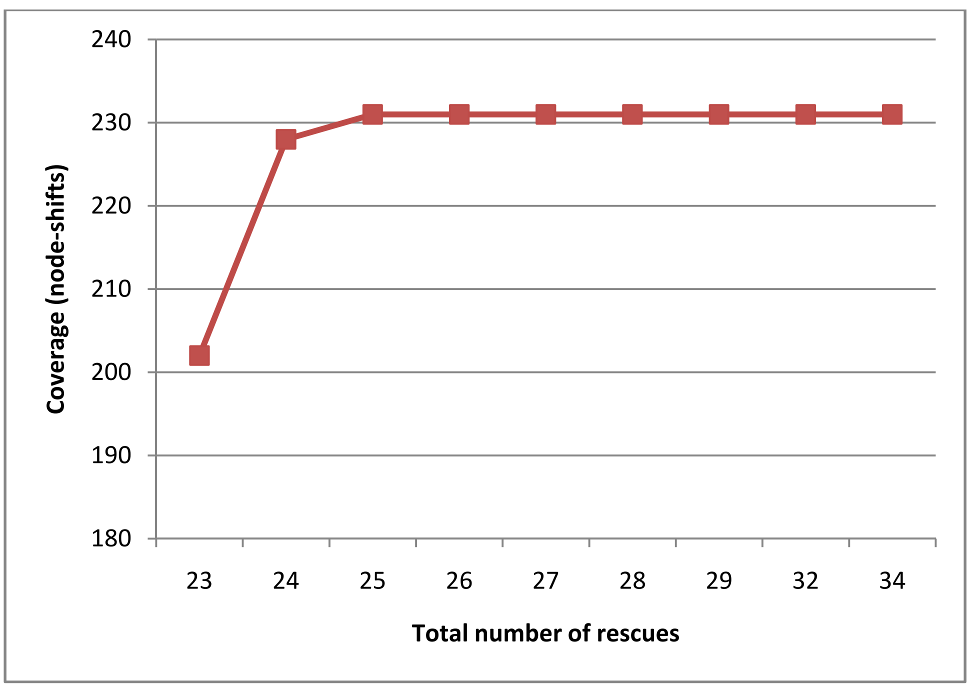

Figure 10: Coverage for Scenario 2

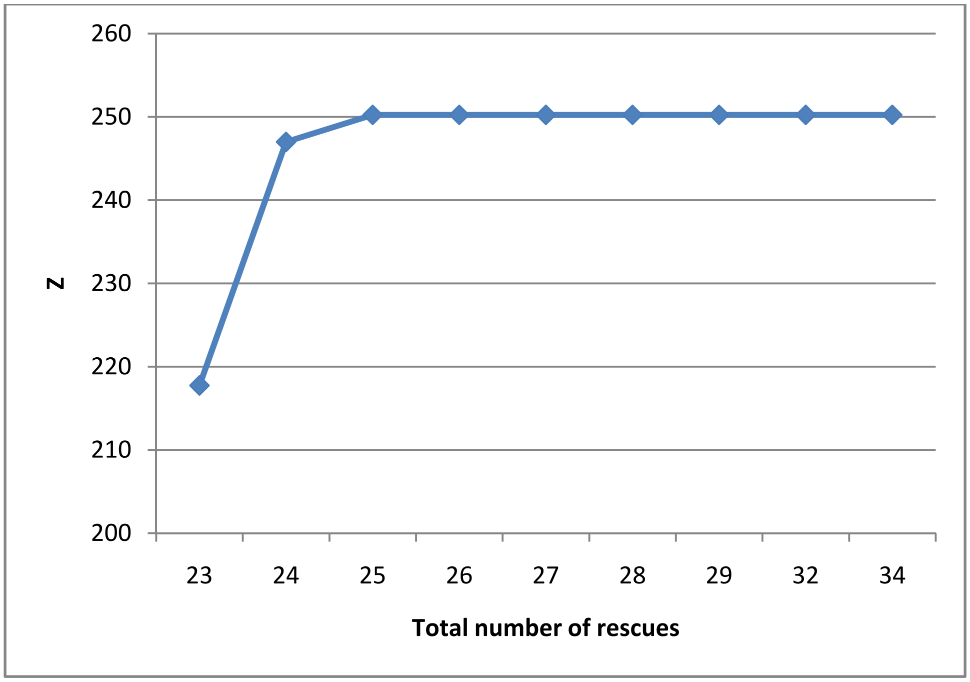

Figure 11: Objective function for Scenario 2

After a careful observation of the constraints, the reason causing this result is identified. Under the service reliability of 90%, the $e_{i,\tau}^{fire}$ and $e_{i,\tau}^{rescue}$ values are both 2 for each CIs. It implies that for a CI to be covered there must be at least two fire fighting vehicles and two rescues located at the fire stations within the service standard. The observation of the constraints shows that there are a few CIs with just one fire station in the range of the service standard. To cover these CIs, those fire stations must have a capacity of four to accommodate four vehicles simultaneously.

6.3 Scenario 3: Increase the Capacities of the Fire Stations

As the analysis of the results in Scenario 2 shows, the factors limiting the improvement of the coverage are not only the number of the ERS units (in this case the total rescues), but also the capacities of the fire stations. In this scenario, both factors are considered to investigate how the improvement will be. To overcome the capacity constraints of the fire stations, the capacities of the fire stations are expanded to four if the original capacities are less than 4. If the original capacities are greater than or equal to 4, the capacities remains unchanged. This adjustment is described in the following equations.

$$\sum x_j^v \leq \max\left\{4, B_j\right\} \qquad \forall j \in J \tag{6.12}$$

Based on the existing situation, the capacities of 26 fire stations, including the new fire station need to be increased. These fire stations are shown in the following map. Those red stars represent fire stations at which the capacities are increased and those green stars represent fire stations whose capacities remain unchanged.

Figure 12: Fire stations with a capacity less than 4

In the following figures, it is shown that after increasing the minimum capacity of each fire station to 4, the coverage is improved when the total number of the rescues is increased. There is a significant increase when the capacities of all fire stations are expanded to a minimum of 4 even if the total number of rescues remains at 23 vehicles. The reason is that all the fire station has the capacity to accommodate 2 fire fighting vehicles and 2 rescues to cover those CIs within the range of service standard. Therefore, one single fire station can aggregate enough ERS vehicles to cover as many CIs as possible. While in the previous situation, at least two fire stations are necessary to cover an area, but these two fire stations have their own covering range

and their covering range may not overlap. For instance, two fire stations, each with a capacity of two vehicles, share two fire fighting vehicles and two rescues to provide enough resources to cover an area. Fire station 1 has six and fire station 2 has eight CI nodes within the service standard. However, the overlapped coverage of these two fire stations is just four. So the maximum coverage can only be four. Right now, if both of them can accommodate enough resources, the maximum coverage becomes four. This is because all four vehicles are allocated to fire station 2 to provide this maximum coverage of four nodes. In other words, the covering efficiency of a single fire station is higher than that of multiple fire stations.

From the Figure 13, another interesting observation is the way the coverage increases. The coverage increases only when two rescues have been added to the model. For example, when the total number of rescues is increased from 23 to 24, the coverage is improved from 294 to 306. However, when the number of rescues is increased from 24 to 25, there is no improvement to the coverage. The reason is simply due to the condition of coverage defined by $e_{i,\tau}^{rescue}=2$.

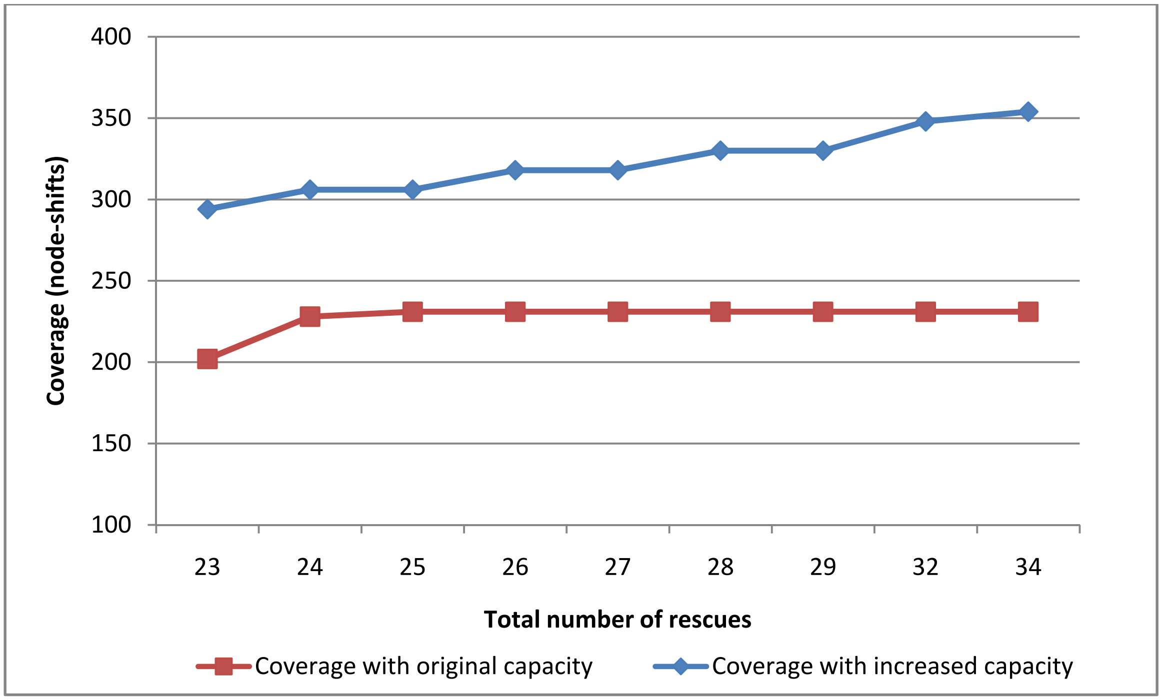

Figure 13: Comparison of coverage between Scenario 2 and Scenario 3

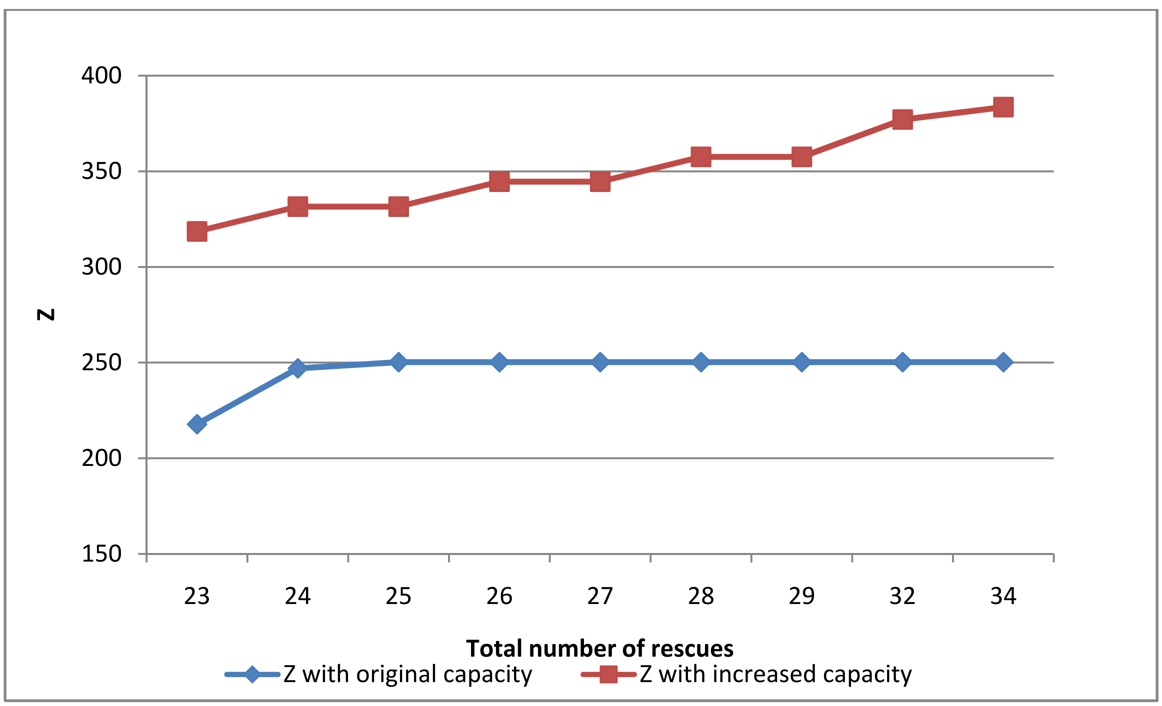

Figure 14: Comparison of objective values between Scenario 2 and Scenario 3

Figure 15 compares the coverage with and without capacity increase under the same fleet of 23 rescues. In the figure, the red dots are the CIs whose coverage for all the three shifts change from covered to uncovered, the green dots are CIs whose coverage remain and the blue dots are the CIs whose coverage change from uncovered to covered. From the map, it is observed that those CIs becoming covered after capacity increase are located in the north and downtown area of El Paso.

Figure 15: Coverage compared with/without capacity increasing ($p^{rescue} = 23$)

6.4 Other Potential Applications

In this chapter, some applications of this improved probabilistic covering mixed integer programming model have been presented. These are just some illustrative applications. There are still some other applications not discussed in detail.

For example, if the EPFD is granted a specific budget to expand its operations. How to allocate this money to achieve maximum improvement in the coverage? There are several possible alternatives. One alternative is to build one or more fire stations. Another one is to purchase more rescue or fire fighting vehicles. The third alternative is to expand the capacity of one or more specific fire stations. If the budget permits, the EPFD can even select a combination of the above alternatives. The application of the improved mixed integer programming model for such scenario will require detailed economic information about the alternatives such as: (i) the cost of a new fire station, (ii) the cost of a new vehicle including its operation and maintenance and the salaries for the new crew members (fire fighters), and (iii) the cost of expanding an existing fire station. If this information is known, the model can be modified for each alternative to find out the respective improvement in coverage, and cost-benefit analysis performed to find out the best alternative. The model can be modified for a fixed budge *M*. The objective function of maximizing coverage remains unchanged but the following constraints are added to the model:

$$\sum_{m \in NS} u_m \times C^m + \sum_{v \in V} g^v \times C^v + \sum_{j \in J} \beta_j \times C^j \le M \tag{6.13}$$

$$\sum_{v \in V} x_m^v \le B_m u_m \qquad \forall m \in NS \tag{6.14}$$

$$\sum_{j \in J+NS} x_j^v \le P^v + g^v \qquad \forall v \in V \tag{6.15}$$

$$\sum_{v \in V} x_j^v \le B_j + \beta_j \qquad \forall v \in V, \forall j \in J \tag{6.16}$$

$$u_m = \{0,1\} \tag{6.17}$$

$$g^v \ge 0 \tag{6.18}$$

$$w_j \ge 0 \tag{6.19}$$

where: NS = the set of candidate new fire stations;

u_m = 1 if the candidate fire station m is selected; otherwise 0;

B_m = the capacity for the candidate fire station m; this integer non-negative value is supposed to be known;

g^v = the number of vehicle type V; non-negative integer;

β_j = the increase in capacity for the existing fire station j; non-negative integer;

C^m = the cost to construct the candidate fire station m;

C^v = the total cost, including purchasing cost, operation and maintenance cost of vehicle type V;

C^j = the cost to increase the capacity of existing fire station j by 1 vehicle;

The new decision variables are u_m, $\forall m \in NS$; g^v, $\forall v \in V$; β_j, $\forall j \in J$. Constraint (6.13) guarantees that the total cost is not greater than the available budget. Constraint (6.14) is to guarantee that if the candidate fire station m is not selected, the capacity of that fire station is set to 0, so that no vehicles can be allocated to that fire station. Constraint (6.15) states that the total number of vehicles includes the possible new purchase. Constraint (6.16) states that the

capacity for an existing fire station may be increased. Constraint (6.17) is a binary integer constraint on decision variable. Constraints (6.18) and (6.19) are nonnegative constraints.

Another possible application is taking the resource redundancy into consideration. In the optimized results, the fire fighting vehicles and rescues are not allocated to every fire station. There exist fire stations at which no vehicles are allocated. For example, in the scenario in Chapter 5, only 18 fire stations are allocated resources and the rest of 16 fire stations are not used. How to deal with those fire stations? Note that, in the day to day operations, the EPFD still needs to allocate the remaining vehicles to these stations to respond to the routine calls from residents and businesses. The remaining fire fighting vehicles may be distributed among the remaining fire stations with this consideration. An optimization model may be formulated for this task. One possible consideration is to use those fire stations as backups to cover CIs. In this way, this model can be further extended as a probabilistic backup covering mixed integer programming model.

An additional possible application is distinguishing the weights of the CIs ($w_{i,\tau}$ in Equation (3.4)) at the different times of a day, or months of a year. For example, during the summer and winter holidays, the schools are usually closed. At these periods, the importance to cover the schools is not as great as that in the middle of a semester. However, how to decide the weights between CIs, schools and hospitals is another research.

There may be some other applications for this model. Those applications mentioned above are just to show the flexibility of this model. Also, this model is not limited to the

application to cover the critical infrastructures. It can be easily adjusted or extended to fit to coverage of other facilities.

7. CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER RESEARCH

7.1 Conclusions and Contributions

In this thesis, the research topic of optimization of ERS operations in the case of allocating emergency service vehicles to serve CIs is presented.

Based on the research work done in the field of siting ERSs, an improved probabilistic covering mixed integer programming model was implemented in this research to simultaneously allocate multiple types of emergency service vehicles: fire fighting vehicles and rescues, to cover the CIs. The proposed improved model addresses several problems encountered in reality:

1) The operation of the ERSs are divided into multiple shifts in a day and correspondingly the CIs at different shift has a different weights of importance;
2) The travel times between CIs and base stations are time dependent due to the traffic at the different times of a day;
3) The spatial and temporal distribution of competing demands.

A CI is deemed sufficiently covered only if all types of vehicles are each available within their service standards, and with specified reliability levels.

Compared with the existing models, this model has several contributions to the research of sitting emergency service. This model extends the operation of ERS departments from one shift to three shifts which are more realistic. This model also considers the effects of the

congestion on the travel time. The frequency of competing demands for the ERS resources from residents and business around the different fire stations in a region are no longer assumed uniform in this model. A more accurate calculation method of busy factors is developed to reflect the real situations.

Moreover, this model has been applied to the City of El Paso with two types of vehicles, 138 CIs and 38 fire stations. This model can also be further applied in allocating other types of ERS vehicle, or in other cities.

7.2 Recommendation for Further Research

Further research can be expanded to address the problem of allocating limited funds to achieve maximum improvement in the coverage. A possible formulation to solve this problem has already presented in Section 6.4.

Another recommendation is to utilize the remaining ERS vehicles to provide coverage for the background demands. These background demands are routine calls from residents and businesses.

In addition, to acquire better estimation of the travel times between the CIs and fire stations, other traffic simulation software, such as Paramics (Quadstone, 2008) and DYNASMART-P (Federal Highway Administration, 2008), can be used to achieve more accurate estimations. However, significantly extra efforts are needed to convert the existing

transportation planning map into the format the software requires. The simulation model must also be calibrated and validated before application.

This model can also be applied to future land-use, demographic and travel time data (e.g. El Paso 2030 model) to find out the coverage at that time so as to response to the corresponding changes.

REFERENCES

1. Berlin, G. N. and J. C. Liebman. (1971). "Mathematical Analysis of Emergency Ambulance Location." *Socio-Econ Planning Science*, 8, 323-328.

2. Bennett, V. L., D. J. Eaton and R. L. Church. (1982). "Selecting Sites for Rural Health Workers." *Social Sciences and Medicine*, 16, 63-72.

3. Bianchi, G. and R. Church. (1988). "A Hybrid FLEET Model for Emergency Medical Service System Design." *Social Sciences in Medicine*, 26, 163-171.

4. Caliper. (2006). *TransCAD Transportation GIS Software User's Guide.* Caliper Corp., Newton, MA.

5. Caliper. (2005). *Travel Demand Modeling with TransCAD 4.8*. Caliper Corp., Newton, MA.

6. Cheu, R. L., Y. Huang and B. Huang. (2007). "Allocating Emergency Service Vehicles to Serve Critical Transportation Infrastructures." *Journal of Intelligent Transportation Systems*, 12(1), 38-49.

7. Church, R. L. and C. ReVelle. (1974). "The maximal Covering Location Problem." *Papers of the Regional Science Association*, 32, 101-118.

8. Daskin, M. S. (1983). “A Maximum Expected Covering Location Model: Formulation, Properties and Heuristic Solution.” *Transportation Science*, 17, 48-69.

9. Daskin, M. S. (1995). *Network and Discrete Location: Models, Algorithms and Applications*. *New York: John Wiley & Sons, Inc*.

10. Eaton, D. J. and Daskin, M. S. (1979). “Location Techniques for Emergency Medical Service Vehicles.” *Policy Research Report 34*, Lyndon B. Johnson School of Public Affairs, University of Texas, Austin.

11. Eaton, D. J., et al. (1980). *Analysis of Emergency Medical Service in Austin, Texas.* Policy Research Report 41, Lyndon B. Johnson School of Public Affairs, University of Texas, Austin.

12. Eaton, D. J., R. Church, V. Bennett and B. Namon. (1981). “On Deployment of Health Resources in Rural Colombia.” *TIMS Studies in the Management Sciences*, 17, 331-359.

13. Eaton, D. J., M. S. Daskin, B. Bulloch and G. Jansma. (1985). “Determining Emergency Medical Service Vehicle Deployment in Austin, Texas.” *Interfaces*, 15, 96-108.

14. Federal Highway Administration. (2008). *DYNASMART-P User Manual*. U.S. Department of Transportation, U.S.A.

15. Garey, M. R. and D. S. Johnson. (1979). *Computers and Intractability: A Guide to the Theory of NP-Completeness*. W. H. Freeman and Co. New York, NY

16. Hakimi, S. L. (1964). “Optimum Locations of Switching Centers and the Absolute Centers and Medians of a Graph.” *Operations Research*, 12, 450-459.

17. Hogan, K. and C. ReVelle. (1986). “Concept and Application of Backup Coverage,.” *Management Science*, 32, 1434-1444.

18. Huang, Y, R. L. Cheu and Y. Y. Fan. (2007). “Optimal Allocation of Multiple Emergency Service Resources for Protecting Critical Transportation Infrastructure.” *Transportation Research Record – Journal of the Transportation Research Board*, No. 2022, 1-8.

19. ILOG. (2007). *ILOG CPLEX 11.1 Interactive Optimizer Reference Manual.* ILOG S.A., Mountain View, CA.

20. Jarvis, J. P., K. A. Stevenson and T. R. Willemain. (1975). *A Simple Procedure for the Allocation of Ambulances in Semi-Rural Areas*. Technical Report, Operation Research Center, Massachusetts Institute of Technology.

21. Kariv, O. and S. L. Hakimi. (1979). “An Algorithmic Approach to Network Location Problem. I: the P-centers.” *SIAM Journal on Applied Mathematics*, 37, 513-538.

22. Marianov, V. and C. ReVelle. (1991). “The Standard Response Fire Protection Siting Problem.” *Finnish Operation Research Society*, INFOR, 29, 116-129.

23. Marianov, V. and C. ReVelle. (1992). “The Capacitated Standard Response Fire Protection Siting Problem: Deterministic and Probabilistic Models.” *Annals of Operations Research*, 40, 302-322.

24. Martinich, J. S. (1988). “A Vertex-Closing Approach to the P-Center Problem.” *Naval Research Logistics*, 35, 185-201.

25. Minieka, E. (1977). “The Centers and Medians of a Graph.” *Operations Research*, 25, 641-650.

26. Miller, H. J. and S-L. Shaw. (2001). *Geographic Information System for Transportation: Principle and Application.* Oxford, UK: Oxford University Press.

27. Moore, G. C. and C. ReVelle. (1982). “The Hierarchical Service Location Problem.” *Management Science*, 28, 775-780.

28. Owen, S. H., and M. S. Daskin. (1998). “Strategic Facility Location: A Review.” *European Journal of Operational Research*, 111(3), 423-447.

29. Paramics. (2003). *Paramics Training Manual*. Quadstone, Ltd., Edinburgh, UK.

30. Repede, J. and J. Bernardo. (1981). "Developing and Validating a Decision Support System for Locating Emergency Medical Vehicles in Louisville, Kentucky." *European Journal of Operational Research*, 75, 567-581.

31. ReVelle, C. and K. Hogan. (1988). "A Reliability Constrained Siting Model with Local Estimates of Busy Fractions." *Environment and Planning B: Planning and Design*, 15, 143-152.

32. ReVelle, C. and K. Hogan. (1989). "The Maximal Covering Location Problem and α-Reliable P-Center Problem: Derivatives of the Probabilistic Location Set Covering Problem." *Annals of Operations Research*, Vol. 18, 155-174.

33. ReVelle, C. and V. Marianov. (1991). "A probabilistic FLEET model with individual vehicle reliability requirements." *European Journal of Operation Research*, Vol. 53, No.1, 93-105.

34. ReVelle, C. and S. Snyder. (1995). "Integrated Fire and Ambulance Siting: a Deterministic Model." *Socio-Econ Planning Science*, 29, 264-271.

35. Schilling, D., D. Elzinga, J. Cohon, R. Church, and C. ReVelle. (1979). "The TEAM/FLEET models for simultaneous facility and equipment siting." *Transportation Science*, 13(2), 163-175.

36. Schilling, D. A., J. Vaidyanathan and R. Barkhi. (1993). “A Review of Covering Problems in Facility Location.” *Location Science*, 1, 25-55.

37. Tavakoli, A. and C. Lightner. (2004). “Implementing a Mathematical Model for Location EMS Vehicles in Fayetteville, NC.” *Computer and Operations Research*, 31, 1549-1563.

38. Toregas, C., R. Swain, C. ReVelle, and L. Bergman. (1071). “The location of emergency service facilities.” *Operations Research*, 19(6), 1363-1373.

39. Toregas, C., and C. ReVelle. (1973). “Binary Logic solutions to a class of location problems.” *Geographical Analysis*, v5, 145-155.

40. Walker, W. E. “1974). “Using the Set-Covering Problem to Assign Fire companies to Fire Stations.” *Operations Research*, 22, 275-277.

APPENDIX

Improved Mixed Integer Programming Model for Section 5.5

Maximize
0.85 Y1M + 0.85 Y2M + 0.85 Y3M + 0.85 Y4M + 0.85 Y5M + 0.85 Y6M + 0.85 Y7M + 0.85 Y8M + 0.85 Y9M + 0.85 Y10M + 0.85 Y11M + 0.85 Y12M + 0.85 Y13M + 0.85 Y14M + 0.85 Y15M + 0.85 Y16M + 0.85 Y17M + 0.85 Y18M + 0.85 Y19M + 0.85 Y20M + 0.85 Y21M + 0.85 Y22M + 0.85 Y23M + 0.85 Y24M + 0.85 Y25M + 0.85 Y26M + 0.85 Y27M + 0.85 Y28M + 0.85 Y29M + 0.85 Y30M + 0.85 Y31M + 0.85 Y32M + 0.85 Y33M + 0.85 Y34M + 0.85 Y35M + 0.85 Y36M + 0.85 Y37M + 0.85 Y38M + 0.85 Y39M + 0.85 Y40M + 0.85 Y41M + 0.85 Y42M + 0.85 Y43M + 0.85 Y44M + 0.85 Y45M + 0.85 Y46M + 0.85 Y47M + 0.85 Y48M + 0.85 Y49M + 0.85 Y50M + 0.85 Y51M + 0.85 Y52M + 0.85 Y53M + 0.85 Y54M + 0.85 Y55M + 0.85 Y56M + 0.85 Y57M + 0.85 Y58M + 0.85 Y59M + 0.85 Y60M + 0.85 Y61M + 0.85 Y62M + 0.85 Y63M + 0.85 Y64M + 0.85 Y65M + 0.85 Y66M + 0.85 Y67M + 0.85 Y68M + 0.85 Y69M + 0.85 Y70M + 0.85 Y71M + 0.85 Y72M + 0.85 Y73M + 0.85 Y74M + 0.85 Y75M + 0.85 Y76M + 0.85 Y77M + 0.85 Y78M + 0.85 Y79M + 0.85 Y80M + 0.85 Y81M + 0.85 Y82M + 0.85 Y83M + 0.85 Y84M + 0.85 Y85M + 0.85 Y86M + 0.85 Y87M + 0.85 Y88M + 0.85 Y89M + 0.85 Y90M + 0.85 Y91M + 0.85 Y92M + 0.85 Y93M + 0.85 Y94M + 0.85 Y95M + 0.85 Y96M + 0.85 Y97M + 0.85 Y98M + 0.85 Y99M + 0.85 Y100M + 0.85 Y101M + 0.85 Y102M + 0.85 Y103M + 0.85 Y104M + 0.85 Y105M + 0.85 Y106M + 0.85 Y107M + 0.85 Y108M + 0.85 Y109M + 0.85 Y110M + 0.85 Y111M + 0.85 Y112M + 0.85 Y113M + 0.85 Y114M + 0.85 Y115M + 0.85 Y116M + 0.85 Y117M + 0.85 Y118M + 0.85 Y119M + 0.85 Y120M + 0.85 Y121M + 0.85 Y122M + 0.85 Y123M + 0.85 Y124M + 0.85 Y125M + 0.85 Y126M + 0.85 Y127M + 0.85 Y128M + 0.85 Y129M + 0.85 Y130M + 0.85 Y131M + 0.85 Y132M + 0.85 Y133M + 0.85 Y134M + 0.85 Y135M + 0.85 Y136M + 0.85 Y137M + 0.85 Y138M + 1.4 Y1A + 1.4 Y2A + 1.4 Y3A + 1.4 Y4A + 1.4 Y5A + 1.4 Y6A + 1.4 Y7A + 1.4 Y8A + 1.4 Y9A + 1.4 Y10A + 1.4 Y11A + 1.4 Y12A + 1.4 Y13A + 1.4 Y14A + 1.4 Y15A + 1.4 Y16A + 1.4 Y17A + 1.4 Y18A + 1.4 Y19A + 1.4 Y20A + 1.4 Y21A + 1.4 Y22A + 1.4 Y23A + 1.4 Y24A + 1.4 Y25A + 1.4 Y26A + 1.4 Y27A + 1.4 Y28A + 1.4 Y29A + 1.4 Y30A + 1.4 Y31A + 1.4 Y32A + 1.4 Y33A + 1.4 Y34A + 1.4 Y35A + 1.4 Y36A + 1.4 Y37A + 1.4 Y38A + 1.4 Y39A + 1.4 Y40A + 1.4 Y41A + 1.4 Y42A + 1.4 Y43A + 1.4 Y44A + 1.4 Y45A + 1.4 Y46A + 1.4 Y47A + 1.4 Y48A + 1.4 Y49A + 1.4 Y50A + 1.4 Y51A + 1.4 Y52A + 1.4 Y53A + 1.4 Y54A + 1.4 Y55A + 1.4 Y56A + 1.4 Y57A + 1.4 Y58A + 1.4 Y59A + 1.4 Y60A + 1.4 Y61A + 1.4 Y62A + 1.4 Y63A + 1.4 Y64A + 1.4 Y65A + 1.4 Y66A + 1.4 Y67A + 1.4 Y68A + 1.4 Y69A + 1.4 Y70A + 1.4 Y71A + 1.4 Y72A + 1.4 Y73A + 1.4 Y74A + 1.4 Y75A + 1.4 Y76A + 1.4 Y77A + 1.4 Y78A + 1.4 Y79A + 1.4 Y80A + 1.4 Y81A + 1.4 Y82A + 1.4 Y83A + 1.4 Y84A + 1.4 Y85A + 1.4 Y86A + 1.4 Y87A + 1.4 Y88A + 1.4 Y89A + 1.4 Y90A + 1.4 Y91A + 1.4 Y92A + 1.4 Y93A + 1.4 Y94A + 1.4 Y95A + 1.4 Y96A + 1.4 Y97A + 1.4 Y98A + 1.4 Y99A + 1.4 Y100A + 1.4 Y101A + 1.4 Y102A + 1.4 Y103A + 1.4 Y104A + 1.4 Y105A + 1.4 Y106A + 1.4 Y107A + 1.4 Y108A + 1.4 Y109A + 1.4 Y110A + 1.4 Y111A + 1.4 Y112A + 1.4 Y113A + 1.4 Y114A + 1.4 Y115A + 1.4 Y116A + 1.4 Y117A + 1.4 Y118A + 1.4 Y119A + 1.4 Y120A + 1.4 Y121A + 1.4 Y122A + 1.4 Y123A + 1.4 Y124A + 1.4 Y125A + 1.4 Y126A + 1.4 Y127A + 1.4 Y128A + 1.4 Y129A + 1.4 Y130A + 1.4 Y131A + 1.4 Y132A + 1.4 Y133A + 1.4 Y134A + 1.4 Y135A + 1.4 Y136A + 1.4 Y137A + 1.4 Y138A + 1 Y1N + 1 Y2N + 1 Y3N + 1 Y4N + 1 Y5N + 1 Y6N + 1 Y7N + 1 Y8N + 1 Y9N + 1 Y10N + 1 Y11N + 1 Y12N + 1 Y13N + 1

Y14N + 1 Y15N + 1 Y16N + 1 Y17N + 1 Y18N + 1 Y19N + 1 Y20N + 1 Y21N + 1 Y22N + 1 Y23N + 1 Y24N + 1 Y25N + 1 Y26N + 1 Y27N + 1 Y28N + 1 Y29N + 1 Y30N + 1 Y31N + 1 Y32N + 1 Y33N + 1 Y34N + 1 Y35N + 1 Y36N + 1 Y37N + 1 Y38N + 1 Y39N + 1 Y40N + 1 Y41N + 1 Y42N + 1 Y43N + 1 Y44N + 1 Y45N + 1 Y46N + 1 Y47N + 1 Y48N + 1 Y49N + 1 Y50N + 1 Y51N + 1 Y52N + 1 Y53N + 1 Y54N + 1 Y55N + 1 Y56N + 1 Y57N + 1 Y58N + 1 Y59N + 1 Y60N + 1 Y61N + 1 Y62N + 1 Y63N + 1 Y64N + 1 Y65N + 1 Y66N + 1 Y67N + 1 Y68N + 1 Y69N + 1 Y70N + 1 Y71N + 1 Y72N + 1 Y73N + 1 Y74N + 1 Y75N + 1 Y76N + 1 Y77N + 1 Y78N + 1 Y79N + 1 Y80N + 1 Y81N + 1 Y82N + 1 Y83N + 1 Y84N + 1 Y85N + 1 Y86N + 1 Y87N + 1 Y88N + 1 Y89N + 1 Y90N + 1 Y91N + 1 Y92N + 1 Y93N + 1 Y94N + 1 Y95N + 1 Y96N + 1 Y97N + 1 Y98N + 1 Y99N + 1 Y100N + 1 Y101N + 1 Y102N + 1 Y103N + 1 Y104N + 1 Y105N + 1 Y106N + 1 Y107N + 1 Y108N + 1 Y109N + 1 Y110N + 1 Y111N + 1 Y112N + 1 Y113N + 1 Y114N + 1 Y115N + 1 Y116N + 1 Y117N + 1 Y118N + 1 Y119N + 1 Y120N + 1 Y121N + 1 Y122N + 1 Y123N + 1 Y124N + 1 Y125N + 1 Y126N + 1 Y127N + 1 Y128N + 1 Y129N + 1 Y130N + 1 Y131N + 1 Y132N + 1 Y133N + 1 Y134N + 1 Y135N + 1 Y136N + 1 Y137N + 1 Y138N

subject to

X2L + X2P + X2Q - 2 Y1M >= 0
X1L + X1P + X1Q + X2L + X2P + X2Q - 2 Y2M >= 0
X2L + X2P + X2Q - 2 Y3M >= 0
X2L + X2P + X2Q - 2 Y4M >= 0
X1L + X1P + X1Q + X33L + X33P + X33Q - 2 Y5M >= 0
X1L + X1P + X1Q - 2 Y6M >= 0
X1L + X1P + X1Q - 2 Y7M >= 0
X2L + X2P + X2Q - 2 Y8M >= 0
X2L + X2P + X2Q - 2 Y9M >= 0
X2L + X2P + X2Q - 2 Y10M >= 0
X2L + X2P + X2Q - 2 Y11M >= 0
X2L + X2P + X2Q - 2 Y12M >= 0
X2L + X2P + X2Q + X4L + X4P + X4Q - 2 Y13M >= 0
X4L + X4P + X4Q - 2 Y14M >= 0
X4L + X4P + X4Q - 2 Y15M >= 0
X4L + X4P + X4Q - 2 Y16M >= 0
X4L + X4P + X4Q - 2 Y17M >= 0
X29L + X29P + X29Q - 2 Y18M >= 0
X4L + X4P + X4Q - 2 Y19M >= 0
X4L + X4P + X4Q - 2 Y20M >= 0
X4L + X4P + X4Q - 2 Y21M >= 0
X4L + X4P + X4Q - 2 Y22M >= 0
X4L + X4P + X4Q - 2 Y23M >= 0
X29L + X29P + X29Q - 2 Y24M >= 0
 - 2 Y25M >= 0
 - 2 Y26M >= 0
 - 2 Y27M >= 0
 - 2 Y28M >= 0
X29L + X29P + X29Q - 2 Y29M >= 0
X34L + X34P + X34Q - 2 Y30M >= 0

X18L + X18P + X18Q + X19L + X19P + X19Q + X20L + X20P + X20Q - 2 Y31M >= 0
X27L + X27P + X27Q - 2 Y32M >= 0
X18L + X18P + X18Q + X19L + X19P + X19Q + X20L + X20P + X20Q + X21L + X21P + X21Q - 2 Y33M >= 0
X27L + X27P + X27Q - 2 Y34M >= 0
X28L + X28P + X28Q - 2 Y35M >= 0
X28L + X28P + X28Q + X29L + X29P + X29Q - 2 Y36M >= 0
X27L + X27P + X27Q - 2 Y37M >= 0
X27L + X27P + X27Q - 2 Y38M >= 0
X23L + X23P + X23Q + X25L + X25P + X25Q + X34L + X34P + X34Q - 2 Y39M >= 0
X15L + X15P + X15Q + X16L + X16P + X16Q + X17L + X17P + X17Q + X18L + X18P + X18Q - 2 Y40M >= 0
X28L + X28P + X28Q - 2 Y41M >= 0
X19L + X19P + X19Q + X20L + X20P + X20Q + X23L + X23P + X23Q - 2 Y42M >= 0
X27L + X27P + X27Q - 2 Y43M >= 0
X23L + X23P + X23Q + X25L + X25P + X25Q + X34L + X34P + X34Q - 2 Y44M >= 0
X23L + X23P + X23Q + X25L + X25P + X25Q + X34L + X34P + X34Q - 2 Y45M >= 0
X28L + X28P + X28Q - 2 Y46M >= 0
X27L + X27P + X27Q + X28L + X28P + X28Q - 2 Y47M >= 0
X20L + X20P + X20Q + X22L + X22P + X22Q + X23L + X23P + X23Q - 2 Y48M >= 0
X27L + X27P + X27Q + X28L + X28P + X28Q - 2 Y49M >= 0
X15L + X15P + X15Q + X16L + X16P + X16Q + X17L + X17P + X17Q + X19L + X19P + X19Q + X21L + X21P + X21Q - 2 Y50M >= 0
X25L + X25P + X25Q + X34L + X34P + X34Q - 2 Y51M >= 0
X15L + X15P + X15Q + X17L + X17P + X17Q + X19L + X19P + X19Q + X21L + X21P + X21Q - 2 Y52M >= 0
X15L + X15P + X15Q + X17L + X17P + X17Q + X21L + X21P + X21Q - 2 Y53M >= 0
X27L + X27P + X27Q - 2 Y54M >= 0
X26L + X26P + X26Q - 2 Y55M >= 0
X28L + X28P + X28Q - 2 Y56M >= 0
X17L + X17P + X17Q + X21L + X21P + X21Q + X22L + X22P + X22Q - 2 Y57M >= 0
X26L + X26P + X26Q - 2 Y58M >= 0
X28L + X28P + X28Q - 2 Y59M >= 0
X17L + X17P + X17Q + X21L + X21P + X21Q + X22L + X22P + X22Q - 2 Y60M >= 0
X17L + X17P + X17Q + X21L + X21P + X21Q + X22L + X22P + X22Q - 2 Y61M >= 0
X22L + X22P + X22Q + X23L + X23P + X23Q + X25L + X25P + X25Q - 2 Y62M >= 0
X28L + X28P + X28Q - 2 Y63M >= 0
X15L + X15P + X15Q + X16L + X16P + X16Q - 2 Y64M >= 0
X25L + X25P + X25Q - 2 Y65M >= 0
X28L + X28P + X28Q - 2 Y66M >= 0
X22L + X22P + X22Q + X24L + X24P + X24Q - 2 Y67M >= 0
X24L + X24P + X24Q - 2 Y68M >= 0
X24L + X24P + X24Q - 2 Y69M >= 0
X24L + X24P + X24Q - 2 Y70M >= 0
X8L + X8P + X8Q - 2 Y71M >= 0
X8L + X8P + X8Q - 2 Y72M >= 0
- 2 Y73M >= 0
X8L + X8P + X8Q - 2 Y74M >= 0
X14L + X14P + X14Q - 2 Y75M >= 0
X8L + X8P + X8Q - 2 Y76M >= 0

X8L + X8P + X8Q - 2 Y77M >= 0
X14L + X14P + X14Q - 2 Y78M >= 0
X6L + X6P + X6Q + X8L + X8P + X8Q - 2 Y79M >= 0
X14L + X14P + X14Q - 2 Y80M >= 0
X14L + X14P + X14Q - 2 Y81M >= 0
X6L + X6P + X6Q + X8L + X8P + X8Q - 2 Y82M >= 0
X14L + X14P + X14Q - 2 Y83M >= 0
X7L + X7P + X7Q - 2 Y84M >= 0
 - 2 Y85M >= 0
X14L + X14P + X14Q - 2 Y86M >= 0
X6L + X6P + X6Q - 2 Y87M >= 0
X13L + X13P + X13Q - 2 Y88M >= 0
X13L + X13P + X13Q + X14L + X14P + X14Q - 2 Y89M >= 0
X12L + X12P + X12Q + X13L + X13P + X13Q - 2 Y90M >= 0
X12L + X12P + X12Q + X13L + X13P + X13Q - 2 Y91M >= 0
X6L + X6P + X6Q - 2 Y92M >= 0
X12L + X12P + X12Q + X13L + X13P + X13Q - 2 Y93M >= 0
X12L + X12P + X12Q + X13L + X13P + X13Q - 2 Y94M >= 0
X12L + X12P + X12Q + X13L + X13P + X13Q - 2 Y95M >= 0
X12L + X12P + X12Q + X13L + X13P + X13Q - 2 Y96M >= 0
X12L + X12P + X12Q + X13L + X13P + X13Q - 2 Y97M >= 0
X12L + X12P + X12Q + X13L + X13P + X13Q - 2 Y98M >= 0
X10L + X10P + X10Q + X12L + X12P + X12Q - 2 Y99M >= 0
X10L + X10P + X10Q + X12L + X12P + X12Q - 2 Y100M >= 0
X10L + X10P + X10Q - 2 Y101M >= 0
X11L + X11P + X11Q - 2 Y102M >= 0
X10L + X10P + X10Q - 2 Y103M >= 0
X9L + X9P + X9Q - 2 Y104M >= 0
X9L + X9P + X9Q - 2 Y105M >= 0
X15L + X15P + X15Q + X16L + X16P + X16Q + X17L + X17P + X17Q + X18L + X18P + X18Q - 2 Y106M >= 0
X15L + X15P + X15Q + X16L + X16P + X16Q + X17L + X17P + X17Q - 2 Y107M >= 0
X15L + X15P + X15Q - 2 Y108M >= 0
 - 2 Y109M >= 0
X27L + X27P + X27Q - 2 Y110M >= 0
 - 2 Y111M >= 0
X17L + X17P + X17Q + X21L + X21P + X21Q + X22L + X22P + X22Q - 2 Y112M >= 0
X15L + X15P + X15Q + X17L + X17P + X17Q + X21L + X21P + X21Q - 2 Y113M >= 0
X17L + X17P + X17Q + X21L + X21P + X21Q - 2 Y114M >= 0
X15L + X15P + X15Q + X16L + X16P + X16Q + X17L + X17P + X17Q + X18L + X18P + X18Q + X19L + X19P + X19Q + X21L + X21P + X21Q - 2 Y115M >= 0
X23L + X23P + X23Q + X25L + X25P + X25Q + X34L + X34P + X34Q - 2 Y116M >= 0
X34L + X34P + X34Q - 2 Y117M >= 0
X2L + X2P + X2Q - 2 Y118M >= 0
X16L + X16P + X16Q + X18L + X18P + X18Q - 2 Y119M >= 0
X15L + X15P + X15Q + X16L + X16P + X16Q + X17L + X17P + X17Q + X18L + X18P + X18Q - 2 Y120M >= 0
X15L + X15P + X15Q + X16L + X16P + X16Q + X17L + X17P + X17Q + X18L + X18P + X18Q + X19L + X19P + X19Q - 2 Y121M >= 0
 - 2 Y122M >= 0

- 2 Y123M >= 0
- 2 Y124M >= 0
X15L + X15P + X15Q - 2 Y125M >= 0
X18L + X18P + X18Q + X19L + X19P + X19Q - 2 Y126M >= 0
X16L + X16P + X16Q + X17L + X17P + X17Q + X18L + X18P + X18Q + X19L + X19P + X19Q - 2 Y127M >= 0
X15L + X15P + X15Q + X16L + X16P + X16Q + X17L + X17P + X17Q + X19L + X19P + X19Q + X21L + X21P + X21Q - 2 Y128M >= 0
X23L + X23P + X23Q + X25L + X25P + X25Q - 2 Y129M >= 0
X24L + X24P + X24Q - 2 Y130M >= 0
X12L + X12P + X12Q - 2 Y131M >= 0
- 2 Y132M >= 0
X1L + X1P + X1Q + X33L + X33P + X33Q - 2 Y133M >= 0
X26L + X26P + X26Q + X27L + X27P + X27Q - 2 Y134M >= 0
X26L + X26P + X26Q - 2 Y135M >= 0
X29L + X29P + X29Q - 2 Y136M >= 0
- 1 Y137M >= 0
X32L + X32P + X32Q - 2 Y138M >= 0

X2L + X2P + X2Q - 2 Y1A >= 0
X1L + X1P + X1Q + X2L + X2P + X2Q - 2 Y2A >= 0
X2L + X2P + X2Q - 2 Y3A >= 0
X2L + X2P + X2Q - 2 Y4A >= 0
X1L + X1P + X1Q + X33L + X33P + X33Q - 2 Y5A >= 0
X1L + X1P + X1Q - 2 Y6A >= 0
X1L + X1P + X1Q - 2 Y7A >= 0
X2L + X2P + X2Q - 2 Y8A >= 0
X2L + X2P + X2Q - 2 Y9A >= 0
X2L + X2P + X2Q - 2 Y10A >= 0
X2L + X2P + X2Q - 2 Y11A >= 0
X2L + X2P + X2Q - 2 Y12A >= 0
X2L + X2P + X2Q + X4L + X4P + X4Q - 2 Y13A >= 0
X4L + X4P + X4Q - 2 Y14A >= 0
X4L + X4P + X4Q - 2 Y15A >= 0
X4L + X4P + X4Q - 2 Y16A >= 0
X4L + X4P + X4Q - 2 Y17A >= 0
X29L + X29P + X29Q - 2 Y18A >= 0
X4L + X4P + X4Q - 2 Y19A >= 0
X4L + X4P + X4Q - 2 Y20A >= 0
X4L + X4P + X4Q - 2 Y21A >= 0
X4L + X4P + X4Q - 2 Y22A >= 0
X4L + X4P + X4Q - 2 Y23A >= 0
X29L + X29P + X29Q - 2 Y24A >= 0
- 2 Y25A >= 0
- 2 Y26A >= 0
- 2 Y27A >= 0
- 2 Y28A >= 0
X29L + X29P + X29Q - 2 Y29A >= 0
X34L + X34P + X34Q - 2 Y30A >= 0
X18L + X18P + X18Q + X19L + X19P + X19Q + X20L + X20P + X20Q - 2 Y31A >= 0

X27L + X27P + X27Q - 2 Y32A >= 0
X18L + X18P + X18Q + X19L + X19P + X19Q + X20L + X20P + X20Q + X21L + X21P + X21Q - 2 Y33A >= 0
X27L + X27P + X27Q - 2 Y34A >= 0
X28L + X28P + X28Q - 2 Y35A >= 0
X28L + X28P + X28Q + X29L + X29P + X29Q - 2 Y36A >= 0
X27L + X27P + X27Q - 2 Y37A >= 0
X27L + X27P + X27Q - 2 Y38A >= 0
X23L + X23P + X23Q + X25L + X25P + X25Q + X34L + X34P + X34Q - 2 Y39A >= 0
X15L + X15P + X15Q + X16L + X16P + X16Q + X17L + X17P + X17Q + X18L + X18P + X18Q - 2 Y40A >= 0
X28L + X28P + X28Q - 2 Y41A >= 0
X19L + X19P + X19Q + X20L + X20P + X20Q + X23L + X23P + X23Q - 2 Y42A >= 0
X27L + X27P + X27Q - 2 Y43A >= 0
X23L + X23P + X23Q + X25L + X25P + X25Q + X34L + X34P + X34Q - 2 Y44A >= 0
X23L + X23P + X23Q + X25L + X25P + X25Q + X34L + X34P + X34Q - 2 Y45A >= 0
X28L + X28P + X28Q - 2 Y46A >= 0
X27L + X27P + X27Q + X28L + X28P + X28Q - 2 Y47A >= 0
X20L + X20P + X20Q + X22L + X22P + X22Q + X23L + X23P + X23Q - 2 Y48A >= 0
X27L + X27P + X27Q + X28L + X28P + X28Q - 2 Y49A >= 0
X15L + X15P + X15Q + X16L + X16P + X16Q + X17L + X17P + X17Q + X19L + X19P + X19Q + X21L + X21P + X21Q - 2 Y50A >= 0
X25L + X25P + X25Q + X34L + X34P + X34Q - 2 Y51A >= 0
X15L + X15P + X15Q + X17L + X17P + X17Q + X19L + X19P + X19Q + X21L + X21P + X21Q - 2 Y52A >= 0
X15L + X15P + X15Q + X17L + X17P + X17Q + X21L + X21P + X21Q - 2 Y53A >= 0
X27L + X27P + X27Q - 2 Y54A >= 0
X26L + X26P + X26Q - 2 Y55A >= 0
X28L + X28P + X28Q - 2 Y56A >= 0
X17L + X17P + X17Q + X21L + X21P + X21Q + X22L + X22P + X22Q - 2 Y57A >= 0
X26L + X26P + X26Q - 2 Y58A >= 0
X27L + X27P + X27Q + X28L + X28P + X28Q - 2 Y59A >= 0
X17L + X17P + X17Q + X21L + X21P + X21Q + X22L + X22P + X22Q - 2 Y60A >= 0
X17L + X17P + X17Q + X21L + X21P + X21Q + X22L + X22P + X22Q - 2 Y61A >= 0
X22L + X22P + X22Q + X23L + X23P + X23Q + X25L + X25P + X25Q - 2 Y62A >= 0
X28L + X28P + X28Q - 2 Y63A >= 0
X15L + X15P + X15Q + X16L + X16P + X16Q - 2 Y64A >= 0
X25L + X25P + X25Q - 2 Y65A >= 0
X28L + X28P + X28Q - 2 Y66A >= 0
X22L + X22P + X22Q + X24L + X24P + X24Q - 2 Y67A >= 0
X24L + X24P + X24Q - 2 Y68A >= 0
X24L + X24P + X24Q - 2 Y69A >= 0
X24L + X24P + X24Q - 2 Y70A >= 0
X8L + X8P + X8Q - 2 Y71A >= 0
X8L + X8P + X8Q - 2 Y72A >= 0
- 2 Y73A >= 0
X8L + X8P + X8Q - 2 Y74A >= 0
X14L + X14P + X14Q - 2 Y75A >= 0
X8L + X8P + X8Q - 2 Y76A >= 0
X8L + X8P + X8Q - 2 Y77A >= 0

X14L + X14P + X14Q - 2 Y78A >= 0
X6L + X6P + X6Q + X8L + X8P + X8Q - 2 Y79A >= 0
X14L + X14P + X14Q - 2 Y80A >= 0
X14L + X14P + X14Q - 2 Y81A >= 0
X6L + X6P + X6Q + X8L + X8P + X8Q - 2 Y82A >= 0
X14L + X14P + X14Q - 2 Y83A >= 0
X7L + X7P + X7Q - 2 Y84A >= 0
- 2 Y85A >= 0
X14L + X14P + X14Q - 2 Y86A >= 0
X6L + X6P + X6Q - 2 Y87A >= 0
X13L + X13P + X13Q - 2 Y88A >= 0
X13L + X13P + X13Q + X14L + X14P + X14Q - 2 Y89A >= 0
X12L + X12P + X12Q + X13L + X13P + X13Q - 2 Y90A >= 0
X12L + X12P + X12Q + X13L + X13P + X13Q - 2 Y91A >= 0
X6L + X6P + X6Q - 2 Y92A >= 0
X12L + X12P + X12Q + X13L + X13P + X13Q - 2 Y93A >= 0
X12L + X12P + X12Q + X13L + X13P + X13Q - 2 Y94A >= 0
X12L + X12P + X12Q + X13L + X13P + X13Q - 2 Y95A >= 0
X12L + X12P + X12Q + X13L + X13P + X13Q - 2 Y96A >= 0
X12L + X12P + X12Q + X13L + X13P + X13Q - 2 Y97A >= 0
X12L + X12P + X12Q + X13L + X13P + X13Q - 2 Y98A >= 0
X10L + X10P + X10Q + X12L + X12P + X12Q - 2 Y99A >= 0
X10L + X10P + X10Q + X12L + X12P + X12Q - 2 Y100A >= 0
X10L + X10P + X10Q - 2 Y101A >= 0
X11L + X11P + X11Q - 2 Y102A >= 0
X10L + X10P + X10Q - 2 Y103A >= 0
X9L + X9P + X9Q - 2 Y104A >= 0
X9L + X9P + X9Q - 2 Y105A >= 0
X15L + X15P + X15Q + X16L + X16P + X16Q + X17L + X17P + X17Q + X18L + X18P + X18Q - 2 Y106A >= 0
X15L + X15P + X15Q + X16L + X16P + X16Q + X17L + X17P + X17Q - 2 Y107A >= 0
X15L + X15P + X15Q - 2 Y108A >= 0
- 2 Y109A >= 0
X27L + X27P + X27Q - 2 Y110A >= 0
- 2 Y111A >= 0
X17L + X17P + X17Q + X21L + X21P + X21Q + X22L + X22P + X22Q - 2 Y112A >= 0
X15L + X15P + X15Q + X17L + X17P + X17Q + X21L + X21P + X21Q - 2 Y113A >= 0
X17L + X17P + X17Q + X21L + X21P + X21Q - 2 Y114A >= 0
X15L + X15P + X15Q + X16L + X16P + X16Q + X17L + X17P + X17Q + X18L + X18P + X18Q + X19L + X19P + X19Q + X21L + X21P + X21Q - 2 Y115A >= 0
X23L + X23P + X23Q + X25L + X25P + X25Q + X34L + X34P + X34Q - 2 Y116A >= 0
X34L + X34P + X34Q - 2 Y117A >= 0
X2L + X2P + X2Q - 2 Y118A >= 0
X16L + X16P + X16Q + X18L + X18P + X18Q - 2 Y119A >= 0
X15L + X15P + X15Q + X16L + X16P + X16Q + X17L + X17P + X17Q + X18L + X18P + X18Q - 2 Y120A >= 0
X15L + X15P + X15Q + X16L + X16P + X16Q + X17L + X17P + X17Q + X18L + X18P + X18Q + X19L + X19P + X19Q - 2 Y121A >= 0
- 2 Y122A >= 0
- 2 Y123A >= 0

- 2 Y124A >= 0
X15L + X15P + X15Q - 2 Y125A >= 0
X18L + X18P + X18Q + X19L + X19P + X19Q - 2 Y126A >= 0
X16L + X16P + X16Q + X17L + X17P + X17Q + X18L + X18P + X18Q + X19L + X19P + X19Q - 2 Y127A >= 0
X15L + X15P + X15Q + X16L + X16P + X16Q + X17L + X17P + X17Q + X19L + X19P + X19Q + X21L + X21P + X21Q - 2 Y128A >= 0
X23L + X23P + X23Q + X25L + X25P + X25Q - 2 Y129A >= 0
X24L + X24P + X24Q - 2 Y130A >= 0
X12L + X12P + X12Q - 2 Y131A >= 0
- 2 Y132A >= 0
X1L + X1P + X1Q + X33L + X33P + X33Q - 2 Y133A >= 0
X26L + X26P + X26Q + X27L + X27P + X27Q - 2 Y134A >= 0
X26L + X26P + X26Q - 2 Y135A >= 0
X29L + X29P + X29Q - 2 Y136A >= 0
- 2 Y137A >= 0
X32L + X32P + X32Q - 2 Y138A >= 0

X2L + X2P + X2Q - 2 Y1N >= 0
X1L + X1P + X1Q + X2L + X2P + X2Q - 2 Y2N >= 0
X2L + X2P + X2Q - 2 Y3N >= 0
X2L + X2P + X2Q - 2 Y4N >= 0
X1L + X1P + X1Q + X33L + X33P + X33Q - 2 Y5N >= 0
X1L + X1P + X1Q - 2 Y6N >= 0
X1L + X1P + X1Q - 2 Y7N >= 0
X2L + X2P + X2Q - 2 Y8N >= 0
X2L + X2P + X2Q - 2 Y9N >= 0
X2L + X2P + X2Q - 2 Y10N >= 0
X2L + X2P + X2Q - 2 Y11N >= 0
X2L + X2P + X2Q - 2 Y12N >= 0
X2L + X2P + X2Q + X4L + X4P + X4Q - 2 Y13N >= 0
X4L + X4P + X4Q - 2 Y14N >= 0
X4L + X4P + X4Q - 2 Y15N >= 0
X4L + X4P + X4Q - 2 Y16N >= 0
X4L + X4P + X4Q - 2 Y17N >= 0
X29L + X29P + X29Q - 2 Y18N >= 0
X4L + X4P + X4Q - 2 Y19N >= 0
X4L + X4P + X4Q - 2 Y20N >= 0
X4L + X4P + X4Q - 2 Y21N >= 0
X4L + X4P + X4Q - 2 Y22N >= 0
X4L + X4P + X4Q - 2 Y23N >= 0
X29L + X29P + X29Q - 2 Y24N >= 0
- 2 Y25N >= 0
- 2 Y26N >= 0
- 2 Y27N >= 0
- 2 Y28N >= 0
X29L + X29P + X29Q - 2 Y29N >= 0
X34L + X34P + X34Q - 2 Y30N >= 0
X18L + X18P + X18Q + X19L + X19P + X19Q + X20L + X20P + X20Q - 2 Y31N >= 0
X27L + X27P + X27Q - 2 Y32N >= 0

X18L + X18P + X18Q + X19L + X19P + X19Q + X20L + X20P + X20Q + X21L + X21P + X21Q - 2 Y33N >= 0
X27L + X27P + X27Q - 2 Y34N >= 0
X28L + X28P + X28Q - 2 Y35N >= 0
X28L + X28P + X28Q + X29L + X29P + X29Q - 2 Y36N >= 0
X27L + X27P + X27Q - 2 Y37N >= 0
X27L + X27P + X27Q - 2 Y38N >= 0
X23L + X23P + X23Q + X25L + X25P + X25Q + X34L + X34P + X34Q - 2 Y39N >= 0
X15L + X15P + X15Q + X16L + X16P + X16Q + X17L + X17P + X17Q + X18L + X18P + X18Q - 2 Y40N >= 0
X28L + X28P + X28Q - 2 Y41N >= 0
X19L + X19P + X19Q + X20L + X20P + X20Q + X23L + X23P + X23Q - 2 Y42N >= 0
X27L + X27P + X27Q - 2 Y43N >= 0
X23L + X23P + X23Q + X25L + X25P + X25Q + X34L + X34P + X34Q - 2 Y44N >= 0
X23L + X23P + X23Q + X25L + X25P + X25Q + X34L + X34P + X34Q - 2 Y45N >= 0
X28L + X28P + X28Q - 2 Y46N >= 0
X27L + X27P + X27Q + X28L + X28P + X28Q - 2 Y47N >= 0
X20L + X20P + X20Q + X22L + X22P + X22Q + X23L + X23P + X23Q - 2 Y48N >= 0
X27L + X27P + X27Q + X28L + X28P + X28Q - 2 Y49N >= 0
X15L + X15P + X15Q + X16L + X16P + X16Q + X17L + X17P + X17Q + X19L + X19P + X19Q + X21L + X21P + X21Q - 2 Y50N >= 0
X25L + X25P + X25Q + X34L + X34P + X34Q - 2 Y51N >= 0
X15L + X15P + X15Q + X17L + X17P + X17Q + X19L + X19P + X19Q + X21L + X21P + X21Q - 2 Y52N >= 0
X15L + X15P + X15Q + X17L + X17P + X17Q + X21L + X21P + X21Q - 2 Y53N >= 0
X27L + X27P + X27Q - 2 Y54N >= 0
X26L + X26P + X26Q - 2 Y55N >= 0
X28L + X28P + X28Q - 2 Y56N >= 0
X17L + X17P + X17Q + X21L + X21P + X21Q + X22L + X22P + X22Q - 2 Y57N >= 0
X26L + X26P + X26Q - 2 Y58N >= 0
X27L + X27P + X27Q + X28L + X28P + X28Q - 2 Y59N >= 0
X17L + X17P + X17Q + X21L + X21P + X21Q + X22L + X22P + X22Q - 2 Y60N >= 0
X17L + X17P + X17Q + X21L + X21P + X21Q + X22L + X22P + X22Q - 2 Y61N >= 0
X22L + X22P + X22Q + X23L + X23P + X23Q + X25L + X25P + X25Q - 2 Y62N >= 0
X28L + X28P + X28Q - 2 Y63N >= 0
X15L + X15P + X15Q + X16L + X16P + X16Q - 2 Y64N >= 0
X25L + X25P + X25Q - 2 Y65N >= 0
X28L + X28P + X28Q - 2 Y66N >= 0
X22L + X22P + X22Q + X24L + X24P + X24Q - 2 Y67N >= 0
X24L + X24P + X24Q - 2 Y68N >= 0
X24L + X24P + X24Q - 2 Y69N >= 0
X24L + X24P + X24Q - 2 Y70N >= 0
X8L + X8P + X8Q - 2 Y71N >= 0
X8L + X8P + X8Q - 2 Y72N >= 0
- 2 Y73N >= 0
X8L + X8P + X8Q - 2 Y74N >= 0
X14L + X14P + X14Q - 2 Y75N >= 0
X8L + X8P + X8Q - 2 Y76N >= 0
X8L + X8P + X8Q - 2 Y77N >= 0
X14L + X14P + X14Q - 2 Y78N >= 0

X6L + X6P + X6Q + X8L + X8P + X8Q - 2 Y79N >= 0
X14L + X14P + X14Q - 2 Y80N >= 0
X14L + X14P + X14Q - 2 Y81N >= 0
X6L + X6P + X6Q + X8L + X8P + X8Q - 2 Y82N >= 0
X14L + X14P + X14Q - 2 Y83N >= 0
X7L + X7P + X7Q - 2 Y84N >= 0
- 2 Y85N >= 0
X14L + X14P + X14Q - 2 Y86N >= 0
X6L + X6P + X6Q - 2 Y87N >= 0
X13L + X13P + X13Q - 2 Y88N >= 0
X13L + X13P + X13Q + X14L + X14P + X14Q - 2 Y89N >= 0
X12L + X12P + X12Q + X13L + X13P + X13Q - 2 Y90N >= 0
X12L + X12P + X12Q + X13L + X13P + X13Q - 2 Y91N >= 0
X6L + X6P + X6Q - 2 Y92N >= 0
X12L + X12P + X12Q + X13L + X13P + X13Q - 2 Y93N >= 0
X12L + X12P + X12Q + X13L + X13P + X13Q - 2 Y94N >= 0
X12L + X12P + X12Q + X13L + X13P + X13Q - 2 Y95N >= 0
X12L + X12P + X12Q + X13L + X13P + X13Q - 2 Y96N >= 0
X12L + X12P + X12Q + X13L + X13P + X13Q - 2 Y97N >= 0
X12L + X12P + X12Q + X13L + X13P + X13Q - 2 Y98N >= 0
X10L + X10P + X10Q + X12L + X12P + X12Q - 2 Y99N >= 0
X10L + X10P + X10Q + X12L + X12P + X12Q - 2 Y100N >= 0
X10L + X10P + X10Q - 2 Y101N >= 0
X11L + X11P + X11Q - 2 Y102N >= 0
X10L + X10P + X10Q - 2 Y103N >= 0
X9L + X9P + X9Q - 2 Y104N >= 0
X9L + X9P + X9Q - 2 Y105N >= 0
X15L + X15P + X15Q + X16L + X16P + X16Q + X17L + X17P + X17Q + X18L + X18P + X18Q - 2 Y106N >= 0
X15L + X15P + X15Q + X16L + X16P + X16Q + X17L + X17P + X17Q - 2 Y107N >= 0
X15L + X15P + X15Q - 2 Y108N >= 0
- 2 Y109N >= 0
X27L + X27P + X27Q - 2 Y110N >= 0
- 2 Y111N >= 0
X17L + X17P + X17Q + X21L + X21P + X21Q + X22L + X22P + X22Q - 2 Y112N >= 0
X15L + X15P + X15Q + X17L + X17P + X17Q + X21L + X21P + X21Q - 2 Y113N >= 0
X17L + X17P + X17Q + X21L + X21P + X21Q - 2 Y114N >= 0
X15L + X15P + X15Q + X16L + X16P + X16Q + X17L + X17P + X17Q + X18L + X18P + X18Q + X19L + X19P + X19Q + X21L + X21P + X21Q - 2 Y115N >= 0
X23L + X23P + X23Q + X25L + X25P + X25Q + X34L + X34P + X34Q - 2 Y116N >= 0
X34L + X34P + X34Q - 2 Y117N >= 0
X2L + X2P + X2Q - 2 Y118N >= 0
X16L + X16P + X16Q + X18L + X18P + X18Q - 2 Y119N >= 0
X15L + X15P + X15Q + X16L + X16P + X16Q + X17L + X17P + X17Q + X18L + X18P + X18Q - 2 Y120N >= 0
X15L + X15P + X15Q + X16L + X16P + X16Q + X17L + X17P + X17Q + X18L + X18P + X18Q + X19L + X19P + X19Q - 2 Y121N >= 0
- 2 Y122N >= 0
- 2 Y123N >= 0
- 2 Y124N >= 0

X15L + X15P + X15Q - 2 Y125N >= 0
X18L + X18P + X18Q + X19L + X19P + X19Q - 2 Y126N >= 0
X16L + X16P + X16Q + X17L + X17P + X17Q + X18L + X18P + X18Q + X19L + X19P + X19Q - 2 Y127N >= 0
X15L + X15P + X15Q + X16L + X16P + X16Q + X17L + X17P + X17Q + X19L + X19P + X19Q + X21L + X21P + X21Q - 2 Y128N >= 0
X23L + X23P + X23Q + X25L + X25P + X25Q - 2 Y129N >= 0
X24L + X24P + X24Q - 2 Y130N >= 0
X12L + X12P + X12Q - 2 Y131N >= 0
- 2 Y132N >= 0
X1L + X1P + X1Q + X33L + X33P + X33Q - 2 Y133N >= 0
X26L + X26P + X26Q + X27L + X27P + X27Q - 2 Y134N >= 0
X26L + X26P + X26Q - 2 Y135N >= 0
X29L + X29P + X29Q - 2 Y136N >= 0
- 1 Y137N >= 0
X32L + X32P + X32Q - 2 Y138N >= 0

X2R - 2 Y1M >= 0
X1R + X2R - 2 Y2M >= 0
X2R - 2 Y3M >= 0
X2R - 2 Y4M >= 0
X1R + X33R - 2 Y5M >= 0
X1R - 2 Y6M >= 0
X1R - 2 Y7M >= 0
X2R - 2 Y8M >= 0
X2R - 2 Y9M >= 0
X2R - 2 Y10M >= 0
X2R - 2 Y11M >= 0
X2R - 2 Y12M >= 0
X2R + X4R - 2 Y13M >= 0
X4R - 2 Y14M >= 0
X4R - 2 Y15M >= 0
X4R - 2 Y16M >= 0
X4R - 2 Y17M >= 0
X29R - 2 Y18M >= 0
X4R - 2 Y19M >= 0
X4R - 2 Y20M >= 0
X4R - 2 Y21M >= 0
X4R - 2 Y22M >= 0
X4R - 2 Y23M >= 0
X29R - 2 Y24M >= 0
- 2 Y25M >= 0
- 2 Y26M >= 0
- 2 Y27M >= 0
- 2 Y28M >= 0
X29R - 2 Y29M >= 0
X34R - 2 Y30M >= 0
X18R + X19R + X20R - 2 Y31M >= 0
X27R - 2 Y32M >= 0
X18R + X19R + X20R + X21R - 2 Y33M >= 0

X27R - 2 Y34M >= 0
X28R - 2 Y35M >= 0
X28R + X29R - 2 Y36M >= 0
X27R - 2 Y37M >= 0
X27R - 2 Y38M >= 0
X23R + X25R + X34R - 2 Y39M >= 0
X15R + X16R + X17R + X18R - 2 Y40M >= 0
X28R - 2 Y41M >= 0
X19R + X20R + X23R - 2 Y42M >= 0
X27R - 2 Y43M >= 0
X23R + X25R + X34R - 2 Y44M >= 0
X23R + X25R + X34R - 2 Y45M >= 0
X28R - 2 Y46M >= 0
X27R + X28R - 2 Y47M >= 0
X20R + X22R + X23R - 2 Y48M >= 0
X27R + X28R - 2 Y49M >= 0
X15R + X16R + X17R + X19R + X21R - 2 Y50M >= 0
X25R + X34R - 2 Y51M >= 0
X15R + X17R + X19R + X21R - 2 Y52M >= 0
X15R + X17R + X21R - 2 Y53M >= 0
X27R - 2 Y54M >= 0
X26R - 2 Y55M >= 0
X28R - 2 Y56M >= 0
X17R + X21R + X22R - 2 Y57M >= 0
X26R - 2 Y58M >= 0
X28R - 2 Y59M >= 0
X17R + X21R + X22R - 2 Y60M >= 0
X17R + X21R + X22R - 2 Y61M >= 0
X22R + X23R + X25R - 2 Y62M >= 0
X28R - 2 Y63M >= 0
X15R + X16R - 2 Y64M >= 0
X25R - 2 Y65M >= 0
X28R - 2 Y66M >= 0
X22R + X24R - 2 Y67M >= 0
X24R - 2 Y68M >= 0
X24R - 2 Y69M >= 0
X24R - 2 Y70M >= 0
X8R - 2 Y71M >= 0
X8R - 2 Y72M >= 0
- 2 Y73M >= 0
X8R - 2 Y74M >= 0
X14R - 2 Y75M >= 0
X8R - 2 Y76M >= 0
X8R - 2 Y77M >= 0
X14R - 2 Y78M >= 0
X6R + X8R - 2 Y79M >= 0
X14R - 2 Y80M >= 0
X14R - 2 Y81M >= 0
X6R + X8R - 2 Y82M >= 0
X14R - 2 Y83M >= 0

X7R - 2 Y84M >= 0
- 2 Y85M >= 0
X14R - 2 Y86M >= 0
X6R - 2 Y87M >= 0
X13R - 2 Y88M >= 0
X13R + X14R - 2 Y89M >= 0
X12R + X13R - 2 Y90M >= 0
X12R + X13R - 2 Y91M >= 0
X6R - 2 Y92M >= 0
X12R + X13R - 2 Y93M >= 0
X12R + X13R - 2 Y94M >= 0
X12R + X13R - 2 Y95M >= 0
X12R + X13R - 2 Y96M >= 0
X12R + X13R - 2 Y97M >= 0
X12R + X13R - 2 Y98M >= 0
X10R + X12R - 2 Y99M >= 0
X10R + X12R - 2 Y100M >= 0
X10R - 2 Y101M >= 0
X11R - 2 Y102M >= 0
X10R - 2 Y103M >= 0
X9R - 2 Y104M >= 0
X9R - 2 Y105M >= 0
X15R + X16R + X17R + X18R - 2 Y106M >= 0
X15R + X16R + X17R - 2 Y107M >= 0
X15R - 2 Y108M >= 0
- 2 Y109M >= 0
X27R - 2 Y110M >= 0
- 2 Y111M >= 0
X17R + X21R + X22R - 2 Y112M >= 0
X15R + X17R + X21R - 2 Y113M >= 0
X17R + X21R - 2 Y114M >= 0
X15R + X16R + X17R + X18R + X19R + X21R - 2 Y115M >= 0
X23R + X25R + X34R - 2 Y116M >= 0
X34R - 2 Y117M >= 0
X2R - 2 Y118M >= 0
X16R + X18R - 2 Y119M >= 0
X15R + X16R + X17R + X18R - 2 Y120M >= 0
X15R + X16R + X17R + X18R + X19R - 2 Y121M >= 0
- 2 Y122M >= 0
- 2 Y123M >= 0
- 2 Y124M >= 0
X15R - 2 Y125M >= 0
X18R + X19R - 2 Y126M >= 0
X16R + X17R + X18R + X19R - 2 Y127M >= 0
X15R + X16R + X17R + X19R + X21R - 2 Y128M >= 0
X23R + X25R - 2 Y129M >= 0
X24R - 2 Y130M >= 0
X12R - 2 Y131M >= 0
- 2 Y132M >= 0
X1R + X33R - 2 Y133M >= 0

X26R + X27R - 2 Y134M >= 0
X26R - 2 Y135M >= 0
X29R - 2 Y136M >= 0
- 2 Y137M >= 0
X32R - 2 Y138M >= 0

X2R - 2 Y1A >= 0
X1R + X2R - 2 Y2A >= 0
X2R - 2 Y3A >= 0
X2R - 2 Y4A >= 0
X1R + X33R - 2 Y5A >= 0
X1R - 2 Y6A >= 0
X1R - 2 Y7A >= 0
X2R - 2 Y8A >= 0
X2R - 2 Y9A >= 0
X2R - 2 Y10A >= 0
X2R - 2 Y11A >= 0
X2R - 2 Y12A >= 0
X2R + X4R - 2 Y13A >= 0
X4R - 2 Y14A >= 0
X4R - 2 Y15A >= 0
X4R - 2 Y16A >= 0
X4R - 2 Y17A >= 0
X29R - 2 Y18A >= 0
X4R - 2 Y19A >= 0
X4R - 2 Y20A >= 0
X4R - 2 Y21A >= 0
X4R - 2 Y22A >= 0
X4R - 2 Y23A >= 0
X29R - 2 Y24A >= 0
- 2 Y25A >= 0
- 2 Y26A >= 0
- 2 Y27A >= 0
- 2 Y28A >= 0
X29R - 2 Y29A >= 0
X34R - 2 Y30A >= 0
X18R + X19R + X20R - 2 Y31A >= 0
X27R - 2 Y32A >= 0
X18R + X19R + X20R + X21R - 2 Y33A >= 0
X27R - 2 Y34A >= 0
X28R - 2 Y35A >= 0
X28R + X29R - 2 Y36A >= 0
X27R - 2 Y37A >= 0
X27R - 2 Y38A >= 0
X23R + X25R + X34R - 2 Y39A >= 0
X15R + X16R + X17R + X18R - 2 Y40A >= 0
X28R - 2 Y41A >= 0
X19R + X20R + X23R - 2 Y42A >= 0
X27R - 2 Y43A >= 0
X23R + X25R + X34R - 2 Y44A >= 0

X23R + X25R + X34R - 2 Y45A >= 0
X28R - 2 Y46A >= 0
X27R + X28R - 2 Y47A >= 0
X20R + X22R + X23R - 2 Y48A >= 0
X27R + X28R - 2 Y49A >= 0
X15R + X16R + X17R + X19R + X21R - 2 Y50A >= 0
X25R + X34R - 2 Y51A >= 0
X15R + X17R + X19R + X21R - 2 Y52A >= 0
X15R + X17R + X21R - 2 Y53A >= 0
X27R - 2 Y54A >= 0
X26R - 2 Y55A >= 0
X28R - 2 Y56A >= 0
X17R + X21R + X22R - 2 Y57A >= 0
X26R - 2 Y58A >= 0
X27R + X28R - 2 Y59A >= 0
X17R + X21R + X22R - 2 Y60A >= 0
X17R + X21R + X22R - 2 Y61A >= 0
X22R + X23R + X25R - 2 Y62A >= 0
X28R - 2 Y63A >= 0
X15R + X16R - 2 Y64A >= 0
X25R - 2 Y65A >= 0
X28R - 2 Y66A >= 0
X22R + X24R - 2 Y67A >= 0
X24R - 2 Y68A >= 0
X24R - 2 Y69A >= 0
X24R - 2 Y70A >= 0
X8R - 2 Y71A >= 0
X8R - 2 Y72A >= 0
- 2 Y73A >= 0
X8R - 2 Y74A >= 0
X14R - 2 Y75A >= 0
X8R - 2 Y76A >= 0
X8R - 2 Y77A >= 0
X14R - 2 Y78A >= 0
X6R + X8R - 2 Y79A >= 0
X14R - 2 Y80A >= 0
X14R - 2 Y81A >= 0
X6R + X8R - 2 Y82A >= 0
X14R - 2 Y83A >= 0
X7R - 2 Y84A >= 0
- 2 Y85A >= 0
X14R - 2 Y86A >= 0
X6R - 2 Y87A >= 0
X13R - 2 Y88A >= 0
X13R + X14R - 2 Y89A >= 0
X12R + X13R - 2 Y90A >= 0
X12R + X13R - 2 Y91A >= 0
X6R - 2 Y92A >= 0
X12R + X13R - 2 Y93A >= 0
X12R + X13R - 2 Y94A >= 0

X12R + X13R - 2 Y95A >= 0
X12R + X13R - 2 Y96A >= 0
X12R + X13R - 2 Y97A >= 0
X12R + X13R - 2 Y98A >= 0
X10R + X12R - 2 Y99A >= 0
X10R + X12R - 2 Y100A >= 0
X10R - 2 Y101A >= 0
X11R - 2 Y102A >= 0
X10R - 2 Y103A >= 0
X9R - 2 Y104A >= 0
X9R - 2 Y105A >= 0
X15R + X16R + X17R + X18R - 2 Y106A >= 0
X15R + X16R + X17R - 2 Y107A >= 0
X15R - 2 Y108A >= 0
- 2 Y109A >= 0
X27R - 2 Y110A >= 0
- 2 Y111A >= 0
X17R + X21R + X22R - 2 Y112A >= 0
X15R + X17R + X21R - 2 Y113A >= 0
X17R + X21R - 2 Y114A >= 0
X15R + X16R + X17R + X18R + X19R + X21R - 2 Y115A >= 0
X23R + X25R + X34R - 2 Y116A >= 0
X34R - 2 Y117A >= 0
X2R - 2 Y118A >= 0
X16R + X18R - 2 Y119A >= 0
X15R + X16R + X17R + X18R - 2 Y120A >= 0
X15R + X16R + X17R + X18R + X19R - 2 Y121A >= 0
- 2 Y122A >= 0
- 2 Y123A >= 0
- 2 Y124A >= 0
X15R - 2 Y125A >= 0
X18R + X19R - 2 Y126A >= 0
X16R + X17R + X18R + X19R - 2 Y127A >= 0
X15R + X16R + X17R + X19R + X21R - 2 Y128A >= 0
X23R + X25R - 2 Y129A >= 0
X24R - 2 Y130A >= 0
X12R - 2 Y131A >= 0
- 2 Y132A >= 0
X1R + X33R - 2 Y133A >= 0
X26R + X27R - 2 Y134A >= 0
X26R - 2 Y135A >= 0
X29R - 2 Y136A >= 0
- 2 Y137A >= 0
X32R - 2 Y138A >= 0

X2R - 2 Y1N >= 0
X1R + X2R - 2 Y2N >= 0
X2R - 2 Y3N >= 0
X2R - 2 Y4N >= 0
X1R + X33R - 2 Y5N >= 0

X1R - 2 Y6N >= 0
X1R - 2 Y7N >= 0
X2R - 2 Y8N >= 0
X2R - 2 Y9N >= 0
X2R - 2 Y10N >= 0
X2R - 2 Y11N >= 0
X2R - 2 Y12N >= 0
X2R + X4R - 2 Y13N >= 0
X4R - 2 Y14N >= 0
X4R - 2 Y15N >= 0
X4R - 2 Y16N >= 0
X4R - 2 Y17N >= 0
X29R - 2 Y18N >= 0
X4R - 2 Y19N >= 0
X4R - 2 Y20N >= 0
X4R - 2 Y21N >= 0
X4R - 2 Y22N >= 0
X4R - 2 Y23N >= 0
X29R - 2 Y24N >= 0
- 2 Y25N >= 0
- 2 Y26N >= 0
- 2 Y27N >= 0
- 2 Y28N >= 0
X29R - 2 Y29N >= 0
X34R - 2 Y30N >= 0
X18R + X19R + X20R - 2 Y31N >= 0
X27R - 2 Y32N >= 0
X18R + X19R + X20R + X21R - 2 Y33N >= 0
X27R - 2 Y34N >= 0
X28R - 2 Y35N >= 0
X28R + X29R - 2 Y36N >= 0
X27R - 2 Y37N >= 0
X27R - 2 Y38N >= 0
X23R + X25R + X34R - 2 Y39N >= 0
X15R + X16R + X17R + X18R - 2 Y40N >= 0
X28R - 2 Y41N >= 0
X19R + X20R + X23R - 2 Y42N >= 0
X27R - 2 Y43N >= 0
X23R + X25R + X34R - 2 Y44N >= 0
X23R + X25R + X34R - 2 Y45N >= 0
X28R - 2 Y46N >= 0
X27R + X28R - 2 Y47N >= 0
X20R + X22R + X23R - 2 Y48N >= 0
X27R + X28R - 2 Y49N >= 0
X15R + X16R + X17R + X19R + X21R - 2 Y50N >= 0
X25R + X34R - 2 Y51N >= 0
X15R + X17R + X19R + X21R - 2 Y52N >= 0
X15R + X17R + X21R - 2 Y53N >= 0
X27R - 2 Y54N >= 0
X26R - 2 Y55N >= 0

X28R - 2 Y56N >= 0
X17R + X21R + X22R - 2 Y57N >= 0
X26R - 2 Y58N >= 0
X27R + X28R - 2 Y59N >= 0
X17R + X21R + X22R - 2 Y60N >= 0
X17R + X21R + X22R - 2 Y61N >= 0
X22R + X23R + X25R - 2 Y62N >= 0
X28R - 2 Y63N >= 0
X15R + X16R - 2 Y64N >= 0
X25R - 2 Y65N >= 0
X28R - 2 Y66N >= 0
X22R + X24R - 2 Y67N >= 0
X24R - 2 Y68N >= 0
X24R - 2 Y69N >= 0
X24R - 2 Y70N >= 0
X8R - 2 Y71N >= 0
X8R - 2 Y72N >= 0
- 2 Y73N >= 0
X8R - 2 Y74N >= 0
X14R - 2 Y75N >= 0
X8R - 2 Y76N >= 0
X8R - 2 Y77N >= 0
X14R - 2 Y78N >= 0
X6R + X8R - 2 Y79N >= 0
X14R - 2 Y80N >= 0
X14R - 2 Y81N >= 0
X6R + X8R - 2 Y82N >= 0
X14R - 2 Y83N >= 0
X7R - 2 Y84N >= 0
- 2 Y85N >= 0
X14R - 2 Y86N >= 0
X6R - 2 Y87N >= 0
X13R - 2 Y88N >= 0
X13R + X14R - 2 Y89N >= 0
X12R + X13R - 2 Y90N >= 0
X12R + X13R - 2 Y91N >= 0
X6R - 2 Y92N >= 0
X12R + X13R - 2 Y93N >= 0
X12R + X13R - 2 Y94N >= 0
X12R + X13R - 2 Y95N >= 0
X12R + X13R - 2 Y96N >= 0
X12R + X13R - 2 Y97N >= 0
X12R + X13R - 2 Y98N >= 0
X10R + X12R - 2 Y99N >= 0
X10R + X12R - 2 Y100N >= 0
X10R - 2 Y101N >= 0
X11R - 2 Y102N >= 0
X10R - 2 Y103N >= 0
X9R - 2 Y104N >= 0
X9R - 2 Y105N >= 0

X15R + X16R + X17R + X18R - 2 Y106N >= 0
X15R + X16R + X17R - 2 Y107N >= 0
X15R - 2 Y108N >= 0
- 2 Y109N >= 0
X27R - 2 Y110N >= 0
- 2 Y111N >= 0
X17R + X21R + X22R - 2 Y112N >= 0
X15R + X17R + X21R - 2 Y113N >= 0
X17R + X21R - 2 Y114N >= 0
X15R + X16R + X17R + X18R + X19R + X21R - 2 Y115N >= 0
X23R + X25R + X34R - 2 Y116N >= 0
X34R - 2 Y117N >= 0
X2R - 2 Y118N >= 0
X16R + X18R - 2 Y119N >= 0
X15R + X16R + X17R + X18R - 2 Y120N >= 0
X15R + X16R + X17R + X18R + X19R - 2 Y121N >= 0
- 2 Y122N >= 0
- 2 Y123N >= 0
- 2 Y124N >= 0
X15R - 2 Y125N >= 0
X18R + X19R - 2 Y126N >= 0
X16R + X17R + X18R + X19R - 2 Y127N >= 0
X15R + X16R + X17R + X19R + X21R - 2 Y128N >= 0
X23R + X25R - 2 Y129N >= 0
X24R - 2 Y130N >= 0
X12R - 2 Y131N >= 0
- 2 Y132N >= 0
X1R + X33R - 2 Y133N >= 0
X26R + X27R - 2 Y134N >= 0
X26R - 2 Y135N >= 0
X29R - 2 Y136N >= 0
- 2 Y137N >= 0
X32R - 2 Y138N >= 0

X1L + X2L + X3L + X4L + X5L + X6L + X7L + X8L + X9L + X10L + X11L + X12L + X13L + X14L + X15L + X16L + X17L + X18L + X19L + X20L + X21L + X22L + X23L + X24L + X25L + X26L + X27L + X28L + X29L + X30L + X31L + X32L + X33L + X34L <= 4
X1P + X2P + X3P + X4P + X5P + X6P + X7P + X8P + X9P + X10P + X11P + X12P + X13P + X14P + X15P + X16P + X17P + X18P + X19P + X20P + X21P + X22P + X23P + X24P + X25P + X26P + X27P + X28P + X29P + X30P + X31P + X32P + X33P + X34P <= 33
X1Q + X2Q + X3Q + X4Q + X5Q + X6Q + X7Q + X8Q + X9Q + X10Q + X11Q + X12Q + X13Q + X14Q + X15Q + X16Q + X17Q + X18Q + X19Q + X20Q + X21Q + X22Q + X23Q + X24Q + X25Q + X26Q + X27Q + X28Q + X29Q + X30Q + X31Q + X32Q + X33Q + X34Q <= 8
X1R + X2R + X3R + X4R + X5R + X6R + X7R + X8R + X9R + X10R + X11R + X12R + X13R + X14R + X15R + X16R + X17R + X18R + X19R + X20R + X21R + X22R + X23R + X24R + X25R + X26R + X27R + X28R + X29R + X30R + X31R + X32R + X33R + X34R <= 23

X1L + X1P + X1Q + X1R <= 10

X2L + X2P + X2Q + X2R <= 4

X3L + X3P + X3Q + X3R <= 4

X4L + X4P + X4Q + X4R <= 1

X5L + X5P + X5Q + X5R <= 5

X6L + X6P + X6Q + X6R <= 3

X7L + X7P + X7Q + X7R <= 3

X8L + X8P + X8Q + X8R <= 1

X9L + X9P + X9Q + X9R <= 2

X10L + X10P + X10Q + X10R <= 1
X11L + X11P + X11Q + X11R <= 5

X12L + X12P + X12Q + X12R <= 2

X13L + X13P + X13Q + X13R <= 1

X14L + X14P + X14Q + X14R <= 2

X15L + X15P + X15Q + X15R <= 3

X16L + X16P + X16Q + X16R <= 3

X17L + X17P + X17Q + X17R <= 3

X18L + X18P + X18Q + X18R <= 4

X19L + X19P + X19Q + X19R <= 2

X20L + X20P + X20Q + X20R <= 3

X21L + X21P + X21Q + X21R <= 2

X22L + X22P + X22Q + X22R <= 4

X23L + X23P + X23Q + X23R <= 3

X24L + X24P + X24Q + X24R <= 5

X25L + X25P + X25Q + X25R <= 3

X26L + X26P + X26Q + X26R <= 3

X27L + X27P + X27Q + X27R <= 3

X28L + X28P + X28Q + X28R <= 4

X29L + X29P + X29Q + X29R <= 3

X30L + X30P + X30Q + X30R <= 3

X31L + X31P + X31Q + X31R <= 2

X32L + X32P + X32Q + X32R <= 3

X33L + X33P + X33Q + X33R <= 3

X34L + X34P + X34Q + X34R <= 2

Bounds
Binaries

Y1M	Y50M	Y99M	Y10A	Y59A	Y108A	Y19N	Y68N	Y117N
Y2M	Y51M	Y100M	Y11A	Y60A	Y109A	Y20N	Y69N	Y118N
Y3M	Y52M	Y101M	Y12A	Y61A	Y110A	Y21N	Y70N	Y119N
Y4M	Y53M	Y102M	Y13A	Y62A	Y111A	Y22N	Y71N	Y120N
Y5M	Y54M	Y103M	Y14A	Y63A	Y112A	Y23N	Y72N	Y121N
Y6M	Y55M	Y104M	Y15A	Y64A	Y113A	Y24N	Y73N	Y122N
Y7M	Y56M	Y105M	Y16A	Y65A	Y114A	Y25N	Y74N	Y123N
Y8M	Y57M	Y106M	Y17A	Y66A	Y115A	Y26N	Y75N	Y124N
Y9M	Y58M	Y107M	Y18A	Y67A	Y116A	Y27N	Y76N	Y125N
Y10M	Y59M	Y108M	Y19A	Y68A	Y117A	Y28N	Y77N	Y126N
Y11M	Y60M	Y109M	Y20A	Y69A	Y118A	Y29N	Y78N	Y127N
Y12M	Y61M	Y110M	Y21A	Y70A	Y119A	Y30N	Y79N	Y128N
Y13M	Y62M	Y111M	Y22A	Y71A	Y120A	Y31N	Y80N	Y129N
Y14M	Y63M	Y112M	Y23A	Y72A	Y121A	Y32N	Y81N	Y130N
Y15M	Y64M	Y113M	Y24A	Y73A	Y122A	Y33N	Y82N	Y131N
Y16M	Y65M	Y114M	Y25A	Y74A	Y123A	Y34N	Y83N	Y132N
Y17M	Y66M	Y115M	Y26A	Y75A	Y124A	Y35N	Y84N	Y133N
Y18M	Y67M	Y116M	Y27A	Y76A	Y125A	Y36N	Y85N	Y134N
Y19M	Y68M	Y117M	Y28A	Y77A	Y126A	Y37N	Y86N	Y135N
Y20M	Y69M	Y118M	Y29A	Y78A	Y127A	Y38N	Y87N	Y136N
Y21M	Y70M	Y119M	Y30A	Y79A	Y128A	Y39N	Y88N	Y137N
Y22M	Y71M	Y120M	Y31A	Y80A	Y129A	Y40N	Y89N	Y138N
Y23M	Y72M	Y121M	Y32A	Y81A	Y130A	Y41N	Y90N	
Y24M	Y73M	Y122M	Y33A	Y82A	Y131A	Y42N	Y91N	
Y25M	Y74M	Y123M	Y34A	Y83A	Y132A	Y43N	Y92N	
Y26M	Y75M	Y124M	Y35A	Y84A	Y133A	Y44N	Y93N	
Y27M	Y76M	Y125M	Y36A	Y85A	Y134A	Y45N	Y94N	
Y28M	Y77M	Y126M	Y37A	Y86A	Y135A	Y46N	Y95N	
Y29M	Y78M	Y127M	Y38A	Y87A	Y136A	Y47N	Y96N	
Y30M	Y79M	Y128M	Y39A	Y88A	Y137A	Y48N	Y97N	
Y31M	Y80M	Y129M	Y40A	Y89A	Y138A	Y49N	Y98N	
Y32M	Y81M	Y130M	Y41A	Y90A	Y1N	Y50N	Y99N	
Y33M	Y82M	Y131M	Y42A	Y91A	Y2N	Y51N	Y100N	
Y34M	Y83M	Y132M	Y43A	Y92A	Y3N	Y52N	Y101N	
Y35M	Y84M	Y133M	Y44A	Y93A	Y4N	Y53N	Y102N	
Y36M	Y85M	Y134M	Y45A	Y94A	Y5N	Y54N	Y103N	
Y37M	Y86M	Y135M	Y46A	Y95A	Y6N	Y55N	Y104N	
Y38M	Y87M	Y136M	Y47A	Y96A	Y7N	Y56N	Y105N	
Y39M	Y88M	Y137M	Y48A	Y97A	Y8N	Y57N	Y106N	
Y40M	Y89M	Y138M	Y49A	Y98A	Y9N	Y58N	Y107N	
Y41M	Y90M	Y1A	Y50A	Y99A	Y10N	Y59N	Y108N	
Y42M	Y91M	Y2A	Y51A	Y100A	Y11N	Y60N	Y109N	
Y43M	Y92M	Y3A	Y52A	Y101A	Y12N	Y61N	Y110N	
Y44M	Y93M	Y4A	Y53A	Y102A	Y13N	Y62N	Y111N	
Y45M	Y94M	Y5A	Y54A	Y103A	Y14N	Y63N	Y112N	
Y46M	Y95M	Y6A	Y55A	Y104A	Y15N	Y64N	Y113N	
Y47M	Y96M	Y7A	Y56A	Y105A	Y16N	Y65N	Y114N	
Y48M	Y97M	Y8A	Y57A	Y106A	Y17N	Y66N	Y115N	
Y49M	Y98M	Y9A	Y58A	Y107A	Y18N	Y67N	Y116N	

General

X1L	X11L	X21L	X31L
X1P	X11P	X21P	X31P
X1Q	X11Q	X21Q	X31Q
X1R	X11R	X21R	X31R
X2L	X12L	X22L	X32L
X2P	X12P	X22P	X32P
X2Q	X12Q	X22Q	X32Q
X2R	X12R	X22R	X32R
X3L	X13L	X23L	X33L
X3P	X13P	X23P	X33P
X3Q	X13Q	X23Q	X33Q
X3R	X13R	X23R	X33R
X4L	X14L	X24L	X34L
X4P	X14P	X24P	X34P
X4Q	X14Q	X24Q	X34Q
X4R	X14R	X24R	X34R
X5L	X15L	X25L	
X5P	X15P	X25P	
X5Q	X15Q	X25Q	
X5R	X15R	X25R	
X6L	X16L	X26L	
X6P	X16P	X26P	
X6Q	X16Q	X26Q	
X6R	X16R	X26R	
X7L	X17L	X27L	
X7P	X17P	X27P	
X7Q	X17Q	X27Q	
X7R	X17R	X27R	
X8L	X18L	X28L	
X8P	X18P	X28P	
X8Q	X18Q	X28Q	
X8R	X18R	X28R	
X9L	X19L	X29L	
X9P	X19P	X29P	
X9Q	X19Q	X29Q	
X9R	X19R	X29R	
X10L	X20L	X30L	
X10P	X20P	X30P	
X10Q	X20Q	X30Q	
X10R	X20R	X30R	

End

VITA

Hao Lei was born in Chengdu, Sichuan, China. He received his Bachelor of Science from East China Normal University, Shanghai, China, in the summer of 1999. In the fall of 2000, he entered the graduate school of The University of Electronic Science and Technology of China to pursue a master's degree in computer science. After graduated in April 2003, he worked with Huawei Technologies Co. Ltd., a telecommunication company in Shenzhen, China, and later for another telecommunication company – Alcatel Shanghai Bell Co. Ltd. from May 2005 to August 2006. He entered the Graduate School at The University of Texas at El Paso in the fall of 2006.