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MODELLING MEASUREMENTS AS TIMED INFORMATION PROCESSES IN SIMPLEX DOMAINS

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Abstract – This paper presents a domain-theoretic model for measurements and measuring instruments, by making explicit in simplex-domain structures two important aspects of measurement processes: the notion of standard representation relation, established between the (physical) values that are being measured and the meanings of the readings (semantic values) of the measuring instruments used to measure them, and the time underlying every measurement process, in a way that it is possible to trace the history of every measuring process. We also present the modelling of measurements performed by combined measuring instruments synchronized in time. Finally, the domain-theoretic modelling of a sample measuring process is presented to illustrate the approach.

Keywords: qualitative domains, simplex domains, measurement instruments, fusion processes

1. INTRODUCTION

Scott and Strachey [1] introduced Domain Theory as a mathematical framework for the semantics of programming languages. The main idea is that programming language semantics can be formally specified in terms of objects of domains, conceived as partially ordered sets of objects with certain properties such that the order (called information order) models the notions of approximation between objects and the objects themselves model partial results of computation steps. This means that (partially computed) objects can be compared by the quality of information they carry with respect to some totally computed object (a maximal object in the domain, called total object). In this sense, if an object x approximates another object y, then y contains at least the same information carried by x. The least element or bottom (required for every domain) models the absence of information, representing the beginning of any computational step; in contrast, total objects represent the final result of a finite computation or a limit, when they are produced by infinite computations.

The main subject addressed by Domain Theory is precisely the modelling of computations performed over objects that can only be produced by infinite processes. Therefore, it became the ideal structure for modelling computations over the real numbers [2] and real intervals [3]. Since then, domain-like structures have been used in several applications, most of them in the context of computation and mathematics [4]. Those features of Domain Theory support the main intuition behind our work, the possibility of developing a domain-theoretical model of uncertainty in measuring instruments and processes. In [5], we introduced the simplex and simplicial complex domains – qualitative domains whose objects are, respectively, simplices and simplicial complexes that are coherent, in a certain sense, allowing a domain-theoretical modelling of measurement processes based on those domains. We showed that qualitative domains of coherent simplices can be used to model the steps of (possibly infinite) measuring processes. In [6], we used the same approach to model a simple competitive sensor data fusion [7]-[8], where several subsequent perceptions obtained by a single perception module are fused. In both papers, time is not considered explicitly in the domain-theoretic model.

In this paper, we extend the initial work [5]-[6] done on simplex domains, where the notions of qualitative domains and simplices were firstly combined. We introduce a general model for a measuring instrument (measuring device, sensor etc.) as a simplex domain whose objects are constructed from reading events, labelled with temporal information and interpreted over a semantic domain, allowing for the definition of a temporal order between them, in addition to the information order. A standard representation relation between sources of physical values and semantic domains is defined, so that the notion of a calibrated measuring instrument for each set of physical values can be defined, showing constructively (step-by-step) how information events about a physical value are gathered and interpreted in the semantic domain. An exponential constructor is introduced for modelling the histories of measuring processes.

The new features introduced in this paper can support the definition of some timed simplex-domain...

² The elements of the semantic domain model the set of possible final measurements that shall be taken into consideration by the user. For instance, in agents (or robots), they model the internal representations (internal data structures) used for making decisions or coming to conclusions [7]-[8].
constructors that allow the modelling of composed measurement instruments that are able to synchronize and to produce fused measurements with less measurement uncertainty.

The paper is organized as follows. Section 2 presents basic definitions and summarizes previous work. Section 3 introduces the notion of measuring instrument and the calibration procedure. Reading processes are discussed in Section 4, allowing temporal information. The exponential constructor is introduced in Section 5. Section 6 discusses measuring processes performed by combined instruments. A sample of a measuring process is presented in Section 7 and Section 8 comes with the Conclusion.

2. SIMPLEX DOMAINS

A qualitative domain \( D = (\mathcal{D}, \subseteq) \) is a collection of sets \( \mathcal{D} \) ordered under the inclusion relation \( \subseteq \), such that: (i) \( D \) is non-empty: the empty set \( \emptyset \in \mathcal{D} \) is the bottom of \( D \); (ii) \( D \) is closed under directed unions: \( \forall S \subseteq D (S \text{ directed } \Rightarrow \exists \bar{S} \in D) \); (iii) \( D \) is downward closed: \( \forall a, b (a \in D \land b \subseteq a \Rightarrow b \in D) \). The information order is the inclusion relation. The information utility is called token and the set of tokens is given by \( \mathcal{P} = \{[\alpha] | \alpha \in \mathcal{D} \} = \mathcal{D} \). A set \( x \subseteq [\mathcal{P}] \) is said to be coherent if \( x \in \mathcal{D} \).

A simplex \( \sigma \) is a countable set. A simplex \( \sigma^p \) of dimension \( p \) is a finite set of \( p+1 \) elements. \( \sigma^\omega \) represents a simplex of infinite dimension and \( \sigma^\prime \) is the empty non-dimensional simplex. A simplicial complex is a countable set \( K \) of simplexes such that if \( \sigma \in K \) then \( \sigma \) is a face of all its simplexes (subsets). The dimension of \( K \) is the largest of the dimensions of its simplexes.

Let \( B \) be a countable set of tokens having an associated interpretation in a semantic domain \( \mathcal{S} \) induced by a complete lattice \( \mathcal{S} = (\mathcal{S}, \leq, \bot, \top, \supseteq) \), where \( \bot \) is the least element, \( \top \) is the greatest element and \( \bot \neq \top \).

\( i : B \to \mathcal{S} \) is said to be a token interpretation function if and only if \( \forall \beta \in B (i(\beta) \neq \bot \land i(\beta) \neq \top) \).

The notion of coherence in \( B \) is defined in terms of a token interpretation function \( i \), that is, a simplex of tokens \( \sigma = \{\beta_1, \beta_2, \ldots\} \subseteq B \) is said to be coherent if and only if \( \cup \{i(\beta) | \beta \in \sigma \} \neq \top \), where \( \cup \) means the supremum of the indicated set. The collection of (partial or total) coherent simplexes induced on \( B \) by the interpretation \( i \) is denoted by \( \text{CohSimp}(B, i) \).

The interpretation of a coherent simplex \( \sigma \in \text{CohSimp}(B, i) \) is given by the simplex interpretation function \( i : \text{CohSimp}(B, i) \to \mathcal{S} \), defined by \( i(\sigma) = \{i(\beta) | \beta \in \sigma \} \).

Theorem 1. \( \{ \text{CohSimp}(B, i), \subseteq \} \) is a qualitative domain, called the simplex domain induced on \( B \) by an interpretation function \( i \).

3. MEASURING INSTRUMENTS AND CALIBRATION FUNCTIONS

The set of physical values is conceived as \( \text{cpo} (\mathcal{V}, \subseteq, \bot, \top) \), with elements represented by values laying in a semantic domain \( \mathcal{S} \). It is possible to define many different relations between \( \mathcal{S} \) and \( \mathcal{V} \), but one of them shall be consider as a standard for calibration procedures. To define such standard relation, denoted by \( \ll \subseteq \mathcal{S} \times \mathcal{V} \), consider the sets:

\( V_{\alpha} = \{v \in \mathcal{V} | s \ll v, \mathcal{S}_{\alpha} \mathcal{V} = \{s \in \mathcal{S} | s \ll v \} \}, \) (1)

\( V_{\alpha} = \{v \in \mathcal{V} | e \in X (s \ll v) \}, \) (2)

\( \mathcal{S}_{\alpha} \mathcal{V} = \{s \in \mathcal{S} | e \in Y (s \ll v) \}. \) (3)

Definition 1. A standard representation relation for \( \mathcal{V} \) in \( \mathcal{S} \) is a relation \( \ll \subseteq \mathcal{S} \times \mathcal{V} \) such that:

i. Every well-defined physical value \( v \in \mathcal{V} \) has a well-defined representation in \( \mathcal{S} \), that is:

\( \forall v \in \mathcal{V} (v \ll \Rightarrow \exists s \in \mathcal{S} (s \neq \bot \land s \ll v)). \) (4)

ii. \( \ll \) satisfies the strict-like properties:

\( \forall s (s \ll \bot \Rightarrow s \ll \bot), \forall v (s \ll v \Rightarrow v \ll \bot). \) (5)

iii. \( \ll \) satisfies the order-preserving properties:

\( \forall s_i, s_j \in \mathcal{S} (\forall s_i \neq \emptyset \land \forall s_j \neq \emptyset \land s_i \ll s_j \Rightarrow
\forall v_1, v_2 \in \mathcal{V} (v_1 \ll v_2 \Rightarrow v_1 \ll v_2), \) (6)

\( \forall v_1, v_2 \in \mathcal{V} (v_1 \ll v_2 \Rightarrow \forall s_i \in \mathcal{S}_i (s_i \ll v_1 \land s_i \ll v_2)). \) (7)

iv. \( \ll \) satisfies the continuous-like properties:

\( \forall X \subseteq \mathcal{S} (\forall s \ll v \supseteq \forall s \subseteq X \Rightarrow \forall v \subseteq \supseteq \subseteq X \Rightarrow \forall Y \subseteq \mathcal{S} (\forall \mathcal{V} \subseteq \mathcal{V} \subseteq X \Rightarrow \forall \mathcal{V} \subseteq \mathcal{V} \subseteq Y). \) (8)

Given a cpo of physical values \( \mathcal{V} \) and a semantic domain \( \mathcal{S} \), a measuring instrument for \( \mathcal{V} \) in \( \mathcal{S} \), denoted by \( \mathcal{M}(\mathcal{V}, \mathcal{S}) \), is any simplex domain \( \mathcal{M}(\mathcal{V}, \mathcal{S}) = (\text{CohSimp}(M, i), \subseteq) \), where the countable set \( M \) is called the token internal scale with respective interpretation function \( i : M \to \mathcal{S} \). A coherent simplex

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5 A cpo (or a complete partial order) is a partially ordered set having a least element, where each directed subset has a supremum.
\( \sigma \in M_{(\sigma, \psi)} \) is called coherent simplex of readings, or coherent reading, for short. The set of all (partial or total) coherent readings is denoted by \( \text{CohSimp}(M, i) \). The coherent readings are interpreted in the semantic domain \( \mathcal{S} \) by the simplex interpretation function \( i: \text{CohSimp}(M, i) \rightarrow \mathcal{S} \). If \( \sigma \) is a possible coherent reading in \( M_{(\sigma, \psi)} \), then its interpretation \( i(\sigma) \in \mathcal{S} \) is called the measurement given by \( \sigma \).

An important task is to know whether or not a measurement is calibrated so that a measurement can be reliable associated to a physical value. In this sense, a measuring instrument is considered calibrated if it is capable to interpret its coherent readings according to a standard representation relation.

**Definition 2:** A measuring instrument is calibrated for the standard \( \Leftrightarrow \subseteq \mathcal{S} \times \mathcal{V} \) if and only if the following conditions hold:

\[
\forall \sigma \in M_{(\sigma, \psi)} \exists v \in \mathcal{V} \left( i(\sigma) \Leftrightarrow v \right), \quad (10)
\]

\[
\forall v \in \mathcal{V} \exists \sigma \in M_{(\sigma, \psi)} \left( i(\sigma) \Leftrightarrow v \right). \quad (11)
\]

A calibrated measuring instrument is denoted by \( M_{\sigma_0} \).

Considering a standard \( \Leftrightarrow \subseteq \mathcal{S} \times \mathcal{V} \) and a measuring instrument \( M_{(\sigma, \psi)} = \{ \text{CohSimp}(M, i) \} \), a calibration procedure for \( \Leftrightarrow \) is any operator \( C_{\infty} : [M \rightarrow \mathcal{S}] \rightarrow [M \rightarrow \mathcal{S}] \) that is able to adjust the scale interpretation scale \( i \), such that

\[
\forall \sigma \in M_{\sigma_0} \exists v \in \mathcal{V} \left( i_{\infty}(\sigma) \Leftrightarrow v \right), \quad (12)
\]

\[
\forall v \in \mathcal{V} \exists \sigma \in M_{\sigma_0} \left( i_{\infty}(\sigma) \Leftrightarrow v \right), \quad (13)
\]

where \( i_{\infty} = \left\{ \left( C_{\sigma_0}, i(\beta) \right) \in \mathcal{S} \mid \exists \beta \in \sigma \right\} \) is the calibrated simplex interpretation function. Any measuring instrument is liable to be calibrated if and only one succeeds in finding such a calibration procedure.

4. READING PROCESSES

The two fundamental notions of Domain Theory are those of partial objects and the approximation relation. Partial objects represent partial information about a subject, and the approximation relation orders such objects according to the degree of completeness of their information content. Given an approximation order between the partial coherent readings in a measuring instrument, a directed set of such objects can be understood as a reading process, where each reading in the directed set can be seen as resulting from a particular step in the progressive process of accumulating information about the physical value \( v \).

**Definition 3:** Let \( T = \{ t_0, t_1, \ldots \} \) be a (possibly infinite) sequence of discrete instants of time. A reading process of physical value \( v \in \mathcal{V} \) with time duration \( T \), performed by a measuring instrument \( M_{\infty} \), is a function \( \tau : T \rightarrow \{ \tau \} \), with \( \tau(\tau) = t \Leftrightarrow t = t_0 \), where \( M \) is the token internal scale and \( t_0 \) is the initial time of the process.

We use the notation \( \beta_0 \) to say that \( \tau^* \equiv \beta_0 \). \( \beta_0 \) is called the reading event that occurs at the time \( t \neq t_0 \) and \( \epsilon \) is the null information event that occurs at the beginning of every reading process at the initial time \( t_0 \). The set of readings events that occur at any time \( t \neq t_0 \) is denoted by \( M_t \).

The reading process interpretation function \( i_t : M_t \rightarrow \mathcal{S} \), induced by the interpretation function \( i : M \rightarrow \mathcal{S} \), is given by \( i(i_t) = i(\beta) \). It follows that a simplex of readings \( \sigma \subseteq M_t \) is coherent with respect to a reading process interpretation \( i_t \) if and only if \( \{ \beta_t \in M \mid \exists \epsilon \in \tau \beta \epsilon = \sigma \} \) is a coherent reading with respect to \( i_t \). Then, it is reasonable to use the standard representation relation of the measuring instrument \( M_{\infty} \) to obtain a subset of partial coherent readings that are relevant for the reading process \( \tau^* \) about a given physical value \( v \in \mathcal{V} \).

**Definition 4:** A non-empty coherent reading \( \sigma_0 \subseteq M_t \) is said to be a \( v \)-reading if \( i(\sigma_0) \Leftrightarrow v \), for \( \sigma = \{ \beta_t \in M \mid \exists \epsilon \in \tau \beta \epsilon \in \sigma_0 \} \). The empty simplex \( \sigma'' \) is called a null-reading. They are denoted by \( \sigma' \).

The subset of reading events that are relevant for the reading process of \( v \) is given by \( M'_v = \{ \beta_t \in M_t \mid \exists \epsilon \in \tau \beta \epsilon \in \sigma' \} \subseteq M_t \). The set of \( v \)-readings together with the null-reading is denoted by \( \text{CohSimp}(M'_v, i_t) \). The time duration of any partial \( v \)-reading \( \sigma'' \in \text{CohSimp}(M'_v, i_t) \) is then given by

\[
\Gamma_v(\sigma) = \begin{cases} 0 & \text{if } \sigma = \sigma'', \\ \max \{ t_i \in \tau \beta_t \} - t_0 & \text{if } \sigma \neq \sigma' \text{ is finite}, \\ \text{undefined} & \text{otherwise}. \end{cases} \quad (14)
\]

**Theorem 2:** \( M'_v = \{ \text{CohSimp}(M'_v, i_t) \} \subseteq \mathcal{S} \) is a qualitative domain – the Domain of \( v \)-readings induced on the calibrated measuring instrument \( M_{\infty} \) by a reading process \( \tau^* \) about a physical value \( v \).

A \( v \)-reading simplex in the domain \( M'_v \) is a totally temporally ordered set representing the order of occurrence of its reading events in the reading process \( \tau \).

Define an equivalence relation \( \sim \) on \( M'_v \) as \( \beta_{v'} \sim \beta_{v''}, \Leftrightarrow k' = k'' \). For each \( \beta_{v} \in M' \) in the scale
of the calibrated measuring instrument \( \mathbf{M}_c \), the set
\[ X'_i = \left\{ \beta \mid \exists t \in T \{ \beta \in M'_c \} \right\} \]
is an equivalence class in \( M'_c \). Denote by \( \mathbf{X} = \{ X'_i, \neq \emptyset \mid \beta \in \mathbf{M} \} \) the collection of non-empty equivalence classes. Observe that \( \mathbf{X} \neq \emptyset \), since \( T - \{ t \} \neq \emptyset \) for any \( t, t' \in \mathbf{R} \).

The duration \( \Gamma_k = \mathbf{X} \rightarrow T \) of a \( X'_i \in \mathbf{X} \) is defined as \( \Gamma_k(X'_i) = \max \{ t \in T \mid \beta \in X'_i \} - t_k \). For each \( \beta \in \mathbf{M} \) such that \( X'_i \neq \emptyset \), it is possible to define its maximal occurrence time \( \Gamma_M : \mathbf{M} \rightarrow T \) by
\[
\Gamma_M(\beta) = \max \{ t \in T \mid \beta \in M'_c \}.
\] (15)

Consider an interpretation function defined on \( \mathbf{X} \) as \( I^k : \mathbf{X} \rightarrow \mathbf{X} \), so that \( I^k(X'_i) = \beta(\beta) \). It follows that

**Theorem 3.** \( \mathbf{X}'_i = (\text{CohSimp}(\mathbf{X}, I^k), \subseteq) \) is a qualitative domain of coherent simplicies isomorphic to a sub-domain \( \mathbf{N}^{(i)} = (\text{CohSimp} (\mathbf{N} \subseteq, \iota_k), \subseteq) \) of the calibrated measuring instrument \( \mathbf{M}_c \). \( \mathbf{N}^{(i)} \) is called the \( \iota \)-measuring instrument induced by a reading process \( \tau^c \) on the measuring instrument \( \mathbf{M}_c \).

\( \mathbf{N}^{(i)} \) represents the part of a measuring instrument that shall be involved in a measuring process of a certain physical value \( v \). From Theorem 3, it is possible to associate, to each token \( \beta \in \mathbf{N} \), an information about the time of its latest occurrence, using (15).

5. TRACING MEASURING PROCESSES

The **exponential constructor** is used to model the measuring processes in time, generating the set of all possible histories of all measuring processes. It is essential, e.g., to model sensor data fusion [6][7][8], in order to allow the synchronization of successive and/or parallel sensing processes performed by several sensors.

For a calibrated measuring instrument \( \mathbf{M}_c \), consider a finite \( K \subseteq \text{CohSimp}_c(M, i) \). A simplicial complex \( K \) is said to be coherent if and only if \( \{ \beta \}, \{ \iota \} \in K \Rightarrow i(\iota \mid \beta) \neq T_\emptyset \). The set of coherent complexes is denoted by \( \text{CohComp}(M, i) \).

**Theorem 4.** \( \mathbf{M}_c = (\text{CohComp}(M, i), \subseteq) \) is a qualitative domain, called the exponential of \( \mathbf{M}_c \).

The exponential domain \( \mathbf{M}_c \) of coherent simplicies complexes represents the measuring processes of physical values \( \mathbf{V} \). Any coherent simplicial complex of \( \text{CohComp}(M, i) \) shows the history of the approximation of its maximal elements in \( \mathbf{M}_c \). The total objects in \( \mathbf{M}_c \) are coherent complexes representing complete histories of measurements; partial objects indicates measurements partially performed.

6. COMBINING INSTRUMENTS

In this section, we briefly show some examples of how to obtain a modelling for measuring processes performed by combined measuring instruments (like, for example, in sensor data fusion), based on some special constructors of domains of coherent simplicies that were defined on the basis of the standard coherence spaces constructors [12].

6.1. Competitive Measurements

Competitive measurements perform the fusion of partially redundant information about the same aspect of the subject, obtaining the information using different instruments, in order to simulate a more accurate measuring instrument (by reducing the uncertainty, mainly systematic errors, limited resolution etc. present in the individual measuring instruments).

\( \llbracket \beta \rrbracket, \mathbf{N}_1, \mathbf{N}_2 \subseteq \mathbf{S} \times \mathbf{V} \) are said to be compatible standard representation relations with respect to a standard relation \( \llbracket \beta \rrbracket \subseteq \mathbf{S} \times \mathbf{V} \) if and only if \( \llbracket \beta \rrbracket, \llbracket \beta \rrbracket \subseteq \mathbf{S} \times \mathbf{V} \) and \( \forall \mathbf{v}_1, \mathbf{v}_2 \in \mathbf{V} \), \( \forall \mathbf{s}_1, \mathbf{s}_2 \in \mathbf{S} \),
\[
\mathbf{s}_1 \llbracket \beta \rrbracket \mathbf{v}_1 \land \mathbf{s}_2 \llbracket \beta \rrbracket \mathbf{v}_2 \land \exists \mathbf{v} \mid \mathbf{v} \in \mathbf{V} \Rightarrow \\
\mathbf{s}_1 \llbracket \beta \rrbracket \mathbf{v}_1 \lor \mathbf{s}_2 \llbracket \beta \rrbracket \mathbf{v}_2 \lor \exists \mathbf{v} \mid \mathbf{v} \in \mathbf{V} \Rightarrow \mathbf{s}_1 \llbracket \beta \rrbracket \mathbf{v}_1 \lor \mathbf{s}_2 \llbracket \beta \rrbracket \mathbf{v}_2 \). (16)

Two calibrated measuring instruments \( \mathbf{M}_1 \) and \( \mathbf{M}_2 \) are said to be compatible if and only if their standard representation relations \( \llbracket \beta \rrbracket, \llbracket \beta \rrbracket \subseteq \mathbf{S} \times \mathbf{V} \) are compatible with respect to some standard \( \llbracket \beta \rrbracket \subseteq \mathbf{S} \times \mathbf{V} \). Consider then the disjoint union of the token scales \( M_1 \) and \( M_2 \), given by \( M_1 \cup M_2 = ((1) \times M_1) \cup ((2) \times M_2) \) and an associated interpretation function of indexed tokens \( (i_1, i_2) : M_1 \cup M_2 \rightarrow \mathbf{S}, \) defined by
\[
(i_1, i_2)(n, \beta) = \begin{cases} 
\iota_1(\iota) & \text{if } n = 1; \\
\iota_2(\iota) & \text{if } n = 2.
\end{cases}
\] (17)

A competitive complex \( \Phi \subseteq (M_1 \cup M_2) \) is coherent if \( \exists \{ (i_1, i_2)(n, \beta) \} \subseteq \Phi \neq T_\emptyset \) and its interpretation is \( \iota_{(i_1, i_2)}(\Phi) = \{ (i_1, i_2)(n, \beta) \} \subseteq \Phi \).

**Theorem 4.** The competitive combination of calibrated measuring instruments \( \mathbf{M}_1 \) and \( \mathbf{M}_2 \) is given by a domain \( \mathbf{M}_3 = (\text{CohSimp} (M_1 \cup M_2), (i_1, i_2)) \) which is also calibrated for the standard \( \llbracket \beta \rrbracket \subseteq \mathbf{S} \times \mathbf{V} \).

6.2. Complementary Measurements

Complementary measurements perform the fusion of independent objects about separate
aspects of a subject, obtaining the information using different measuring instruments, in order to simulate a larger, multi-faceted measuring instrument.

Consider two calibrated measuring instruments \( \mathbf{M}_1 \) and \( \mathbf{M}_2 \) and the cartesian product of their token scales \( M_1 \times M_2 \), with an associated interpretation function \( i_1 \times i_2 : M_1 \times M_2 \rightarrow S_1 \times S_2 \), defined by \( (i_1 \times i_2)(\beta_2, \beta_2) = (i_1(\beta_2), i_2(\beta_2)) \), where \( S_1 \times S_2 \) is the semantic domain. A subset \( \Psi \subseteq (M_1 \times M_2) \) is a coherent complementary simplex if \( \bigcup \{(i_1 \times i_2)(\beta_2, \beta_2) \mid \beta_2, \beta_2 \in \Psi \} \neq \{T_{\beta_1}, T_{\beta_2}\} \) and its interpretation is given by

\[
\text{Definition5. The complementary fusion of the measuring instruments } \mathbf{M}_1 \text{ and } \mathbf{M}_2 \text{ is the domain } \mathbf{M}_1 \otimes \mathbf{M}_2 = \{\text{CohSimp} (M_1 \times M_2), (i_1 \times i_2)\} \text{ in each component calibrated for the standards } \ll_1, \ll_2. \]

6.3. Cooperative Measurements

Cooperative measuring processes perform the fusion of independent information about separate aspects of a subject, obtaining the information using different measuring instruments, in order to estimate, at the interpretation level, a single measurement that derives from the original ones. Because of lack of space, we omit here further details of its formulation. See Section 7 for a contextualized example.

7. A SAMPLE MEASURING PROCESS

Consider the case of a box of which we want to measure the height \( H \), the width \( W \) and the area \( A \) of its frontal face\(^6\). To obtain a standard for calibrating measuring instruments for \( H \) and \( W \), consider a semantic domain of real intervals given by \( \mathcal{S} = \{(x_1, x_2) \mid 0 \leq x_1 \leq x_2 \leq 50\} \cup \{0\} \), ordered under the reverse inclusion relation (meaning that the least diameter, the best information), with \( \ll = \{0, 50\} \) (meaning no information) and \( \mathcal{T} = \{0\} \) (contradictory information). The standard representation relations between \( \mathcal{S} \) and the height \( H \) and the width \( W \) of the box are given by \( s \ll H(W) \Leftrightarrow H(W) \in s \).

Assume that we have three calibrated instruments \( \mathbf{M}_1, \mathbf{M}_2, \mathbf{M}_3 \) that we want to combine competitively \((\mathbf{M}_1 \& \mathbf{M}_2)\) to measure the height \( H \), complementarily \((\mathbf{M}_1 \& \mathbf{M}_3) \mathbf{M}_1 \) to measure also the width \( W \). We also shall combine them cooperatively to obtain the area \( A \), at the interpretation level. Each instrument \( \mathbf{M}_j \) can produce readings (tokens) \( \beta^j_1, \beta^j_2, \ldots \) that are interpreted in the semantic domain \( \mathcal{S} \). The interpretation functions of the instruments \( \mathbf{M}_j \) are shown in Table I.

Table II shows one possible reading process for each instrument \( \mathbf{M}_j \), operating during the time instants \( T = \{t_0, t_1, t_2, t_3\} \), the maximal simplex of coherent readings and the respective best semantic values that such processes produce at time \( t_j \).

<table>
<thead>
<tr>
<th>( \beta_j )</th>
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<tr>
<td>( i_1 )</td>
<td>( [2,5] )</td>
<td>( [3,6] )</td>
<td>( [4,7] )</td>
<td>( [5,8] )</td>
</tr>
<tr>
<td>( i_2 )</td>
<td>( [1,5,5,5] )</td>
<td>( [2,5,4,2] )</td>
<td>( [3,8,4,3] )</td>
<td>( [1,5,6,5] )</td>
</tr>
<tr>
<td>( i_3 )</td>
<td>( [0,5,1,5] )</td>
<td>( [0,8,1,8] )</td>
<td>( [1,1,2,1] )</td>
<td>( [1,4,2,4] )</td>
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</tbody>
</table>

6.1. The maximal coherent simplices of readings obtained at time \( t_j \) are registered by each exponential domain \( \mathbf{M}_j \) in the maximal simplicial complexes that represent the complete histories of the reading processes ending at time \( t_j \) in each instrument \( \mathbf{M}_j \). For example, for the instrument \( \mathbf{M}_1 \) in the exponential domain \( \mathbf{M}_1 \), we find that the initial fragments of the history of measuring \( H \) are given by the simplicial complexes shown in Table III. The maximal coherent complex among them has time duration \( t_3 - t_0 \). It represents the complete history of the measuring process, and the interpretation of its maximal simplex \( \{\beta_1^{i_1}, \beta_2^{i_1}, \beta_3^{i_1} \} \) (also shown in Table II) represents the best information gathered about the height \( H \), when using only the measuring instrument \( \mathbf{M}_1 \).

The maximal coherent competitive simplices produced by the fusion process performed by the combination \( \mathbf{M}_1 \& \mathbf{M}_2 \), at each time instant, and the respective interpretations in the semantic domain \( \mathcal{S} \), are shown in Table IV. In Table V, we present the maximal coherent tensor simplices produced by the fusion process performed by the complementary combination \((\mathbf{M}_1 \& \mathbf{M}_3) \mathbf{M}_1 \), at each time instant, and the respective interpretations in the semantic domain \( \mathcal{S} \times \mathcal{S} \).

\( H \) and \( W \) lay in a flat cop of real numbers \( r \leq 50 \).
### TABLE I. Initial fragments of the measuring process of the instrument $\mathbf{M}_i$

<table>
<thead>
<tr>
<th>Time $t_i$</th>
<th>Simplicial Complex</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_0$</td>
<td>${\sigma^i}$</td>
</tr>
<tr>
<td>$t_1$</td>
<td>${\sigma^i, {\beta_{i_1}^j}}$</td>
</tr>
<tr>
<td>$t_2$</td>
<td>${\sigma^i, {\beta_{i_1}^j}, {\beta_{i_2}^j}, {\beta_{i_3}^j}}$</td>
</tr>
<tr>
<td>$t_3$</td>
<td>${\sigma^i, {\beta_{i_1}^j}, {\beta_{i_2}^j}, {\beta_{i_3}^j}, {\beta_{i_4}^j}}$</td>
</tr>
</tbody>
</table>

### TABLE II. Competitive fusion process in the domain $\mathbf{M}_i \& \mathbf{M}_j$, for measuring $H$

<table>
<thead>
<tr>
<th>Time $t_i$</th>
<th>Maximal Competitive Fusion Simplex</th>
<th>Interpretation (1, $\leq H$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_0$</td>
<td>$\sigma^i$</td>
<td>$\perp = [0, 50]$</td>
</tr>
<tr>
<td>$t_1$</td>
<td>${\beta_{i_1}^j, \beta_{i_2}^j}$</td>
<td>$[2, 5, 4, 2]$</td>
</tr>
<tr>
<td>$t_2$</td>
<td>${\beta_{i_1}^j, \beta_{i_2}^j, \beta_{i_3}^j, \beta_{i_4}^j}$</td>
<td>$[3, 8, 4, 2]$</td>
</tr>
<tr>
<td>$t_3$</td>
<td>${\beta_{i_1}^j, \beta_{i_2}^j, \beta_{i_3}^j, \beta_{i_4}^j, \beta_{i_5}^j, \beta_{i_6}^j}$</td>
<td>$[4, 0, 4, 2]$</td>
</tr>
</tbody>
</table>

### TABLE V. Complementary fusion process in the domain $(\mathbf{M}_i \& \mathbf{M}_j) \otimes \mathbf{M}_j$

<table>
<thead>
<tr>
<th>Time $t_i$</th>
<th>Maximal Complementary Fusion Simplex</th>
<th>Interpretation (1, $\leq (H,W)$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_0$</td>
<td>${\sigma^i, \sigma^j}$</td>
<td>$\perp = [0, 50],[0, 50]$</td>
</tr>
<tr>
<td>$t_1$</td>
<td>${\beta_{i_1}^j, \beta_{i_2}^j}$</td>
<td>$[2, 5, 4, 2],[1, 1, 2, 1]$</td>
</tr>
<tr>
<td>$t_2$</td>
<td>${\beta_{i_1}^j, \beta_{i_2}^j, \beta_{i_3}^j}, {\beta_{i_4}^j, \beta_{i_5}^j}$</td>
<td>$[3, 8, 4, 2],[1, 1, 2, 1]$</td>
</tr>
<tr>
<td>$t_3$</td>
<td>${\beta_{i_1}^j, \beta_{i_2}^j, \beta_{i_3}^j, \beta_{i_4}^j, \beta_{i_5}^j, \beta_{i_6}^j}$</td>
<td>$[4, 0, 4, 2],[1, 4, 2, 1]$</td>
</tr>
</tbody>
</table>

Table VI shows the cooperative measurement process performed in the domain $(\mathbf{M}_i \& \mathbf{M}_j) \otimes \mathbf{M}_j$, obtaining the frontal area $A$ using, at the interpretation level, the interval operation $\perp$ defined by $f(H,W) = H \cap W$, that estimates the area $A$.

### TABLE VI. Cooperative fusion process in the domain $(\mathbf{M}_i \& \mathbf{M}_j) \otimes \mathbf{M}_j$, where 1, $\leq A$

<table>
<thead>
<tr>
<th>Time $t_i$</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2500</td>
<td>$[2.75, 8.82]$</td>
</tr>
<tr>
<td>1.0000</td>
<td>$[4.18, 8.82]$</td>
</tr>
<tr>
<td>2.0000</td>
<td>$[5.68, 8.82]$</td>
</tr>
</tbody>
</table>

8. CONCLUSION

This paper introduced a domain-theoretic framework to model measuring processes. We define the notion of calibration of measuring instruments and developed the necessary formal machinery to allow the modelling of measuring processes performed by combined instruments. Such instruments can operate in parallel, and synchronized in time, to attend different kinds of requirements: to obtain better results by reducing the uncertainty; to reduce systematic measurement errors; to simulate a multi-faceted instrument etc. A sample (competitive, complementary, and cooperative) fusion process was modelled, to help figuring out in a concrete way how the elements of the various domains look like in the constructions of the formal model.

REFERENCES


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