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Electroweak Scale Neutrinos

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Abstract

We study a model of neutrino mass generation at the Electroweak scale (EW), where Lepton number is broken by the vacuum expectation value of a Standard Model singlet complex scalar field. No energy scales higher than the EW scale are introduced in the model and neutrinos can still get their tiny masses through the general seesaw mechanism. The neutrino Yukawa couplings need not be more suppressed than that of the electron. As part of our results we present the branching ratios of the possible Higgs decays, which should be detectable at near future colliders.

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Chapter 1

Introduction

For the study of quantum scale phenomena at high energies we employ what is known as the *Standard Model* (SM), which describes the properties of the known fundamental particles and their interactions (except gravity).

The Standard Model is an extension of Quantum Mechanics in the sense that quantum mechanics as originally proposed by Heisenberg and Schrödinger was not consistent with special relativity. And so the Standard model incorporates special relativity and Quantum Mechanics into a theory that explains how quantities such as mass, momentum and energy behave at near light speeds. Keep in mind that in this theory quantum mechanics is united with special relativity and not with general relativity , which is a totally different endeavor, after all special relativity is a theory without forces.

And so since the late 1970's the predictions of the Standard Model have been tested and proved to be correct, but although the predictions made so far have all been verified there are certain observed phenomena that fail to be explained by this theory, such as, the Yukawa couplings, the Hierarchy Problem and the Neutrino Masses. That is why at the very least an extension of the Standard Model must be made in order to successfully account for the problems that lack a solution.

As mentioned previously one of the things that the SM fails to explain is the origin of neutrino masses, so in this thesis we studied a minimal extension of the SM at the Electroweak scale which is taken to be between 10 GeV and 1 TeV. In which neutrino masses are determined, having the origin of both type of masses made through Spontaneous Symmetry Breaking (SSB).

Since the energies scales that are used in our model are at the electroweak scale, it would be possible to get to prove the validity of our model by experimental confirmation at the Large Hadron Colider (LHC). Which is one of the reasons that motivated this work, other models that attempt to study physics beyond the Standard Model lack experimental validity, since they make predictions that would only be possible to verify at energy ranges that are currently unreachable.

The work is developed in the following way: In chapter 2, explanation is given into some aspects of the Standard Model such as the Lagrangian, group structure of the SM, the Higgs Mechanism, etc. On chapter 3, we present some of the properties of neutrino physics. In chapter 4. we talk about some of the problems that justify going beyond the SM. And in chapter 5, we see how the extension to the SM is done. Finally in chapter 6 we present our conclusions.

Chapter 2

The Standard Model

2.1 Lagrangians for scalars, fermions & vectors

In classical mechanics in order to determine the evolution of a physical system one uses the Euler-Langrange equation given by

$$\frac{\partial L(q_i, \dot{q}_i)}{\partial q_i} - \frac{d}{dt}\left(\frac{\partial L(q_i, \dot{q}_i)}{\partial \dot{q}_i}\right) = 0. \tag{2.1}$$

Similarly in a local field theory we have that the Euler-Lagrange equation for fields is re-written as

$$\frac{\partial \mathcal{L}(\varphi, \partial_\mu \varphi)}{\partial_\mu \varphi} - \frac{\partial \mathcal{L}}{\partial \varphi} = 0, \text{ where } L = \int \mathcal{L}(\varphi, \partial_\mu \varphi) d^3x, \tag{2.2}$$

where $\mathcal{L}$ is the Lagrangian density, φ a quantum field and $\partial_\mu\varphi$ its derivative. From now on we shall refer to $\mathcal{L}$ as the Lagrangian.

So far we have determined that nature contains fermion and vector fields, and so the Standard Model contains both. Additionally, in order to obtain masses for the vector fields, the SM requires the introduction of a scalar field, the so called *Higgs Field.* Latter on we will see this in detail, for now we explore the basic machinery needed in the construction of the SM. The first step is to determine the Lagrangian densities for free scalars, fermions and vectors:

For real scalars fields we have

$$\mathcal{L} = \frac{1}{2}\left[(\partial_\mu\phi)^2 - m^2\phi^2\right], \tag{2.3}$$

where $\varphi(x^\mu) \equiv \phi$ denotes a real scalar field of mass m.

For fermions or spin 1/2 particles

$$\mathcal{L} = \bar{\psi}(x)\left(i\gamma^\mu\partial_\mu - m\right)\psi(x), \tag{2.4}$$

$\psi(x)$ is the fermion field.

For bosons or spin 1 particles

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \tag{2.5}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, for some vector field A_μ.

2.2 Quantum Electrodynamics QED - $U(1)$ model -

In order to comprehend the structure of matter at its deepest levels, physicist have been searching for simplicity and symmetry in their theories. Because a symmetry implies a corresponding *invariance* and this invariance under transformations leads to a set of conserved quantities (Noether's Theorem). And since the physical information of a system is contained in the Lagrangian, Lagrangians are constructed to be invariant under transformations, because if a Lagrangian is invariant under some continuous symmetry transformation on its fields, there is a conserved current and thus a conserved charge.

Let us consider the following global transformation on a fermion field

$\psi(x)$of a system consisting of a free fermion with mass m:

$$\psi'(x) = e^{i\theta}\psi(x) = U_\theta\psi(x). \tag{2.6}$$

This is a global $U(1)$ transformation, where $U(1)$ is the Abelian group of all 1×1 unitary matrices and clearly $e^{i\theta} \in U(1)$

$$\mathcal{L}_D(x) = \bar{\psi}(x)\left(i\gamma^\mu\partial_\mu - m\right)\psi(x), \tag{2.7}$$

it turns out that when θ is constant the Lagrangian is invariant under this transformation.

As we apply the global transformation every point in field space transforms in the same way, so for a more general transformation in which every point transforms differently, we use a local gauge transformation which is defined as:

$$\begin{aligned}\psi'(x) &= e^{i\theta(x)}\psi(x),\\ \bar{\psi}'(x) &= e^{-i\theta(x)}\bar{\psi}(x).\end{aligned}$$

Under a local gauge transformation every point in space has a different kind of rotation, as we apply this transformation to the Lagrangian we notice that it is no longer invariant:

$$\begin{aligned}\mathcal{L} \rightarrow \mathcal{L}' &= \bar{\psi}'(x)\left(i\gamma^{\mu}\partial_{\mu} - m\right)\psi'(x) \\ &= \mathcal{L} + j^{\mu}(x)\partial_{\mu}\theta(x)\end{aligned}$$

where $j^{\mu}(x) = \bar{\psi}(x)\gamma^{\mu}\psi(x)$ is the vector current carried by the fermions. So, when we are using a local gauge transformation the Lagrangian that worked for global transformations does not work anymore.

In order to make the Lagrangian gauge invariant we have to add a strength field tensor like the one given by Eq. 2.5, from which we get that

$$\mathcal{L} = \bar{\psi}(x)\left(i\partial\!\!\!/ - m\right)\psi(x) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + q\bar{\psi}(x)\gamma^{\mu}\psi A_{\mu} \tag{2.8}$$

$$\mathcal{L} = \bar{\psi}(x)\left(iD\!\!\!\!/ - m\right)\psi(x) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \tag{2.9}$$

where $q\bar{\psi}\gamma^{\mu}\psi A_{\mu}$ is the interaction term. This new Lagrangian is the complete gauge invariant Lagrangian for an electro-photon system and the field theory for this Lagrangian is Quantum Electrodynamics (QED), where the term $D_{\mu} = \partial_{\mu} + iqA_{\mu}$ is the gauge-covariant derivative and $A_{\mu} \rightarrow A_{\mu} - \frac{1}{q}\partial_{\mu}\theta(x)$. Therefore, electromagnetic dynamics is made invariant by introducing a spin 1 vector boson field A_μ: the photon.

Now we know that we can construct a Lagrangian with fermions and vector fields invariant under a $U(1)$ local symmetry. But we can do even more, we can also add a scalar field:

$$\mathcal{L} = \bar{\psi}(x)\left(i\partial\!\!\!/ - m\right)\psi(x) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + q\bar{\psi}(x)\gamma^{\mu}\psi A_{\mu} + \partial_{\mu}\phi^{*}\partial^{\mu}\phi - m^{2}\phi^{*}\phi \quad (2.10)$$

But once again we notice that $\mathcal{L}'_D \neq \mathcal{L}_D$, so we can't just add new terms and expect the Lagrangian to be gauge invariant. In order for the Lagrangian to be invariant we must replace $\partial_\mu \rightarrow D_\mu$, then $\mathcal{L}' = \mathcal{L}$

2.3 Generalization to Non-Abelian Gauge Theories

On the last section we started out by seeing what would happen to a fermion field if we submitted it to a $U(1)$ transformation, and found that fermions had global symmetry, but lacked local symmetry and in order to make it locally symmetric certain changes needed to be done. Yang and Mills studied this and found that local phase rotations could be generalized to be invariant under any continuous symmetry group or $SU(N)$ group. In the next section we will study one such group $SU(2)$ following the work of (M.E. Peskin; D.V. Schroeder, 1995).

2.3.1 Yang-Mills Gauge Theory - $SU(2)$ model -

The $SU(2)$ gauge theory is a Non-Abelian generalization of the $U(1)$ gauge theory. In the Abelian case we learned what it takes to make a theory gauge invariant, in that case we had single fermion fields but now we will start of with a doublet of a Dirac field

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \text{ and } \bar{\psi} = \left(\bar{\psi}_1, \bar{\psi}_2\right) \tag{2.11}$$

where ψ_1, ψ_2 stand for fermions.

Previously we where applying a $U(1)$ transformation to the Lagrangian, so we transformed fermions as $\psi(x) = e^{i\theta(x)}\psi(x)$. But since we are now dealing with a local $SU(2)$ transformation for a spinor ψ, the transformation will be slightly modified:

$$\psi(x) \rightarrow \psi'(x) = U\psi(x) \tag{2.12}$$

where $U = e^{-i\frac{\sigma^i}{2}\theta^i(x)}$ (for $i = 1, 2, 3$), and σ^i are the Pauli matrices which satisfy the commutation relation $\left[\frac{\sigma^i}{2}, \frac{\sigma^j}{2}\right] = i\epsilon_{ijk}\frac{\sigma^k}{2}$. Analogously to the Abelian case in which $e^{i\theta(x)}$ was the generator of the $U(1)$ group, we have that the generators for the $SU(2)$ group are the Pauli matrices.

As we try to make the Lagrangian invariant under (Eq. 2.12), we find a new complications that we didn't have in the Abelian case, first of all we are now dealing with three orthogonal symmetry motions which do not commute with one another, hence the association of the non-commuting local symmetry terms with a non-Abelian gauge theory.

So, just as in the case of the $U(1)$ group we must first introduce a new expression for the covariant derivative that works for $SU(2)$

$$D_\mu = \partial_\mu - igA^i_\mu \sigma^i. \tag{2.13}$$

But this covariant derivative requires three vector fields, one for each generator, but it is not easy to calculate the derivative explicitly because the exponent does not necessarily commute with its derivative, but fortunately local transformations can be expanded infinitesimally in terms of the generators of the group:

$$U = 1 + i\theta^i(x)\frac{\sigma^i}{2} + \mathcal{O}(\theta^2)$$

so we get that

$$\begin{aligned}
\psi &\longrightarrow \left(1 + i\theta^i\frac{\sigma^i}{2}\right) \\
A^i_\mu &\longrightarrow A^i_\mu + + \frac{1}{g}\partial_\mu\theta^i + \epsilon_{ijk}A^j_\mu\theta^k
\end{aligned}$$

And it is also true that for finite transformations, the covariant derivative has the same transformation law as the field on which it acts. So we find that

$$[D_\mu, D_\nu] = -igF^i_{\mu\nu}\frac{\sigma^i}{2} \tag{2.14}$$

now that we have the form for the derivative we must determine the transformation form of ψ and A^a_μ, from which we get that the kinetic energy term of A^i_μ is:

$$F^i_{\mu\nu} = \partial_\mu A^i_\nu - \partial^i_\nu + g\epsilon_{ijk}A^j_\mu A^k_\nu \tag{2.15}$$

that has the infinitesimal transformation

$$F^i_{\mu\nu} \longrightarrow F^i_{\mu\nu} - \epsilon_{ijk}\theta^j F^k_{\mu\nu} \tag{2.16}$$

Therefore, the complete invariant Lagrangian for a fermion in $SU(2)$ is:

$$\begin{aligned}
\mathcal{L}_B &= -\frac{1}{4}F^i_{\mu\nu}F^{i\mu\nu} \\
\mathcal{L}_F &= \bar{\psi}(x)(i\gamma^\mu D_\mu - m)\psi(x) \\
\mathcal{L} &= \mathcal{L}_F + \mathcal{L}_B
\end{aligned}$$

where $\mathcal{L}_F$ is the Lagrangian for the fermion and $\mathcal{L}_B$ is the kinetic term of the gauge fields.

2.3.2 Generalization to $SU(N)$

We can extend the methodology that we saw for $SU(2)$ to $SU(N)$ starting with a Lagrangian that is very similar to the QED Lagrangian:

$$\mathcal{L} = \bar{\psi}(i\not{D})\psi - m\bar{\psi}\psi - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} \tag{2.17}$$

where $\psi(x)$ will form an n-plet and we will use the set of matrices t^a as generators of the symmetry group, instead of the $\frac{\sigma^i}{2}$ we had in $SU(2)$. The covariant derivative associated to this new transformation is be given by

$$D_\mu = \partial_\mu - igA^a_\mu t^a \tag{2.18}$$

In this frame, fermion $\psi(x)$ and vector fields $A_\mu(x)$ will transform as:

$$\begin{aligned}
\psi(x) &\longrightarrow (1 + i\theta(x)^a t^a)\psi \\
A^a_\mu(x) &\longrightarrow A^a_\mu(x) + \frac{1}{g}\partial_\mu\theta(x) + f^{abc}A^b_\mu\theta(x)^c
\end{aligned}$$

where α^i is a set of parameters coming from expanding a general element of $SU(n)$ in terms of the generators, and f^{abc} is an antisymmetric set of structure constants.

And from the commutator for the covariant derivatives we will get

$$[D_\mu, D_\nu] = -igF^a_{\mu\nu}t^a \tag{2.19}$$

from which we get that the field tensor is defined as

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu. \tag{2.20}$$

Finally, this tensor field has the transformation

$$F^a_{\mu\nu} \longrightarrow F^a_{\mu\nu} - f^{abc}\theta(x)^b F^c_{\mu\nu}, \tag{2.21}$$

adding terms in this form will allow us to have gauge invariant Lagrangians.

2.4 Spontaneous Symmetry Breaking

Spontaneous *Symmetry Breaking* is the mechanism through which the Lagrangian remains symmetric under certain group transformations while the physical vacuum of the field is made non-invariant.

In order to understand how SSB works let's we will begin by first examining the case of a SSB in a classical field theory.

Here we have that the Lagrangian is given by

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4 \tag{2.22}$$

but if we replace m^2 by -μ^2:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 + \frac{1}{2}\mu^2\phi^2 - \frac{\lambda}{4!}\phi^4 \tag{2.23}$$

we can identify the potential of the Lagrangian as

$$V(\phi) = -\frac{1}{2}\mu^2\phi^2 + \frac{\lambda}{4!}\phi^4 \tag{2.24}$$

which is plotted as follows

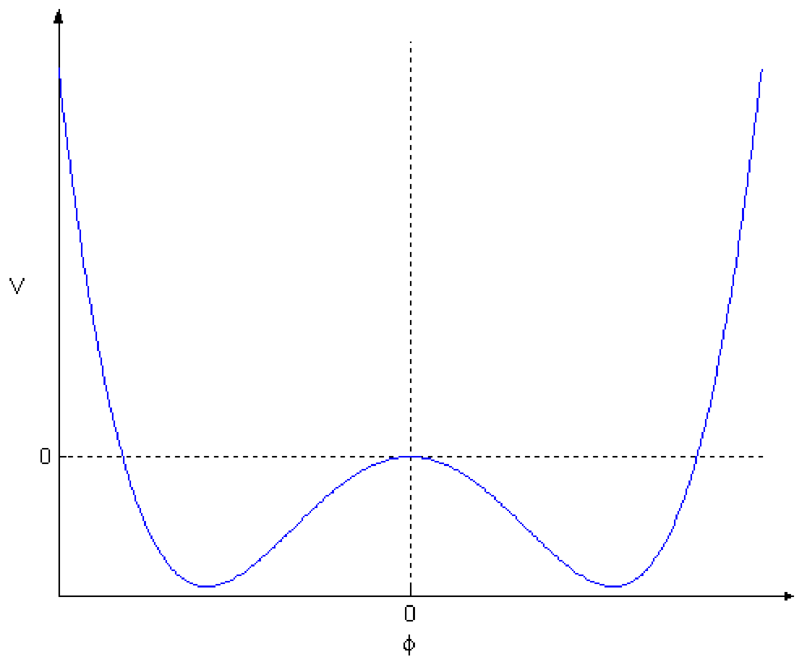

Figure 2.1: Potential for spontaneous symmetry breaking.

So as we minimize the potential we find the minima at

$$\phi_0 = \pm\nu = \pm\sqrt{\frac{6}{\lambda}}\mu \tag{2.25}$$

ν is the vacuum expectation value of ϕ.

Now supposing that the minima is a positive one, we can define

$$\phi(x) = \nu + \sigma(x), \tag{2.26}$$

with this we just add a small oscillation term, which at the minimum has the property that $\sigma = 0$. As we rewrite $\mathcal{L}$ with the new potential we get that

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\sigma)^2 - \frac{1}{2}(2\mu^2)\sigma^2 - \sqrt{\frac{\lambda}{6}}\mu\sigma^3 - \frac{\lambda}{4!}\sigma^4, \tag{2.27}$$

from the Lagrangian we can identify a scalar field of mass $\sqrt{2}\mu$. The form in which we get mass from the breaking of group symmetry is summarized in what is known as *Goldstone's* theorem.

2.4.1 Goldstone's theorem

Goldstone's theorem tells us that a massless particle should appear for every spontaneously broken symmetry in the theory. The massless fields that arise through spontaneous symmetry breaking are called *Goldstone bosons*.

So we start off from a theory that involves fields ϕ_0^a, whose Lagrangian is of the form

$$\mathcal{L} = (\text{terms with derivatives}) - V(\phi). \tag{2.28}$$

By taking ϕ_0^a as a constant field that minimizes V

$$\left[\frac{\partial}{\partial \phi^a} V\right]_{\phi^a(x)=\phi_0^a} = 0. \tag{2.29}$$

So by expanding V around its minimum we get

$$V(\phi) = V(\phi_0) + \frac{1}{2}(\phi - \phi^0)^a(\phi - \phi^0)^b + \left(\frac{\partial^2 V}{\partial \phi^a \partial \phi^b}\right)_{\phi_0} + ... \tag{2.30}$$

where the factor $\left(\frac{\partial^2 V}{\partial \phi^a \partial \phi^b}\right)_{\phi_0} \equiv m_{ab}^2$ is the mass matrix, whose eigenvalues give the masses of the fields.

So now we have a mechanism through which we can get masses of the fields.

2.4.2 The Higgs Mechanism

The mechanism through which by SSB a gauge boson acquires mass is called the *Higgs Mechanism*. We will learn just how it works in a manner similar to how we learned gauge invariance, by starting out studying the Abelian case and then its generalization to the Non-Abelian case.

So, for the abelian case we have a Lagrangian that has a complex scalar field coupled to itself and to the electromagnetic field:

$$\mathcal{L} = -\frac{1}{2}(F_{\mu\nu})^2 + |D_\mu \phi|^2 - V(\phi), \tag{2.31}$$

with $D_\mu = \partial\mu + ieA_\mu$ we have that the Lagrangian is $U(1)$ invariant.

So by choosing a potential like the one used in SSB

$$V(\phi) = -\mu^2 \phi^* \phi + \frac{\lambda}{2}(\phi^* \phi)^2, \tag{2.32}$$

and if $\mu^2 > 0$, we get that the minimum of the potential occurs at

$$\langle \phi \rangle = \phi_0 \left(\frac{\mu^2}{\lambda} \right)^{1/2}. \tag{2.33}$$

Since we are dealing with complex fields, we can decompose $\phi(x)$ as

$$\phi(x) = \phi_0 + \frac{1}{\sqrt{2}}(\phi_1(x) + i\phi_2(x)), \tag{2.34}$$

And the potential is rewritten as

$$V(\phi) = -\frac{1}{2\lambda}\mu^4 + \frac{1}{2} \cdot 2\mu^2\phi_1^2 + \mathcal{O}(\phi^3), \tag{2.35}$$

so just as in the case of the classical field, we notice that ϕ_1 has the mass $m = \sqrt{2}\mu$ and ϕ_2 is a massless Goldstone boson.

You might ask yourselves we haven't done anything new all that has been done is follow the step that we did previously. But now we will work on the kinetic energy term using the decomposed ϕ

$$|D_\mu\phi|^2 = \frac{1}{2}(\partial_\mu\phi_1)^2 + \frac{1}{2}(\partial_\mu\phi_2)^2 + \sqrt{2}e\phi_0 \cdot A_\mu\partial^\mu\phi_2 + e^2\phi_0^2 A_\mu A^\mu + \cdots\,, \tag{2.36}$$

where we notice that the last term is a photon mass term

$$\Delta\mathcal{L} = \frac{1}{2}m_A^2 A_\mu A^\mu, \tag{2.37}$$

where the mass

$$m_A^2 = 2e^2\phi_0^2 \tag{2.38}$$

so we have now found a way to give mass to the photon, our field boson.

Let's now see how the Higgs Mechanism is applied to Non-Abelian gauge symmetries, say an $SU(2)$ gauge field coupled to a scalar field ϕ.

The covariant derivative will be given by

$$D_\mu \phi = (\partial_\mu - igA_\mu^a \tau^a)\phi, \tag{2.39}$$

where $\tau^a = \sigma^a/2$.

And the vacuum expectation value is

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \nu \end{pmatrix} \tag{2.40}$$

So the kinetic energy term will be

$$|D_\mu \phi|^2 = \frac{1}{2} g^2 \begin{pmatrix} 0 & \nu \end{pmatrix} \tau^a \tau^b \begin{pmatrix} 0 \\ \nu \end{pmatrix} A_\mu^a A^{a\mu}$$

using $\tau^a, \tau^b = \frac{1}{2}\delta^{ab}$, we get that the mass term is

$$\Delta \mathcal{L} = \frac{g^2 \nu^2}{8} A_\mu^a A^{a\mu}, \tag{2.41}$$

therefore all three gauge bosons have a mass of the form

$$m_A = \frac{gv}{2}, \tag{2.42}$$

which tells us that the 3 generators of $SU(2)$ are broken equally by the *vev*.

2.5 Standard Model $SU(3) \times SU(2) \times U(1)$

In the Standard Model we incorporate the electromagnetic, weak and strong interactions, so the symmetry group that describes these is

$$SU(3)_C \times SU(2)_W \times U(1)_Y, \tag{2.43}$$

and it is through this theory that we understand the properties of matter particles.

2.5.1 Particle Content

Quarks and Leptons are the particles that make up all of matter. And they are more generally called *fermions* and have spin $\frac{1}{2}$. Where as the bosons or spin 1 particles, are the ones responsible for the forces that influence both quarks and leptons. The electromagnetic, weak and strong interaction are mediated by photons γ, weak bosons $W^{\pm}, Z^0$ and gluons g respectively. These observed particles can be classified as follows:

$$Quarks \quad \begin{pmatrix} u \\ \\ d \end{pmatrix}, \quad \begin{pmatrix} c \\ \\ s \end{pmatrix}, \quad \begin{pmatrix} t \\ \\ b \end{pmatrix}$$

$$Leptons \quad \begin{pmatrix} \nu_e \\ \\ e \end{pmatrix}, \quad \begin{pmatrix} \nu_\mu \\ \\ \mu \end{pmatrix}, \quad \begin{pmatrix} \nu_\tau \\ \\ \tau \end{pmatrix}$$

$$Bosons \quad \begin{pmatrix} \gamma \end{pmatrix}, \quad \begin{pmatrix} W^\pm \\ \\ Z^0 \end{pmatrix}, \quad \begin{pmatrix} g \end{pmatrix}$$

Although the most general interaction between particles is described by the $SU(3) \times SU(2) \times U(1)$ group. The electromagnetic and weak interactions become unified at around 100 GeV into what is known as the electroweak theory. Under the $SU(2)_L$ group, the left-handed quarks and leptons are doublets, while the right-handed fields are singlet's. So the mass terms for fermions are forbidden transitions. However, the *vevs* of the Higgs scalar breaks the electroweak symmetry giving masses to the quarks and leptons. But the problem of neutrino mass is different, so before we learn how neutrinos gain mass let's examine how through SSB the $W^\pm$ and Z become massive.

2.5.2 $SU(2) \times U(1)$ Lagrangian (Electroweak)

This theory is due to the work done by Glashow, Weinberg and Salam (GWS). And what it does is to combine the electromagnetic and weak interactions are into what is known as the electroweak theory. Which is a spontaneously symmetry breaking broken gauge theory.

As we proceed to understand it we start off from an $SU(2)$ gauge symmetry and we add a scalar field to break the symmetry spontaneously, but from this theory we don't get massless gauge bosons. That is why we add an additional $U(1)$ gauge symmetry, so we have the gauge transformation

$$\phi \rightarrow e^{i\alpha^a \tau^a} e^{i\beta/2} \phi, \tag{2.44}$$

where $\tau^a = \frac{\sigma^a}{2}$. If ϕ has the expectation value

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \tag{2.45}$$

the condition that leaves $\langle \phi \rangle$ invariant is

$$\alpha^1 = \alpha^2 = 0 \ , \ \alpha^3 = \beta.$$

With the covariant derivative of ϕ is

$$D_\mu \phi = (\partial_\mu - igA_\mu^a \tau^a - \frac{i}{2} g' B_\mu)\phi, \tag{2.46}$$

where A_μ^a and B_μ are $SU(2)$ and $U(1)$ gauge bosons respectively, g and g' are the coupling constants. The gauge mass terms come from the square of the covariant derivative at the scalar field vacuum expectation value, the relevant terms are

$$\Delta\mathcal{L} = \frac{1}{2} \begin{pmatrix} 0 & v \end{pmatrix} \left(gA_\mu^a \tau^a + \frac{1}{2} g' B_\mu \right) \left(gA^{b\mu} \tau^b + \frac{1}{2} g' B^\mu \right) \begin{pmatrix} 0 \\ v \end{pmatrix}$$

substituting $\tau^a = \sigma^a/2$ we get

$$\Delta\mathcal{L} = \frac{1}{2} \frac{v^2}{4} \left[g^2 (A_\mu^1)^2 + g^2 (A_\mu^2)^2 + (-gA_\mu^3 + g' B_\mu)^2 \right]. \tag{2.47}$$

We will have three massive vector bosons, denoted as

$$
\begin{aligned}
W^{\pm}_{\mu} &= \frac{1}{\sqrt{2}}(A^1_\mu \mp iA^2_\mu) \text{ with mass } m_W = g\frac{v}{2}; \\
Z^0_\mu &= \frac{1}{\sqrt{g^2+g'^2}}(gA^3_\mu - g'B_\mu) \text{ with mass } m_Z = \sqrt{g^2+g'^2}\frac{v}{2};
\end{aligned}
$$

The fourth vector field remains massless

$$
A_\mu = \frac{1}{\sqrt{g^2+g'^2}}(g'A^3_\mu + gB_\mu). \tag{2.48}
$$

For a fermion field belonging to a general $SU(2)$ representation, with $U(1)$ charge Y, the covariant derivative is

$$
D_\mu = \partial_\mu - igA^a_\nu T^a_\nu - ig'YB_\mu \tag{2.49}
$$

We can write this derivative in terms of the mass eigenstate fields as

$$
\begin{aligned}
D_\mu &= \partial_\mu - i\frac{g}{\sqrt{2}}(W^+_\mu T^+ + W^-_\mu T^-) - i\frac{1}{g^2+g'^2}Z_\mu(g^2T^3 - g'^2Y) \\
&- i\frac{gg'}{g^2+g'^2}A_\mu(T^3+Y),
\end{aligned}
$$

where $T^{\pm} = (T^1 \pm iT^2) = \frac{1}{2}(\sigma^1 \pm i\sigma^2) = \sigma^{\pm}$.

We also can identify the coefficient of the electromagnetic interaction as the electron charge e

$$e = \frac{gg'}{g^2 + g'^2}, \tag{2.50}$$

and identify the electric charge quantum number as

$$Q = T^3 + Y. \tag{2.51}$$

Let's define the weak mixing angle θ_w as the angle that satisfy

$$\begin{pmatrix} Z^0 \\ A \end{pmatrix} = \begin{pmatrix} cos\theta_w & -sin\theta_w \\ sin\theta_w & cos\theta_w \end{pmatrix} \begin{pmatrix} A^3 \\ B \end{pmatrix}$$

that is

$$\cos\theta_w = \frac{g}{\sqrt{g^2 + g'^2}} \ , \ \sin\theta_w = \frac{g'}{\sqrt{g^2 + g'^2}}.$$

So, we get

$$g^2 T^3 - g'^2 Y = (g^2 + g'^2) T^3 - g'^2 Q, \tag{2.52}$$

and therefore, we can rewrite the covariant derivative in the following form

$$D_\mu = \partial_\mu - i\frac{g}{\sqrt{2}}(W_\mu^+ T^+ + W_\mu^- T^-) - i\frac{g}{cos\theta_w} Z_\mu (T^3 - sin^2\theta_w Q) - ieA_\mu Q \quad (2.53)$$

where

$$g = \frac{e}{\sin\theta_w}. \quad (2.54)$$

The coupling of all the weak bosons are described in terms of the electron charge and the new parameter θ_w. The couplings due to the introduction of W and Z also involve the masses of this particles, which depends one from each other:

$$m_w = m_Z cos\theta_w. \quad (2.55)$$

So, we find the couplings to bosons, this time we will see the coupling to fermions. In the classical Lagrangian, the kinetic energy term for Dirac fermions splits into separate pieces for the left- and right-handed fields

$$\bar{\psi} i\partial\!\!\!/ \psi = \bar{\psi}_L i\partial\!\!\!/ \psi_L + \bar{\psi}_R i\partial\!\!\!/ \psi_R. \quad (2.56)$$

Here we have two different covariant derivatives and hence, this will lead to two different sets of couplings. We now assign the left-handed fermion fields to doublets in $SU(2)$ and the right-handed fermion fields to singlets in this group. If we specify the values for T^3 we can get Y from Eq. 2.51. This means that the Y will be different for left- and right-handed quarks and leptons. For the right-handed fields $T^3 = 0$. Then, for example, for the right-handed u quark field $Y = +2/3$; for $\bar{e}_R$, $Y = -1$, and for the left-handed fields

$$E_L = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L, \; Q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L,$$

have $T^3 = \pm 1/2$, so we assign $Y = -1/2$ and $Y = +1/6$ respectively. The construction of the mass term would give us some difficulties. We cannot have a term that looks like

$$\Delta\mathcal{L} = -m_e(\bar{e}_L e_R + \bar{e}_R e_L), \tag{2.57}$$

because e_L and e_R have different $SU(2)$ representations and different $U(1)$ charges. This is why we will ignore fermion masses, so the fermion kinetic energy terms for e, ν, u and d are

$$\mathcal{L} = \bar{E}_L(i\not{D})E_L + \bar{e}_R(i\not{D})e_R + \bar{Q}_L(i\not{D})Q_L + \bar{u}_R(i\not{D})u_R + \bar{d}_R(i\not{D})d_R. \tag{2.58}$$

The covariant derivative for each term would be the 2.49 with the specific values for T^a and Y, for example

$$\bar{Q}_L(i\not{D})Q_L = \bar{Q}_L i\gamma^\mu(\partial_\mu - igA^a_\mu\tau^a - ig'\frac{1}{6}B_\mu)Q_L. \quad (2.59)$$

Again, we write the Lagrangian in terms of the vector boson mass eigenstates

$$\begin{aligned}\mathcal{L} &= \bar{E}_L(i\not{\partial})E_L + \bar{e}_R(i\not{\partial})e_R + \bar{Q}_L(i\not{\partial})Q_L + \bar{u}_R(i\not{\partial})u_R + \bar{d}_R(i\not{\partial})d_R \\ &+ g(W^+_\mu J^{\mu+}_W + W^-_\mu J^{\mu-}_W + Z^0_\mu J^\mu_Z) + eA_\mu J^\mu_{EM}\end{aligned}$$

where

$$\begin{aligned}J^{\mu+}_W &= \frac{1}{\sqrt{2}}(\bar{\nu}_L\gamma^\mu e_L + \bar{u}_L\gamma^\mu d_L) \\ J^{\mu-}_W &= \frac{1}{\sqrt{2}}(\bar{e}_L\gamma^\mu\nu_L + \bar{d}_L\gamma^u d_L) \\ J^\mu_Z &= \frac{1}{cos\theta_w}[\bar{\nu}_L\gamma^\mu\left(\frac{1}{2}\right)\nu_L + \bar{e}_L\gamma^\mu\left(-\frac{1}{2} + sin^2\theta_w\right)e_L + \bar{e}_R\gamma^\mu\left(sin^2\theta_w\right)e_R \\ &+ \bar{u}_L\gamma^\mu\left(\frac{1}{2} - \frac{2}{3}sin^2\theta_w\right)u_L + \bar{u}_R\gamma^\mu\left(-\frac{2}{3}sin^2\theta_w\right)u_R \\ &+ \bar{d}_L\gamma^\mu\left(-\frac{1}{2} + \frac{1}{3}sin^2\theta_w\right)d_L + \bar{d}_R\gamma^\mu\left(\frac{1}{3}sin^2\theta_w\right)d_R] \\ J^\mu_{EM} &= \bar{e}\gamma^\nu(-1)e + \bar{u}\gamma^\nu(+\frac{2}{3}u + \bar{d}\gamma^\nu(-\frac{1}{3})d\end{aligned}$$

Chapter 3

Neutrino Physics

3.1 Neutrino Oscillations

We say that there is a neutrino oscillation when a neutrino of some specific flavor is transformed into a neutrino of another flavor. We use the word oscillation because it happens that the probability of the transition oscillates as a function of time. The origin of neutrino oscillations is due to flavor mixing and, although it can be discussed in two scenarios: pure Dirac or seesaw (or Majorana), if we consider that there is no chirality flip we end up being incapable of distinguish these two scenarios.

3.1.1 Atmospheric Neutrino Oscillation

We know that cosmic rays are flowing towards Earth and that they interact with the atmosphere producing pions. These pions decay into what its called atmospheric neutrinos as follows

$$\pi^+ \rightarrow \mu^+ + \nu_\mu \ , \ \mu^+ \rightarrow e^+ + \nu_e + \bar{\nu_\mu}.$$

We also know that the Earth is essentially transparent to neutrinos. So the Super-Kamiokande detector located at Japan discovered that about half of the atmospheric ν_μ neutrinos coming from the other side of the Earth were lost, while those from above were not. But since the Earth is almost transparent to the neutrinos they were supposed to be the same. Thus, the most likely and natural explanation to this problem is the assumption of the neutrino oscillation

$$\nu_\mu \rightarrow \nu_\tau, \tag{3.1}$$

i.e., muon neutrinos "convert" into tau neutrinos, which the Super-Kamiokande detector is unable to detect, whereas ν_e neutrinos do not experience any oscillation. The explanation makes sense because the particles coming from the other side of the Earth have more opportunity to oscillate that those coming from above.

3.1.2 Solar Neutrino Oscillation

There is another way to detect neutrino oscillations. It is from the Sun, the fusion reactions that take place in the Sun produce only electron neutrinos ν_e, but it was found that these can subsequently oscillate into both muon and tau neutrinos; ν_τ and ν_μ.

An experiment run by the Super-Kamiokande combined with the SNO (Sudbury Neutrino Observatory) in Canada combined both their data with in order to confirm that the total number of neutrinos from the Sun agreed with the one predicted by the Standard Model.

Chapter 4

Beyond the Standard Model

4.1 Problems not Solved by SM

4.1.1 Neutrino Masses

The Standard Model fails to predict mass for neutrinos, but experimentally it has been determined that they do in fact have mass. That is why in an extension of the SM must be made in order to account for this problem.

4.1.2 Yukawa Couplings

As we attempt to define masses for gauge bosons we found that the mass for each one of them had a constant term called the Yukawa couplings, whose value is determined experimentally.

4.1.3 Hierarchy Problems

It is expected that the SM is valid until some energy scale Λ and at Λ the theory is replaced by a more fundamental theory, that contains the SM. So, the SM would be a low energy theory. The energy scales at which this grander scheme might work at would be at the Planck mass $M_{pl} \sim 10^19$GeV or the $M_GUT \sim 10^15$GeV for $SU(5)$ GUT. The hierarchy problem is the problem of how to maintain the hierarchy of these mass scales, i.e. $M_w \ll M_GUT, M_pl$.

4.2 Extensions to the SM

We have just seen that the SM fails to explain certain phenomena. So, at the very least and extension of the SM is required in order to explain the problems that are beyond the realm of explanation that can be ascertained in the SM. We shall now see examples of theories that attempt to go beyond the SM and solve or explain the problems mentioned previously.

A natural extension to the Standard Model would be to have a grand unified group at some high energy which breaks down to the these are known as Grand Unifying Theories (GUT).

4.2.1 GUT

A Grand Unified Theory, is made up of one unified gauge group with only one coupling constant, at some high energy scale of unification M_U, with the property that this grand unified group breaks down to the SM

$$\mathcal{G}_U \rightarrow SU(3) \times SU(2) \times U(1). \tag{4.1}$$

At the unification scale, the gauge coupling constant of the unified group is the same as the gauge coupling constants of the SM.

4.2.2 SUSY

In the context of particle physics when one talks about *Supersymmetry*, what is meant is that there is a symmetry that relates a fermion to a boson. Supermultiplets contain both fermions and bosons, and every particle has a supersymmetric counterpart called *superpartners*. However, superpartners have yet to be observed, hence supersymmetry must be a broken symmetry and the superparticles should have masses of the order of the supersymmetry breaking scale.

Among the great triumphs of Supersymmetry is the solution to the hierarchy problem and the unification of the space-time symmetry with the internal symmetry. So it remains to see if superparticles are detected at the LHC.

4.2.3 Extra Dimensions

At the electroweak scale gravity is the weakest of all the fundamental forces. But at very high energies gravity should become as strong as any of the other forces, perhaps at a theory at such energies would be able to unify gravity along with the other forces. A theory that would be supersymmetric to solve the hierarchy problem a kind of supergravity theory. But the symmetries from which the weak, strong and electromagnetic interactions are internal degrees of freedom, whereas gravity originates fro space-time symmetry; so in this sense new space dimensions would be responsible for the new gauge interactions. But this new extra dimension have no been detected in any experiment. So it remains to see if in fact this extra space dimensions do in fact exist. In order, to accept or reject this alternative to the SM.

Chapter 5

Electroweak Scale Neutrinos

Although we already know that neutrinos are massive, the problem of finding the origin of these masses is still open. In this section, in order to give a new approach to this problem it is suggested a minimal extension to the SM keeping in mind the following assumptions:

1. SM principle content and the gauge interactions.

2. The existence of three right-handed (RH) neutrinos with a mass scale of EW size: 10 GeV to 1 TeV.

3. That global $U(1)$ symmetry is spontaneously broken at the EW scale by complex scalar field.

4. All mass scales come from SSB. This assumption leads to the usual SM

Higgs doublet but also to a SM singlet complex scalar field η with lepton number -2.

To start with the physics of this extended model, we write the relevant terms for the Lagrangian for Higgs and neutrino :

$$\mathcal{L}_{\nu y} = -y_{\alpha i}\bar{L}_{\alpha}N_{Ri}\Phi - \frac{1}{2}Z_{ij}\eta\bar{N}^{c}_{Ri}N_{Rj} + h.c. \tag{5.1}$$

where N_R represents the RH neutrinos, and the potential is given by

$$\begin{aligned} V &= \mu_D^2\Phi^\dagger\Phi + \frac{\lambda}{2}(\Phi^\dagger\Phi) + \mu_s^2\eta^*\eta + \lambda'(\eta^*\eta)^2 & (5.2)\\ &+ \kappa(\eta\Phi^\dagger\Phi + h.c) + \lambda(\Phi^\dagger\Phi)(\eta^*\eta). & (5.3)\end{aligned}$$

Now, if ϕ and η are responsible for EW and global $U(1)_L$ symmetry breaking, respectively, we use the following notation

$$\Phi = \begin{pmatrix} 0 \\ \frac{\phi^0+v}{\sqrt{2}} \end{pmatrix} \text{ and } \eta = \frac{\rho+u+i\sigma}{\sqrt{2}}$$

where $v/\sqrt{2}$ and $u/\sqrt{2}$ are the vacuum expectation values (vevs) of Φ and η respectively.

Then, using the next condition:

$$\frac{\partial V}{\partial \phi_0} = 0 \quad , \quad \frac{\partial V}{\partial \rho} = 0, \tag{5.4}$$

we get the conditions for minimization:

$$\mu_D^2 \;=\; -\frac{1}{2}\left(\lambda v^2 + \lambda_m u^2 + 2\sqrt{2}\kappa u\right). \tag{5.5}$$

$$\mu_s^2 \;=\; -\frac{1}{2u}\left(2\lambda' u^3 + \lambda_m u v^2 + \sqrt{2}\kappa v^2\right). \tag{5.6}$$

From which we can derive the mass matrix which is a matrix whose eigenvalues give the masses of the fields

$$M_s^2 = \begin{pmatrix} \frac{\partial^2 V}{\partial \phi^2} & \frac{\partial^2 V}{\partial \phi \partial \rho} \\ \frac{\partial^2 V}{\partial \rho \partial \phi} & \frac{\partial^2 V}{\partial \rho^2} \end{pmatrix} = \begin{pmatrix} \lambda v^2 & vu(\lambda_m - \sqrt{2}r) \\ vu(\lambda_m - \sqrt{2}r) 2\lambda' u^2 & \frac{1}{\sqrt{2}} r v^2 \end{pmatrix}$$

where $r \equiv -\kappa/u$. The mass of the σ (Majoron) field is given by

$$M_\sigma = \frac{rv^2}{\sqrt{2}} \tag{5.7}$$

From the mass matrix above, we can redefine the mass eigenstates as

$$\mathcal{H} = \begin{pmatrix} \phi^0 \\ \rho \end{pmatrix} = \begin{pmatrix} cos(\alpha) & -sin(\alpha) \\ sin(\alpha) & cos(\alpha) \end{pmatrix} \begin{pmatrix} h \\ H \end{pmatrix} = \begin{pmatrix} h\cos\alpha - H\sin\alpha \\ h\sin\alpha + H\cos\alpha \end{pmatrix}$$

If we use this new definitions to rewrite 5.1 we obtain

$$\begin{aligned} \mathcal{L}_{\nu y} &\supset y_{\alpha i}\bar{\nu}_{L1}N_{Ri}\frac{\phi_0}{\sqrt{2}} - \frac{1}{2}Z_{ij}\frac{(\rho + i\sigma)}{\sqrt{2}}\bar{N}_{Rj} + h.c \\ &= \left(-\frac{y_{\alpha i}}{\sqrt{2}}\bar{\nu}_{L1}N_{Ri}(hc_\alpha - Hs_\alpha) + h.c\right) - \left(\frac{i}{2\sqrt{2}}Z_{ij}\bar{N}^c_{Ri}N_{Rj}\sigma + h.c\right) \\ &- \left(\frac{1}{2\sqrt{2}}Z_{ij}\bar{N}^c_{Ri}N_{Rj}(hs_\alpha + Hc_\alpha) + h.c.\right) \end{aligned}$$

this work was originally obtained by Aranda et al., 2008, but are slightly different to the ones presented here do to slight errors made in the calculations, which where corrected during the development of this thesis. If we consider the following neutrino mass matrix

$$m_\nu = \begin{pmatrix} 0 & m_D \\ m_D & M_M \end{pmatrix}$$

where $(m_D)_{\alpha i} = y_{\alpha i} u/\sqrt{2}$. The mass eigenstates are denoted by ν_1 and ν_2 and satisfy

$$\nu_\tau = cos\theta\nu_{L1} + sin\theta\nu_{R2}$$
$$N = -sin\theta\nu_{L1} + cos\theta\nu_{R2}$$

and $\theta = \sqrt{m_D/M_M}$.

We get that the relevant terms for the Lagrangian in 5.8 are the ones containing the term h, from which:

$$\begin{aligned} \mathcal{L} &\supset \left[-\frac{Z}{2\sqrt{2}} h s_\alpha c_\theta^2 (\bar{\nu}^c_{R2}\nu_{R2} + \bar{\nu}_{R2}\nu^c_{R2}) + h.c\right] \\ &+ \left[\frac{y}{\sqrt{2}} h c_\alpha c_\theta^2 (\bar{\nu}_{L1}\nu_{R2} + \bar{\nu}_{R2}\nu_{L1}) h.c.\right] \end{aligned}$$

where we use the rules

$$\begin{aligned}
\psi_L &= P_L\psi \ , \ \psi_R = P_R\psi \\
\bar{\psi}_L &= P_R\bar{\psi} \ , \ \bar{\psi}_R = P_L\bar{\psi} \\
\bar{\psi}^c_L &= P_L\bar{\psi}^c \ , \ \bar{\psi}^c_R = P_R\bar{\psi}^c
\end{aligned}$$

Since we are interested in the coupling of the Higgs Boson h with massive (or heavy) neutrino and massless (or light) neutrino, we will now present the Decay Width of the Higgs into different particles.

5.0.4 First Decay

In this section we consider the decay of the Higgs boson h into a heavy neutrino ν_2 and its antineutrino $\bar{\nu_2}$. Let m_h and m_2 denote the Higgs and neutrino (and antineutrino) masses respectively. We will show that the decay width, Γ for this problem is given by:

$$\Gamma\left(h \rightarrow \bar{\nu_2}\nu_2\right) = \frac{m_h}{128\pi}|Z|^2 c_\theta^4 s_\alpha^2 \left(1 - \frac{4m_2^2}{m_h^2}\right)^{3/2} \tag{5.8}$$

We use the following part of the Lagrangian in 5.8, where ν_2 and $\bar{\nu_2}$ appear:

$$\mathcal{L} \supset \left[h \frac{-Z}{2\sqrt{2}} c_\theta^2 s_\alpha \left(\bar{\nu}_{R_2}^c \nu_{R_2} + \bar{\nu_{R_2}} \nu_{R_2}^c \right) \right] \tag{5.9}$$

where $y_\nu^* = y_\nu$, $Z_{11} = Z$ and the below relations have been used

$$\bar{\nu}_{R_2}^c = \bar{\nu}_2^c P_R, \qquad \nu_{R_2} = P_R \nu_2, \qquad \bar{\nu}_{R_2}^c \nu_{R_2} = \bar{\nu}_2^c P_R \nu_2$$

$$\bar{\nu}_{R_2} = \bar{\nu}_2 P_L, \qquad \nu_{R_2}^c = P_L \nu_2^c, \qquad \bar{\nu}_{R_2} \nu_{R_2}^c = \bar{\nu}_2 P_L \nu_2^c$$

The general formula for fermions decay width is the integral of the following formula

$$d\Gamma = \frac{1}{32\pi^2} \frac{|\mathcal{M}|^2}{m_h^2} |\vec{p_1}| d\omega \tag{5.10}$$

where ω is the solid angle. So, integrating we get

$$\Gamma = \frac{4\pi}{32\pi^2} \frac{|\mathcal{M}|^2}{m_h^2} |\vec{p_1}| \tag{5.11}$$

Hence, in order to find the decay width, the first thing to do is to calculate the term $|\mathcal{M}|^2$. It follows from Feynman's rules that

$$i\mathcal{M} = \bar{u}(\mathbf{p_1})P_R\left(\frac{-Z}{2\sqrt{2}}c_\theta^2 s^\alpha\right)v(\mathbf{p_2}) + \bar{u}(\mathbf{p_1})P_L\left(\frac{-Z}{2\sqrt{2}}c_\theta^2 s^\alpha\right)v(\mathbf{p_2}) \tag{5.12}$$

then

$$|i\mathcal{M}|^2 = \frac{1}{8}|Z|^2 c_\theta^4 s_\alpha^2\left[\bar{u}(\mathbf{p_1})P_R v(\mathbf{p_2})\bar{v}(\mathbf{p_2})P_R u(\mathbf{p_1}) + \bar{u}(\mathbf{p_1})P_L v(\mathbf{p_2})\bar{v}(\mathbf{p_2})P_L u(\mathbf{p_1})\right] \tag{5.13}$$

Now, let $\beta = \frac{1}{8}|Z|^2 c_\theta^4 s_\alpha^2$, then

$$\begin{aligned}
|i\mathcal{M}|^2 &= \beta tr\left[\left(\not{\mathbf{p}}_\mathbf{1} + m_2\right)P_R\left(\not{\mathbf{p}}_\mathbf{2} - m_2\right)P_L + \left(\not{\mathbf{p}}_\mathbf{1} + m_2\right)P_L\left(\not{\mathbf{p}}_\mathbf{2} - m_2\right)P_R\right] \\
&= \beta\frac{1}{2}tr\left[\left(\not{\mathbf{p}}_\mathbf{1} + m_2\right)\left(1+\gamma^5\right)\left(\not{\mathbf{p}}_\mathbf{2} - m_2\right) + \left(\not{\mathbf{p}}_\mathbf{1} + m_2\right)\left(1-\gamma^5\right)\left(\not{\mathbf{p}}_\mathbf{2} - m_2\right)\right] \\
&= 2\frac{\beta}{2}tr\left[\not{\mathbf{p}}_\mathbf{1}\ \not{\mathbf{p}}_\mathbf{2} - m_2^2\right] \\
&= \beta\left[\mathbf{p_1}_\mu\mathbf{p_2}_\nu 4g^{\mu\nu} - 4m_2^2\right] \\
&= 4\beta\left(\mathbf{p_1}\cdot\mathbf{p_2} - m_2^2\right)
\end{aligned}$$

Therefore

$$|i\mathcal{M}|^2 = \frac{1}{2}|Z|^2 c_\theta^4 s_\alpha^2\left(\mathbf{p_1}\cdot\mathbf{p_2} - m_2^2\right) \tag{5.14}$$

To find the dot product $\mathbf{p_1}\cdot\mathbf{p_2}$ in the equation above and also the $\vec{p_1}$ of the width formula, we need to solve the kinematics for this problem. During the decay, the heavy neutrino ν_2 has a momentum $\vec{p}_1$ while the antineutrino $\bar{\nu_2}$ has a momentum $\vec{p}_2$, which is has the same magnitude as $\vec{p}_1$ but in opposite direction. The quadrimomentums for neutrino and antineutrino and their relationships are given as follows

$$
\begin{aligned}
\mathbf{p_1} &= (E_1, \bar{p}_1) = \left(\frac{m_h}{2}, \vec{p}_1\right) \\
\mathbf{p_2} &= (E_2, \bar{p}_2) = \left(\frac{m_h}{2}, -\vec{p}_1\right) \\
\mathbf{p} &= (\mathbf{p_1} + \mathbf{p_2}) \\
\mathbf{p}^2 &= m_h^2
\end{aligned}
$$

Hence

$$
\begin{aligned}
|\vec{p}_1|^2 &= \frac{m_h^2}{4} - m_2^2 \\
|\vec{p}_1| &= \frac{m_h}{2}\left(1 - \frac{4m_2^2}{m_h^2}\right)^{1/2}
\end{aligned}
$$

and

$$
\begin{aligned}
\mathbf{p_1} \cdot \mathbf{p_2} &= E_1 E_2 - \vec{p}_1 \cdot \vec{p}_2 \\
&= \frac{m_h^2}{4} + \left(\frac{m_h^2}{4} - m_2^2 \right) \\
&= \frac{m_h^2}{2} \left(1 - \frac{4m_2^2}{m_h^2} \right)
\end{aligned}
$$

So now, we have everything we need to substitute the missing terms in 5.11:

$$
\begin{aligned}
\Gamma &= \frac{4\pi}{32\pi^2} \frac{1}{m_h^2} \left[\frac{1}{2} |Z|^2 c_\theta^4 s_\alpha^2 \frac{m_h^2}{2} \left(1 - \frac{4m_2^2}{m_h^2} \right) \right] \frac{m_h}{2} \left(1 - \frac{4m_2^2}{m_h^2} \right)^{1/2} \\
&= \frac{m_h}{64\pi} |Z|^2 c_\theta^4 s_\alpha^2 \left(1 - \frac{4m_2^2}{m_h^2} \right)^{3/2}
\end{aligned}
$$

But, since we have that the neutrino and the antineutrino are identical particles so, we we have identical final particles in a decay we use: $\#particles \longrightarrow \frac{1}{\#\text{particles}}$, so we divide the decay with we just got by the number of particles, which is two, and we get our final result

$$
\Gamma = \frac{m_h}{128\pi} |Z|^2 c_\theta^4 s_\alpha^2 \left(1 - \frac{4m_2^2}{m_h^2} \right)^{3/2} \tag{5.15}
$$

5.0.5 Second Decay

For the decay of the higgs into one light ν_1 and one heavy ν_2, we will show that the decay width is given by

$$\Gamma\left(h \to \bar{\nu}_1 \nu_2\right) = \frac{m_h}{16\pi} y_\nu^2 c_\theta^4 c_\alpha^2 \left(1 - \frac{m_2^2}{m_h^2}\right)^2 \tag{5.16}$$

The part of the Lagrangian that we will use is the one that couples ν_1 and ν_2:

$$\mathcal{L} \supset h \frac{y_\nu}{\sqrt{2}} c_\theta^4 c_\alpha^2 \left[\bar{\nu}_{L_1}^c \nu_{R_2} + \bar{\nu}_{R_2}^c \nu_{L1}\right] \tag{5.17}$$

We will also use the following relations and the general formula we saw in the decay above.

$$\bar{\nu}_{L_1}^c \nu_{R_2} = \bar{\nu}_1 P_R \nu_2, \quad \bar{\nu}_{R_2}^c \nu_{L_1} = \bar{\nu}_2 P_L \nu_1.$$

To begin with, again, we need to calculate the term $|i\mathcal{M}|^2$ using Feynman's rules

$$
\begin{aligned}
i\mathcal{M} &= \bar{u}(\mathbf{p_1})P_R\left(\frac{y_\nu}{\sqrt{2}}c_\theta^4 c_\alpha^2\right)v(\mathbf{p_2}) + \bar{u}(\mathbf{p_2})P_L\left(\frac{y_\nu}{\sqrt{2}}\left(S_\theta^2 - C_\theta^2\right)C_\alpha\right)v(\mathbf{p_1}) \\
i\mathcal{M} &= \left(\frac{y_\nu}{\sqrt{2}}c_\theta^4 c_\alpha^2\right)\left[\bar{u}(\mathbf{p_1})P_R v(\mathbf{p_2}) + \bar{u}(\mathbf{p_2})P_L v(\mathbf{p_1})\right].
\end{aligned}
$$

Then we squared it to get the term we need

$$
\begin{aligned}
|i\mathcal{M}|^2 &= \left(y_\nu^2 c_\theta^4 c_\alpha^2\right)\frac{1}{2}\left[\bar{u}(\mathbf{p_1})P_R v(\mathbf{p_2})\bar{v}(\mathbf{p_2})P_R u(\mathbf{p_1}) + \bar{u}(\mathbf{p_2})P_L v(\mathbf{p_1})\bar{v}(\mathbf{p_1})P_L u(\mathbf{p_2})\right] \\
&= \left(y_\nu^2 c_\theta^4 c_\alpha^2\right)\frac{1}{2}tr\left[\not{\mathbf{p}}_\mathbf{1} P_R(\not{\mathbf{p}}_\mathbf{2} + m_2)P_L + \left(\not{\mathbf{p}}_\mathbf{2} - m_2\right)P_L(\not{\mathbf{p}}_\mathbf{1})P_R\right] \\
&= \left(y_\nu^2 c_\theta^4 c_\alpha^2\right)\frac{1}{2}tr[\not{\mathbf{p}}_\mathbf{1}\left(\frac{1+\gamma^5}{2}\right)(\not{\mathbf{p}}_\mathbf{2} + m_2)\left(\frac{1-\gamma^5}{2}\right) \\
&+ (\not{\mathbf{p}}_\mathbf{2} - m_2)\left(\frac{1-\gamma^5}{2}\right)\not{\mathbf{p}}_\mathbf{1}\left(\frac{1+\gamma^5}{2}\right)] \\
&= \left(y_\nu^2 c_\theta^4 c_\alpha^2\right)\frac{1}{8}tr[\left(\not{\mathbf{p}}_\mathbf{1} + \not{\mathbf{p}}_\mathbf{2}\gamma^5\right)\left(\not{\mathbf{p}}_\mathbf{2} - \not{\mathbf{p}}_\mathbf{2}\gamma^5 + m_2 - m_2\gamma^5\right) \\
&+ \left(\not{\mathbf{p}}_\mathbf{2} - \not{\mathbf{p}}_\mathbf{2}\gamma^5 - m_2 + m_2\gamma^5\right)\left(\not{\mathbf{p}}_\mathbf{1} + \not{\mathbf{p}}_\mathbf{1}\gamma^5\right)] \\
&= \left(y_\nu^2 c_\theta^4 c_\alpha^2\right)\frac{1}{8}tr\left[4\ \not{\mathbf{p}}_\mathbf{1}\ \not{\mathbf{p}}_\mathbf{2}\right] \\
&= \left(y_\nu^2 c_\theta^4 c_\alpha^2\right)\frac{1}{2}tr\left[\mathbf{p_1}\mu\mathbf{p_2}\nu\gamma^\mu\gamma^\nu\right] \\
&= \left(y_\nu^2 c_\theta^4 c_\alpha^2\right)\frac{1}{2}\mathbf{p_1}\mu\mathbf{p_2}\nu 4\gamma^{\mu\nu} \\
&= 2\left(y_\nu^2 c_\theta^4 c_\alpha^2\right)\mathbf{p_1}\cdot\mathbf{p_2}.
\end{aligned}
$$

So, we have that

$$|i\mathcal{M}|^2 = 2\left(y_\nu^2 c_\theta^4 c_\alpha^2\right) \mathbf{p_1}\cdot\mathbf{p_2}. \tag{5.18}$$

Again, to find the term $\mathbf{p_1}\cdot\mathbf{p_2}$ n term of what we got, that is, heavy neutrino mass and momentums, we need to work with the dynamics of the problem. This time, the heavy neutrino has momentum $\vec{p}_1$ while the light one has momentum $\vec{p}_2$, so we have

$$\begin{aligned}
\mathbf{p_1} &= (E_1, \vec{p}_1) = \left(\frac{m_h^2 - m_2^2}{2m_h}, \vec{p}_1\right) \\
\mathbf{p_2} &= (E_2, \vec{p}_2) = \left(\frac{m_h^2 + m_2^2}{2m_h}, -\vec{p}_1\right) \\
\mathbf{p} &= (\mathbf{p_1} + \mathbf{p_2}) \\
\mathbf{p}^2 &= m_h^2
\end{aligned}$$

Then we find that

$$\begin{aligned}
|\bar{p}_1|^2 &= \frac{(m_h^2 - m_2^2)(m_h^2 + m_2^2)}{4m_h^2} \\
|\bar{p}_1|^2 &= \frac{m_h^2}{4}\left[1 - \frac{m_2^4}{m_h^4}\right].
\end{aligned}$$

And for the dot product we get

$$
\begin{aligned}
\mathbf{p_1} \cdot \mathbf{p_2} &= E_1 E_2 - \vec{p}_1 \cdot \vec{p}_2 \\
&= \left(\frac{m_h^2 - m_2^2}{2m_h} \right) \left(\frac{m_h^2 + m_2^2}{2m_h} \right) + \frac{m_h^2}{4} \left[1 - \frac{m_2^4}{m_h^4} \right] \\
&= \frac{m_h^2}{2} \left[1 - \frac{m_2^4}{m_h^4} \right].
\end{aligned}
$$

Finally, we just need to replace every term into the general form for the decay width to find that

$$
\Gamma = \frac{m_h}{16\pi} y_\nu^2 c_\theta^4 c_\alpha^2 \left(1 - \frac{m_2^4}{m_h^4} \right)^{3/2}.
$$

Chapter 6

Results

In the previous chapter we saw the explicit way in which the higgs would decay into neutrinos, and from those results and further decays for other particles we made graphs of the branching ratios to see what the probability would be for the Higgs to decay into different types of particles.

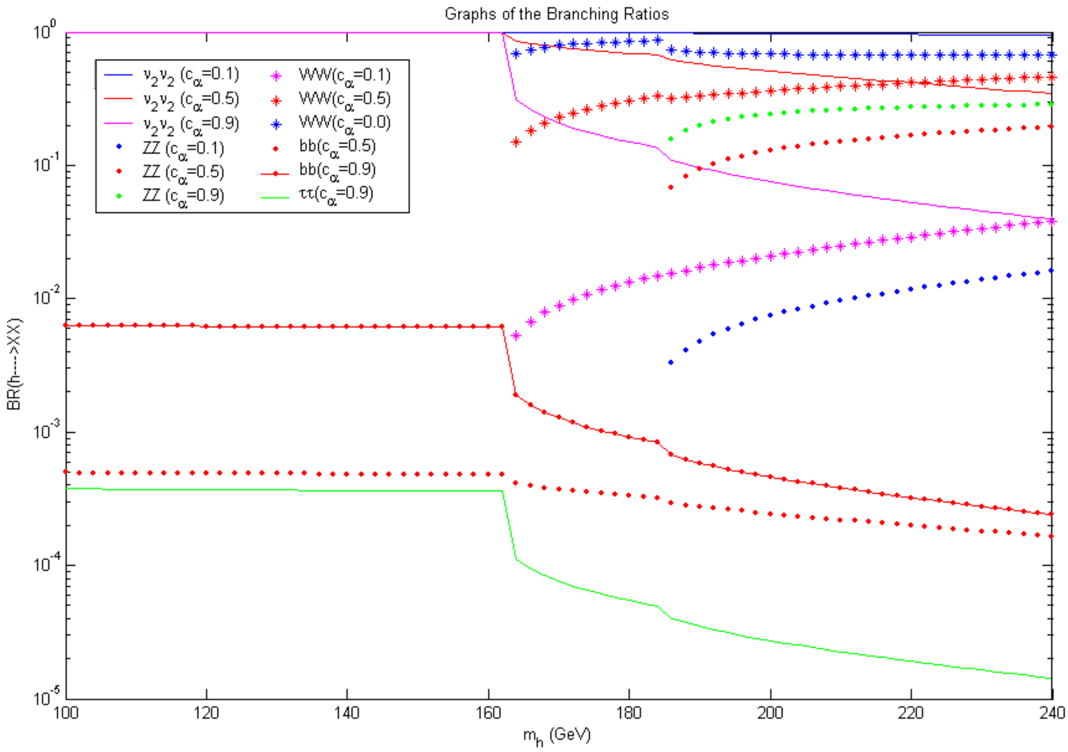

Figure 6.1: Dominant branching ratios for Higgs decays for $m_2 = 10$ GeV. For the three values of $\cos\theta = 0.1$, 0.5 and 0.9

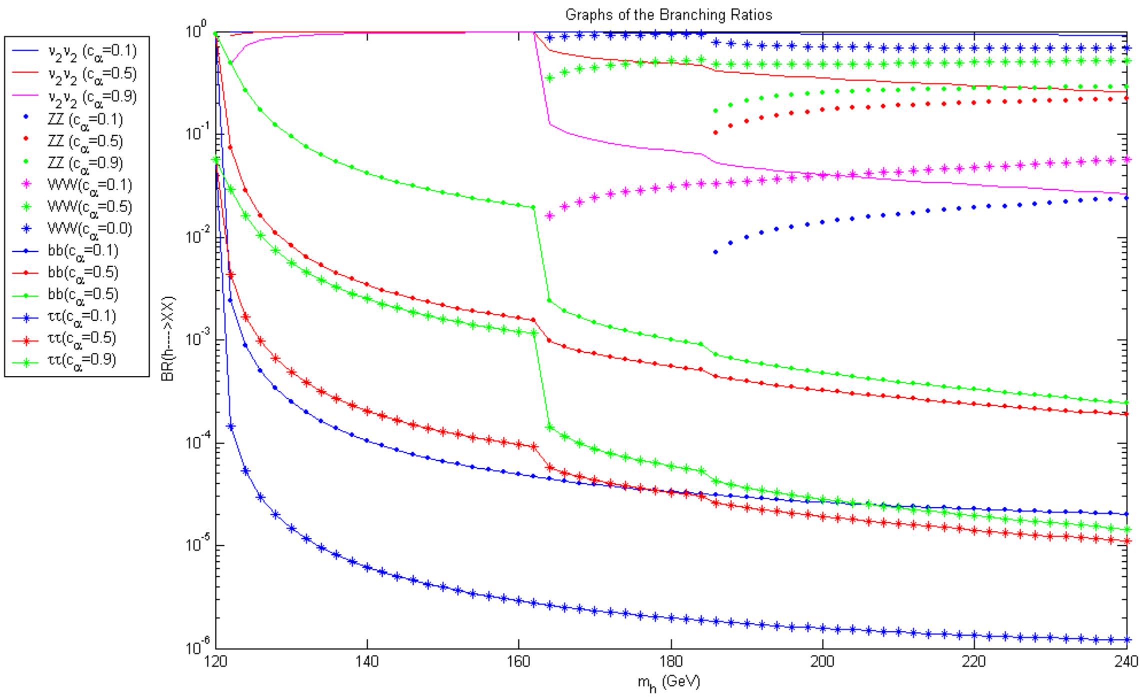

Figure 6.2: Dominant branching ratios for Higgs decay for $m_2 = 60$ GeV. For the three values of $\cos\theta = 0.1$, 0.5 and 0.9

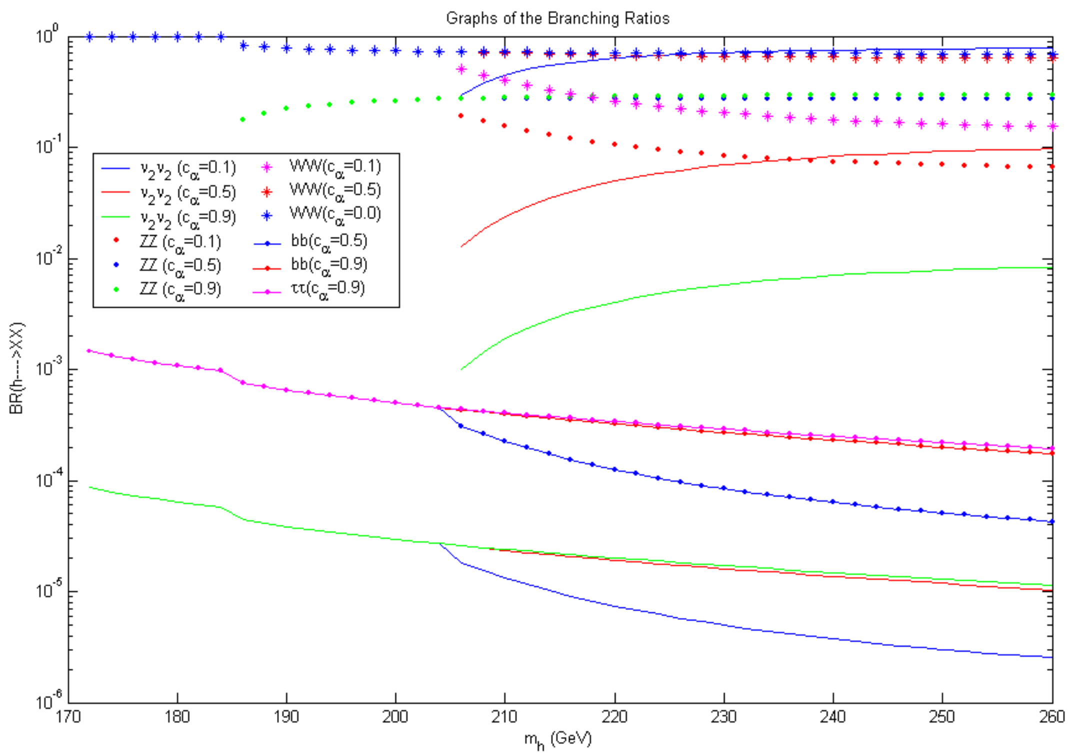

Figure 6.3: Dominant branching ratios for Higgs decay for $m_2 = 100$ GeV. For the three values of $\cos\theta = 0.1$, 0.5 and 0.9

Chapter 7

Conclusions

We calculated the branching ratios of higgs decays into different particles and studied possible scenarios in which through different values for mass of the higgs we get different results for measurable particles.

Our results indicate that:

1. It is possible to generate small neutrino masses without introducing higher energy scales.

2. And the ascertained phenomenology is testable at present and near future colliders.

The results demonstrate that by taking a minima extension of the SM it is possible to account for possible phenomenological results. It remains to see whether or not the signatures proposed with this model are in fact measured in future particle accelerators.

Bibliography

[1] Aranda, A., Blanno O., & Diaz-Cruz J., (**2008**), *A Model of Neutrino and Higgs Physics at the Electroweak Scale*, Phys.Lett.B.

[2] Erhard, S. & Ion-Olimiu S.E., (**2007**), *Approaches to Fundamental Physics*, Springer Heidelberg.

[3] Morii, T., Lim C., & Mukherjee S., (**2004**), *The Physics of the Standard Model and Beyond*, World Scientific Publishing Co.

[4] Peskin, M. & Schroeder D., (**1995**), *An Introduction to Quantum Field Theory*, Westview Press.

[5] Sarkar, U., (**2008**), *Particle and Astroparticle Physics*, Taylor and Francis Group, New York.

[6] Ryder, L.H., (**1985**), *Quantum Field Theory*, Cambridge University Press.

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