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Vladik Kreinovich
The University of Texas at El Paso, vladik@utep.edu

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Astronomical Tests of Relativity: Beyond Parameterized Post-Newtonian Formalism (PPN), to Testing Fundamental Principles

Vladik Kreinovich

University of Texas at El Paso, El Paso, TX 79968, USA
email: vladik@utep.edu

Abstract. By the early 1970s, the improved accuracy of astrometric and time measurements enabled researchers not only to experimentally compare relativistic gravity with the Newtonian predictions, but also to compare different relativistic gravitational theories (e.g., the Brans-Dicke Scalar-Tensor Theory of Gravitation). For this comparison, Kip Thorne and others developed the Parameterized Post-Newtonian Formalism (PPN), and derived the dependence of different astronomically observable effects on the values of the corresponding parameters.

Since then, all the observations have confirmed General Relativity. In other words, the question of which relativistic gravitation theory is in the best accordance with the experiments has been largely settled. This does not mean that General Relativity is the final theory of gravitation: it needs to be reconciled with quantum physics (into quantum gravity), it may also need to be reconciled with numerous surprising cosmological observations, etc. It is, therefore, reasonable to prepare an extended version of the PPN formalism, that will enable us to test possible quantum-related modifications of General Relativity.

In particular, we need to include the possibility of violating fundamental principles that underlie the PPN formalism but that may be violated in quantum physics, such as scale-invariance, T-invariance, P-invariance, energy conservation, spatial isotropy violations, etc. In this paper, we present the first attempt to design the corresponding extended PPN formalism, with the (partial) analysis of the relation between the corresponding fundamental physical principles.

Keywords. Gravitation, relativity, celestial mechanics, etc.

1. Introduction

One of the main motivations for the development of General Relativity was the discrepancy between the astronomical observations and the predictions of Newton's theory: namely, the 43 sec/100 years difference in the perihelion of Mercury. The first confirmation of General Relativity also came from astronomy, as the 1919 eclipse observations of near-solar objects that confirmed the gravity-based bending the light paths. Until the early 1970s, astronomical and time measurements have been used to compare different predictions of General Relativity with the Newtonian ones – and in all the case, General relativity was confirmed.

By the early 1970s, the improved accuracy of astrometric and time measurements enabled the researchers not only to experimentally compare relativistic gravity with the Newtonian predictions, but also to compare different relativistic gravitational theories (e.g., the Brans-Dicke Scalar-Tensor Theory of Gravitation). For this comparison, Kip Thorne (Thorne & Will, 1971) and others developed the Parameterized Post-Newtonian Formalism (PPN), and derived the dependence of different astronomically observable effects on the values of the corresponding parameters; see, e.g., Brumberg (1991), Will (1993).

Since then, all the observations have confirmed General Relativity. In other words, the question of which relativistic gravitation theory is in the best accordance with the experiments has been largely settled. This does not mean that General Relativity is the final theory of gravitation: it needs to be reconciled with quantum physics (into quantum gravity), it may also need to be reconciled with numerous surprising cosmological observations, etc. It is, therefore, reasonable to prepare an extended version of the PPN formalism, that will enable us to test possible quantum-related modifications of General Relativity.

In particular, we need to include the possibility of violating fundamental principles that underlie the PPN formalism but that may be violated in quantum physics, such as scale-invariance, T-invariance, P-invariance, energy conservation, spatial isotropy violations, etc. We present the first attempt to design the corresponding extended PPN formalism, with the (partial) analysis of the relation between the corresponding fundamental physical principles.

2. Possible Violations of T-Invariance

Derivation of possible terms. One of the assumptions behind most terms of the PPN formalism is T-invariance, i.e., invariance with respect to changing time direction $t \to -t$. The largest non-T-invariant terms usually considered in celestial mechanics are radiation c^{-5} terms in binary systems (such as pulsars) Will (2001).

However, it is well known that quantum physics is not T-invariant: interaction experiments have shown that weak interactions are not T-invariant. It is, therefore, reasonable to consider possible effects of T-non-invariance on c^{-3} and c^{-4} terms in celestial mechanics. As usual, the c^{-4} terms in $ds^2 = g_{\alpha\beta}dx^\alpha dx^\beta$ mean c^{-4} terms in g_{00}, c^{-3} terms in g_{0i} ($1 \leqslant i \leqslant 3$), and c^{-2} terms in g_{ij}.

Following the general ideas of PPN (see, e.g., Will (2001)), we assume that the terms g_{ij} analytically depend on the masses m_a of the celestial bodies (of order c^{-2}), on their velocities $\mathbf{v}_a$ (of order c^{-1}), on the inverse distances r_a^{-1} between the current point and the a-th body, r_{ab}^{-1} between the bodies, on the corresponding unit vectors $\mathbf{e}_a$ and $\mathbf{e}_{ab}$ (and on other terms like pressure p). The terms should be dimensionless and rotation-invariant.

The only T-non-invariant quantity is $\mathbf{v}_a$ of order c^{-1}. Every term must contain masses (since it must tend to 0 when $m_a \to 0$), so with m_a of order c^{-2} and $\mathbf{v}_a$, we have c^{-3}. In g_{ij}, we look for c^{-2} terms, so there are no T-non-invariant terms there.

Similarly, the values g_{0i} can contain $\mathbf{v}_a$ at most linearly – else they are $\sim c^{-4}$. Terms containing $\mathbf{v}_a$ linearly are T-invariant, so the new terms must contain no velocities at all. These terms should contain m_a and no other relativistic terms – else they would be c^{-4}. We need to add r_a^{-1} to make these terms dimensionless and $\mathbf{e}_a$ to make it a vector, so we get

$$\delta g_{0i} = \delta_2 \cdot \sum \frac{m_a \cdot e_{a,i}}{r_a}$$

for some parameter δ_2.

The values g_{00} can contain $\mathbf{v}_a$ at most quadratically. Quadratic terms are T-invariant, so new terms must be linear in $\mathbf{v}_a$. Adding r_a^{-1} to make it dimensionless and multiplying by $\mathbf{e}_a$ to make it a scalar, we get

$$\delta g_{00} = \delta_1 \cdot \sum \frac{m_a \cdot (\mathbf{e}_a \cdot \mathbf{v}_a)}{r_a}$$

for some parameter δ_1.

Effect on light. Light is determined by c^{-2} terms in $g_{\alpha\beta}$, so the new terms have no effect on light.

Additional coordinate transformations. Are the new terms coordinate-invariant? To find out, we need to add the possibility of T-non-invariant coordinate transformation $x^{\alpha'} = x^\alpha + \xi^\alpha(x^\beta)$ to the usual PPN transformations Will (2001). These transformations lead to $\delta g_{\alpha\beta} = -\xi_{\alpha,\beta} - \xi_{\beta,\alpha}$. To maintain PPN approximation, we must consider terms up to c^{-2} in x^i and up to c^{-3} in x^0. The term ξ^i contains m_a of order c^{-2}, so it cannot contain any non-T-invariant terms $\mathbf{v}_a$ (which would add the order c^{-1}); thus, the ξ^i terms are T-invariant. The ξ^0 terms must contain $\mathbf{v}_a$ at most linearly; linear terms are T-invariant, so the only new terms do not contain $\mathbf{v}_a$ at all. To make the resulting terms in $g_{\alpha\beta}$ dimensionless, we must multiply m_a by $\log(r_a)$ (to get r_a^{-1} in the derivative). Thus, we get the additional coordinate transformation $x_0' = x_0 + \xi_0$, with

$$\xi_0 = \alpha \cdot \sum_a m_a \cdot \ln(r_a),$$

for some parameter α.

This transformation leads to terms

$$\delta g_{00} = -2\xi_{0,0} = -2\alpha \cdot \sum \frac{m_a \cdot (\mathbf{e}_a \cdot \mathbf{v}_a)}{r_a}$$

and

$$\delta g_{0i} = -\xi_{0,i} = -\alpha \cdot \sum \frac{m_a \cdot e_{a,i}}{r_a},$$

i.e., to $\delta_1' = \delta_1 - 2\alpha$ and $\delta_2' = \delta_2 - \alpha$. One can easily conclude that the necessary and sufficient condition for a metric to be T-invariant in some coordinate system (i.e., to have α for which $\delta_1' = \delta_2' = 0$) is $\delta_1 = 2\delta_2$.

The coordinate-invariant combination of new parameters is $\delta_1' \stackrel{\text{def}}{=} \delta_1 - 2\delta_2$.

The existence of a Lagrange function. When can these new terms come from a Lagrange function L? The new terms must contain m_a (else they do not tend to 0 as $m_a \to 0$), they must be contain at least one other m_b – else there are no distances to make them dimensionless, and they must be of order $\geqslant c^{-6}$. Thus, they can contain $\mathbf{v}_a$ at most quadratically. Quadratic terms are T-invariant, so the only possible non-T-invariant terms contain $\mathbf{v}_a$ linearly. Using dimensionless-ness and rotation-invariance, we conclude that the only possible term is

$$\sum_{a\neq b} \frac{m_a \cdot m_b \cdot (\mathbf{v}_a \cdot \mathbf{e}_{ab})}{r_{ab}},$$

but this term is a full time derivative of the expression

$$\sum_{a\neq b} m_a \cdot m_b \cdot \ln(r_{ab})$$

and therefore, does not affect the Lagrange equations of motion.

Thus, the Lagrange function exists if and only if the metric is T-invariant.

Lorentz-invariance. By applying Lorentz transformation, we can see that the new terms are Lorentz-invariant if and only if $\delta_1 = 2\delta_2$, i.e., if the metric is T-invariant.

Thus, T-non-invariant effects are ether-dependent, i.e., depend on the velocity $\mathbf{w}$ of the system's center of mass with respect to the stationary system.

Effects on the restricted 2-body problem. Following Brumberg (1991), we find the additional term

$$\delta L = -\frac{1}{2} \cdot \delta_1' \cdot \frac{m \cdot (\mathbf{r} \cdot \mathbf{w})}{r^2}$$

in the Lagrange function of the restricted 2-body problem, then compute the average $[\delta L]$ over fast changing angular variables:

$$[\delta L] = \frac{1}{2} \cdot \delta_1' \cdot \frac{m}{a} \cdot \frac{1 - \sqrt{1 - e^2}}{e} \cdot E,$$

where

$$E = w_x[\cos\Omega \sin\omega - \sin\Omega \cos i \sin\omega] + w_y[\sin\Omega \cos\omega - \cos\Omega \cos i \sin\omega] + w_z \sin i \sin\omega$$

Thus, we get the following formulas for the osculating elements:

$$\frac{da}{dt} = 0; \quad \frac{de}{dt} = \delta_1' \cdot \frac{\sqrt{1-e^2} \cdot (1 - \sqrt{1-e^2}) \cdot m}{2 \cdot n \cdot a^3 \cdot e^2} \cdot E_e,$$

where

$$E_e = w_x[\cos\Omega \sin\omega + \sin\Omega \cos i \cos\omega] - w_y[\sin\Omega \sin\omega + \cos\Omega \cos i \cos\omega] - w_z \sin i \cos\omega$$

$$\frac{di}{dt} = -\delta_1' \cdot \frac{\cot i \cdot \left(1 - \sqrt{1-e^2}\right) \cdot m}{2 \cdot n \cdot a^3 \cdot \sqrt{1-e^2} \cdot e} E_e - \delta_1' \cdot \frac{(1 - \sqrt{1-e^2}) \cdot m}{2 \cdot n \cdot a^3 \cdot \sqrt{1-e^2} \cdot e \cdot \sin i} \cdot E_i,$$

where

$$E_i = w_x[\sin\Omega \sin\omega + \cos\Omega \cos i \cos\omega] - w_y[\sin\Omega \sin\omega + \cos\Omega \cos i \cos\omega] - w_z \sin i \cos\omega$$

etc.

In particular, for the perihelion shift, we get

$$\frac{d\pi}{dt} = k \cdot \delta_1' \cdot w \cdot \frac{m}{n \cdot a^3},$$

with $k \approx 1$, so the shift per orbit cycle $\Delta\pi \sim k \cdot \delta_1' \cdot w$ does not depend on the distance a to the Sun. For the accuracy of $0.01''$ per 100 years, and with $w \approx 700$ km/s, we get $|\delta_1'| \leqslant 3 \cdot 10^{-5}$.

Comment. In the Lunar motion, the effect of new terms is also negligible.

T-non-invariance without scale-invariance. If we do not assume that the metric is dimensionless (scale-invariant), then for the gravitational acceleration $\mathbf{a}$ of a body we get a general formula $\mathbf{a} = \mathbf{f}(m_a, \mathbf{r}, \mathbf{r}_a, \mathbf{v}, \mathbf{v}_a)$. The only requirements are that the formula is rotation-invariant and that $\mathbf{f} = 0$ when all $m_a = 0$.

From the physical viewpoint, it is natural to add a requirement of energy conservation, i.e., that it is impossible to have a closed cycle and gain some work while returning all the bodies to their original locations with original velocities.

Under this additional assumption, our first conclusion is that the radial motion in a central field is T-invariant. Indeed, if it was not T-invariant, then we could reverse $\mathbf{v}$ and get a different acceleration. Then, by letting the body go closer to the center and back (or vice versa), we would be able to gain energy.

Our second conclusion is that under P-invariance (under $\mathbf{x} \to -\mathbf{x}$), circular motion in a central field is T-invariant. Indeed, if it was not, then we could reverse the velocities and get a different acceleration. Then, by letting a body go first in one circular direction and then back, we would gain energy.

Since for planets, orbits are almost circular, we can thus conclude that under energy conversation, P-invariance is equivalent to T-invariance (modulo eccentricity e). If we additionally assume that $\mathbf{f}$ is analytical with respect to m_a, $\mathbf{v}$, and $\mathbf{v}_a$, and Lorentz-covariant, then we see that the smallest non-T-invariant terms are of order c^{-5} – as in radiation effect.

3. Possible Violations of P-Invariance

Similarly to the case of T-non-invariance, one can show that in the PPN-order approximation, the only P-non-invariant term is

$$\delta g_{0i} = \varepsilon \cdot \sum \frac{m_a}{r_a^2} \cdot (\mathbf{v}_a \times \mathbf{r}_a)_i.$$

This expression has been described before: it is the Chern-Simons term in Yunes & Pretorius (2009) coming from supersymmetry; we show that this is the only possible P-non-invariant terms of PPN order.

The above formula is in agreement with the above conclusion that all P-asymmetric terms are T-invariant, and that, therefore, PT-invariance implies P- and T-invariance.

Here, no new coordinate transformations are possible. The Lagrange function for an N-body problem exists if and only if the metric is P-invariant, and the motion is Lorentz-invariant if and only if it is P-invariant.

The secular effects in the 2-body problem are (here, $\mathbf{w}$ is the same as above):

$$\frac{da}{dt} = \frac{de}{dt} = \frac{d\mathcal{M}}{dt} = 0; \quad \frac{di}{dt} = \varepsilon \cdot \frac{m}{a^2\sqrt{1-e^2}} \cdot (w_x \cdot \cos(\Omega) + w_y \cdot \sin\Omega);$$

$$\frac{d\Omega}{dt} = -\varepsilon \cdot \frac{m}{a^2\sqrt{1-e^2}} \cdot (\cot(i)(w_x \cdot \sin(\Omega) - w_y \cdot \cos(\Omega)) - w_z);$$

$$\frac{d\omega}{dt} = \varepsilon \cdot \frac{m}{a^2\sqrt{1-e^2}} \cdot (\cot(i) \cdot \cos(i) \cdot (w_x \sin\Omega - w_y \cos\Omega) - w_z \cdot \cos(i)).$$

The effects are of the usual form m/a^2. Thus, $\varepsilon \leqslant$ accuracy of measuring perihelion shift, i.e., $|\varepsilon| \leqslant 0.01$.

4. Possible Violations of Equivalence Principle and Their Relation to Non-Conservation of Energy

In general, we can distinguish between the inertial mass m^I, and active m^A and passive m^P gravitational masses. Under this distinction, the force $\mathbf{F}_1$ with which the 2nd body attracts the 1st one is equal to

$$\mathbf{F}_1 = m_1^I \cdot \mathbf{a}_1 = -G \cdot \frac{m_1^P \cdot m_2^A}{r_{12}^3} \cdot \mathbf{r}_{12}.$$

What if we assume that energy is preserved? First, we can connect two bodies with an elastic rod. In general, the resulting 2-body system moves with the force

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 \sim (m_1^P \cdot m_2^A - m_2^P \cdot m_1^A).$$

If $\mathbf{F} \neq 0$, we can get the immobile combination moving and thus, get energy out of nothing. Thus, if energy is preserved, we have $\mathbf{F} = 0$ and hence, $m^A/m^P = \text{const}$ (i.e.,

$m_a \propto m_P$), and

$$\mathbf{a}_1 = -G \cdot \frac{m_1^A}{m_1^I} \cdot \frac{m_2^A}{r_{12}^3} \cdot \mathbf{r}_{12}.$$

We know that every particle a annihilates with its antiparticle $\widetilde{a}$ into a pair of photons: $a + \widetilde{a} \leftrightarrow 2\gamma$. It is reasonable to assume that gravity is C-invariant, i.e., that $m_a = m_{\widetilde{a}}$. Thus, if $m^I \not\propto m^A$, we can make the following experiment with an originally immobile combination of a and $\widetilde{a}$:

- first, let this combination move towards the gravity source;
- after a while, annihilate a and $\widetilde{a}$, turn the photons into $b + \widetilde{b}$, and move the new combination $b + \widetilde{b}$ back to the original location;
- once in the original location, annihilate b and $\widetilde{b}$, and turn the photons into $a + \widetilde{a}$.

If $m_a^A/m_a^I \neq m_b^A/m_b^I$, the accelerations are different, so the system gains velocity (hence energy). In other words, if $m^I \not\propto m^A$, energy is not preserved.

Thus, in the presence of C-invariance, energy conservation implies the equivalence principle.

5. Other Possible Effects

Other possible effects include cosmology, Finsler (non-Riemannian) space-time, etc.; e.g., for torsion $S^\alpha_{\beta\gamma}$, instead of the formula $T^{\alpha\beta}_{;\beta} = 0$ (with which the derivation of relativistic celestial mechanic effects starts (Brumberg, 1991)), we have $T^{\alpha\beta}_{;\beta} + S_\beta T^{\alpha\beta} = 0$, where $S_\beta \stackrel{\text{def}}{=} S^\alpha_{\alpha\beta}$. The general PPN-type dependence is

$$S_0 = \beta_T \cdot \sum \frac{m_a \cdot (\mathbf{e}_a \cdot \mathbf{v}_a)}{r_a^2} \text{ and } S_i = \beta_T \cdot \sum \frac{m_a \cdot e_{ai}}{r_a^2};$$

additional T- and P-non-invariant terms are also possible. Interestingly, we now have a class of theories including Newton's gravity and intermediate theories. The fact that one of the terms is Newtonian simplifies the computations of the celestial mechanical effects of torsion.

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